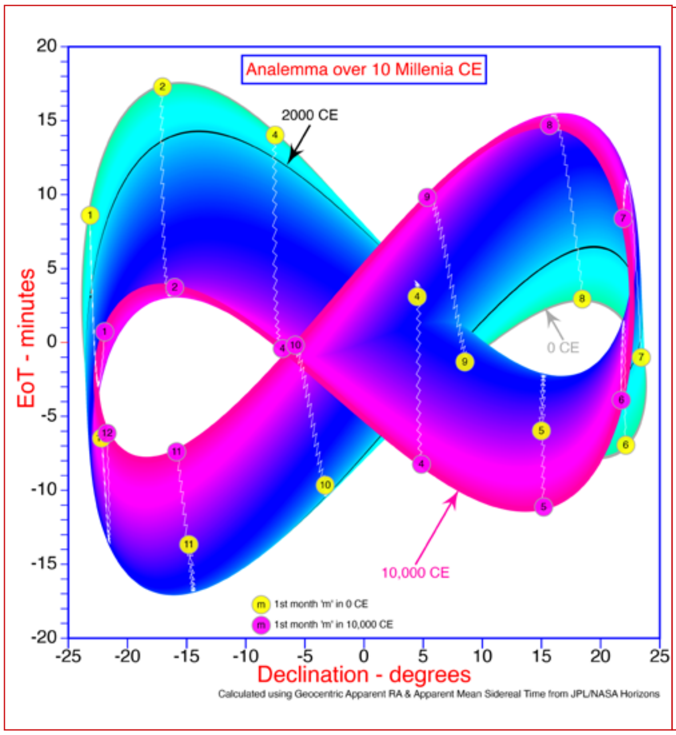
This File is essentially the same as that published in book form two years ago on my 80th birthday.

The book was published not-for-profit and now, on my 82nd birthday, I am making it available to whoever wants it !

A number of corecctions have been made,
some images have been enlarged for clarity
and Silvio Magnani's Ora del Sole, Ora dell'Orologio Dial has been added on Page 155

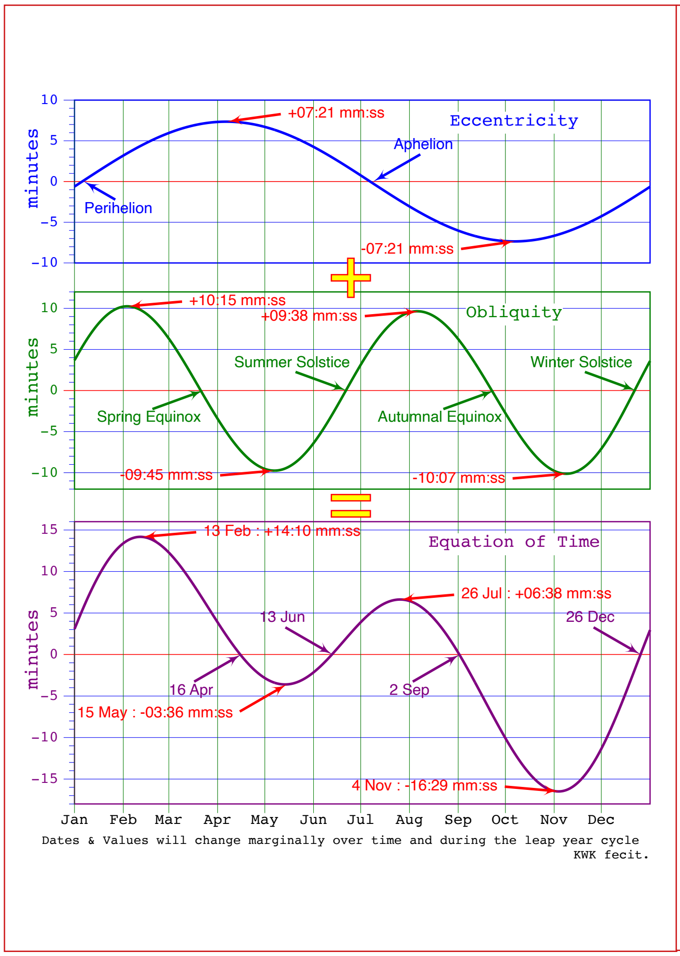
All the items in the Table of Contents may clicked to navigate to that item and one may return to the Table of Contents from the bottom of every page.

Go now to "Table of Contents"

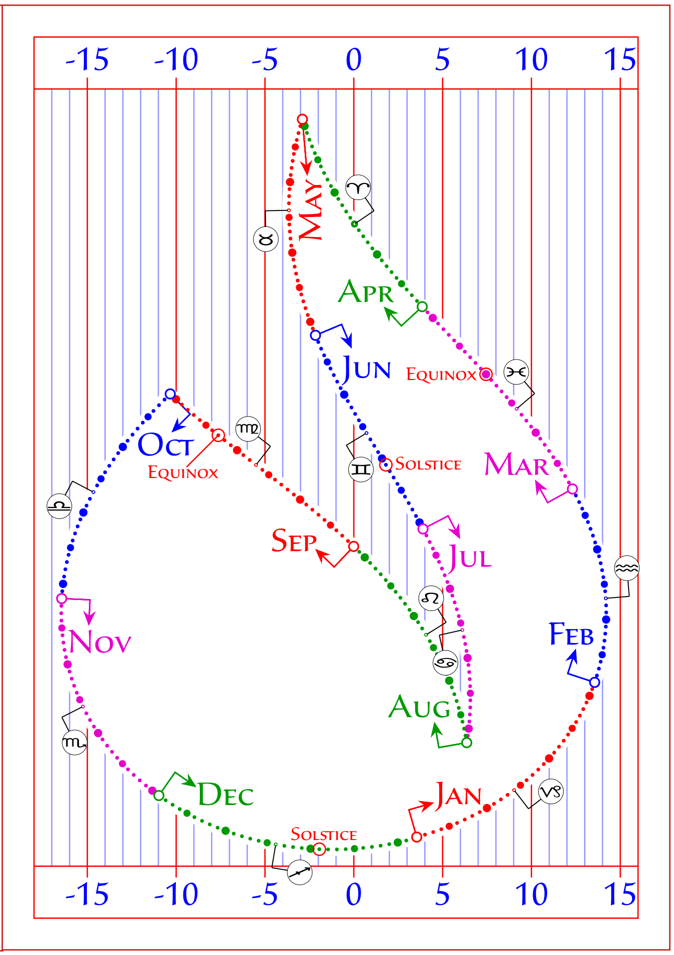


The Analemma over ten thousand years

A French Equation Calculator in the British Museum



The Components of the Equation of Time



The Author’s Intrinsic Representatio of the Equation of Time

The Equation of Time - Æquātiō Diērum

A Journal, a Reference & a Picture Book

by Kevin Karney

with chapters by John Davis

published 2024
minor corrections 2026

Published by the Author via Lulu Publishing Inc.

First Published 2024,
Corrections made 2026,

Set in 9-12 pt Minion Pro and Courier New and laid-out in Adobe InDesign

Library of Congress Catalogue Number 2024907288
A CIP catalogue record for this book is available from the British Library
ISBN 978-1-4457-4541-1

Preface

This is a journal covering about 50 years of my interest in the Equation of Time, during which time I have been gathering pictures, charts and ideas on that subject.

As a distillation of those years, the book should ***not*** be seen as a text book - but rather a book of references, a picture book and a catalogue of some of my ideas.

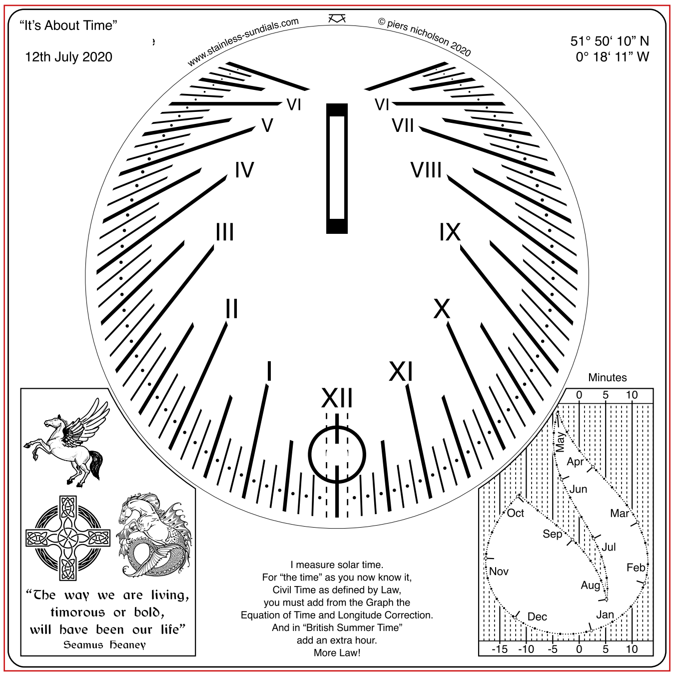
It covers as many parts of the subject that I have stumbled upon. Some are covered in detail, some less so - mainly reflecting my own degree of interest in that particular topic. In some chapters, particular interests of mine, though marginally involved with the main topic, have been added as footnotes at the end of the chapter.

The book started when my son demanded to know what was the outcome from all my work on an obscure subject of little practical use. It started as a web-site - but the internet is a transitory affair. Hence a return to paper, or to a readily available .pdf file for whose who have moved into the modern world....

Contrary to normal practice, wherever possible, I have included references in line, rather than at the end of the document. Unlike paper, internet references tend to be transitory - so many may not work in the future.

The internet has proved to be such a wonderful source of images and I have been downloading hundreds of them over the last 20 years - without bothering much about their source. So I apologise if I have used something that is copyright. and will make suitable acknowlegement in any future editions.

A picture is worth a 1,000 words, but a video is worth 10,000. There are videos, expanding a number of diagrams in the book. A list of these is given in the last chapter, together with access instructions.



A Spot-On Dial with a graph of the Equation of Time and Longitudinal Error, delineated by the author

There are a number of abbreviations in the text...

- BSS is the British Sundial Society
- NASS is the North American Sundial Society
- AHS is the Antiquarian Horological Society.
- EoT or Equation is used for the Equation of Time. This is formally defined by the International Astronomical Union. However, in this document, I use EoT as defined in France and usually by gnomonists. This is the negative of the formal definition, (thus $\mathrm{EoT}_{\text{this document}} = -\,\mathrm{EoT}_{\text{IAU}}$)
- UTC (Universal Time Coordinated) and GMT (Greenwich Mean Time) are used interchangeably, which is not strictly the case. Similarly, the 'Spring Equinox' and the '1st Point of Aries' are similarly treated as the same.
- 'Horizons' refers to the JPL/NASA Horizons web application. The routines therein are used in the preparation of the Astronomical Almanac. I have used this as my gold standard, (but see 'Quoted Accuracies' on page i.)

https://ssd.jpl.nasa.gov/horizons/app.html

The journals and conferences of BSS, NASS and AHS have been constant inspiration. Many of their members have provided advice and friendship. In particular, I would like to acknowledge ....

- Dr Werner Riegler & Dr John Davis for extra material.
- Prof Frederick W. Sawyer III for his magisterial knowledge of dialing and the dissemination thereof.
- Sir Mark Lennox-Boyd for early encouragement and and inspiration.
- Denis Savoie, Anthony Turner for technical inspiration and information.
- Mike Shaw, Martin Jenkins, Prof Woody Sullivan, Chris Lusby Taylor, Douglas Bateman, Geoff Parsons, the late Andrew James and many other BSS and NASS members for friendship.
- Piers Nicholson, a good friend, who paid me good money to delineate and illustrate some 40 Spot-On stainless steel sundials - now installed in the UK, USA, Australia and Sweden & Germany. See the illustration.
- Pete Swanstrom, Dr Frank King, Brain Albinson, Renate & Antonio Palamà and many others for extra images, ideas, corrections & permission to publish.
- lastly, my darling wife Elizabeth, who has put up with all my 'tuts' and expletives when things went awry with the preparation of this book.

This book has been published at exactly Solar Noon on the author's 80th Birthday...

return to "Table of Contents"

Table of Contents

PART 1 - EQUATION OF TIME - GENERAL

- Introduction 1
  - Preamble 1
  - The Name 1
  - Formal Definitions and Caveat 1
  - The Longitudinal Effect 1
- The Equation of Time in the History of Time Keeping 3
  - Early days - Solar Time 3
  - The Golden Age of the Equation of Time 4
  - Following the Golden Age 5
  - In Summary 6
- Understanding the Equation of Time 9
  - Preamble 9
  - Finding the Mean Sun's Longitude 10
  - Visualizing the Eccentricity Effect 12
  - Finding the Sun's True Longitude 14
  - The Eccentricity Effect 17
  - Visualizing the Obliquity Effect 18
  - Calculating the Sun's Right Ascension 20
  - Recapitulation 20
  - Combining the Effects 21
  - Further Analysis 22
- Calculating the Equation of Time 25
  - Why Bother to Calculate at All 25
  - A Minimalistic Approach 25
  - Kepler's Method 25
  - The Astronomical Almanac Method 28
  - The Fourier Method 30
  - Meeus' Algorithms 32
  - Other Sources 32
  - Horizons 32
- The Analemma 35
  - History 35
  - Analemma Basics 35
  - Variation in the Analemma over Time 37
  - Noon Marks 38
  - The Analemma on Sundials 39
  - The Analemma on Objects 39
  - The Analemma in the Sky 40
  - The Mars Analemma 41
  - Analemmas on other Planets 42
  - Delineating an Analemma 43
  - Analemmas as Sundials. 44
  - Symmetrical Analemmas 46
- Variation in the Equation of Time 47
  - Preamble 47
  - Major Components of Change 47
  - Minor Changes - The Calender & Leap Years 49
  - Minor Changes : Topographical -v- Geocentric 50
  - Minor Changes : The Time Zone Error 50
  - The Problem with Equation Tables 51
  - A Really Long Term Look 52
- The Equation of Time as Shown on Sundials 53
  - Introduction 53
  - Historical EoT Tables 53
  - Identifying Sources 57
  - Closing Remarks 61
- More on the Equation of Time on Sundials 63
  - Tompion and Flamsteed 63
  - Thomas Tuttell 64
  - Portable Dials 64
  - Vertical Dials 66
  - Horizontal Dials 67
  - Slate Horizontal Dials 67
  - Graphical Presentations 69
  - Clockmakers 69
  - 20th Century Tables 70
  - Printer's Errors 70
  - Conclusions 71
- Early Days & Ptolemy's Invention of the Equation of Time 73
  - Note on Terminology 73
  - Brief Astronomical History 73
  - Hipparchos and the Sun's Motion 74
  - Ptolemy and Unequal Days 74
  - How Good was the Theory? 75
  - The Almagest 76
  - Ptolemy's Legacy 76
- The Equation of Time in Astronomy & Navigation 79
  - Early Astronomical Tables 79
  - Marine Navigation 80
  - The Genius of Christiaan Huygens 80
  - Instructions Concerning the Use of Pendulum-Watches for Finding the Longitude at Sea 81
  - Huygens' Legacy 83
  - Analysis of Huygen's EoT Table 84
  - Later Astronomical Tables: The French View 85
  - Later Astronomical Tables: The Anglo-Saxon View 85
  - Solar Compass Background 86
  - Burts Solar Compass - 1835 86
  - The Astro Compass 87
  - The Abrams & Cole Compasses - WW II 87
  - The Bagnold Sun Compass 88
  - The Howard Sun Compass -1980 89
- Displaying the Equation of Time 91
  - ... As a Full Table 91
  - ... As a Partial Table 93
  - ... As a 1 minute 'Victorian' Table 94
  - ... As a ¯½ minute 'Victorian' Table 95
  - ... As a Cartesian Graph 95
  - ... As a Polar Graph 97
  - ... As an Intrinsic Graph 97
  - ... As a Figurative Object 99
- EoT Incidentals 101
  - On Which Date is the Latest Sunset? 101
  - Islington's Equation 102
  - A Gnomonist's Riddle 102
  - Equation of Time Nomograms 103
- Mechanically Generating the Equation of Time 109
  - Mechanical Possibilities 109
  - Generating a Sine Curve Mechanically 109
  - The Differential Adder 110
  - The Pulley Adder 110
  - The Whiffletree Adder 112
  - Descovitch Method 113
  - Lusby Taylor Method 113
  - Combining Cams & Adders 115
  - Riegler's Methods 116

return to "Table of Contents"

PART 2 - PICTURES OF AND REFERENCES FOR EQUATION CORRECTED SUNDIALS

- Why Bother to Correct a Sundial ... 117
  - Do Not Bother ... 117
  - Do Bother. ... 117
  - Conclusion ... 117
- Equiangular Sun Dials ... 119
  - Introduction ... 119
  - Pilkington's Method ... 119
  - The Huggett Method ... 121
  - The Housewife's Trick Revised - The Scout's Trick ... 121
- Moving Hour Lines Sundials ... 123
  - Introduction ... 123
  - Equatorial Dials ... 123
  - Non-Equatorial Dials With Moving Hour Lines ... 127
- Moving Style Sundials ... 129
  - Analemmatic Mean Time Dials ... 129
  - Moving Style Equiangular Dials ... 130
- Shaped Style Sundials ... 133
  - The Patents ... 133
  - Shaped Style Dials ... 134
  - Polar Style Dials ... 135
  - The Extraordinary Dial of Werner Riegler ... 136
- Curved Hour Lines Sundials ... 137
  - Full Analemmas ... 137
  - Noon Marks ... 138
  - Half Analemmas ... 139
  - Spider Dials ... 141
- Linearly Equispaced Hour Line Dials ... 143
- Dials with Alidades ... 145
- Bifilar Dials and Other Esoterics ... 149
  - The Bifilar ... 149
  - Other Esoteric Designs ... 150
- Mechanically Corrected Sundials ... 157
  - Profiles ... 157
  - Single Cams ... 157
  - Tusi Mechanism ... 158
  - Dual Cams ... 159

PART 3 - EQUATION CORRECTED CLOCKS

- Equation Clocks - Background ... 161
- Equation Clocks - Forerunners ... 161
  - Printed Tables ... 161
  - An Unknown Mechanism ... 162
  - Nicholas Mercator's Clock ... 162
  - Clocks with a Manually Adjusted Hour Ring ... 162
  - John Smith's Tables ... 163
  - Variable Pendulum Length ... 164
- Equation Clocks - Cams ... 167
  - The Invention of the Kidney Cam ... 167
  - Cam Clocks with Rotating Minute Ring ... 167
  - Cam Clocks with a Separate Face Indicating the EoT as Clock Faster/Slower ... 169
  - Cam Clocks Adding the Equation to Mean Time to Give Mean and Apparent Time ... 170
  - The Last Significant Improvement ... 171
  - The Frictionless Last Improvement ... 173
  - Equation Watches ... 174
- Equation Clocks - Gears ... 175
  - Rittenhouse's Clock at Drexel University ... 175
  - The Schwilgué Strasbourg Clock ... 177
  - The Jens Olsen World Astronomical Clock ... 179
  - Scheurenbrand's Festo Harmonices Mundi ... 181
  - Coming Full Circle : The Clock of the Long Now : Gears & Cam ... 182
- Colophon ... 183
  - Background to the book ... 183
  - Contact ... 183

APPENDICES

- Basic Positional Solar Astronomy : Part 1. Essential Parameters and the Equation of Time ... 187
  - History ... 187
  - Background and Approach Taken ... 187
  - Astronomical Nomenclature & Definitions ... 188
  - The Celestial Sphere ... 188
  - Zenith & Meridian ... 189
  - Right Ascension & Declination ... 189
  - What are we trying to Calculate... ... 189
  - Coordinated Universal & Standard Time ... 190
  - Finding Greenwich Mean Sidereal Time ... 190
  - Finding the Sun's Mean Longitude ... 192
  - Introducing the Sun's Right Ascension and the Equation of Time ... 192
  - The Longitude Correction ... 193
  - Summing up ... 193
- Basic Positional Solar Astronomy - Part 2. Calculating the Sun's Right Ascension, Declination & EoT ... 196
  - Calculations Required ... 196
  - Calculating the True Sun's Longitude ... 196
  - Calculating the Right Ascension & Declination ... 198
  - Calculating the Local Hour Angle ... 199
  - Calculating the Sun's Altitude and Azimuth ... 199
  - Finding the Times of Sunrise and Sunset ... 200
  - Worked Example ... 201
  - Appendix 1 - Derivation of Kepler's Law ... 203
  - Appendix 2 - Derivation of Newton Raphson approximation for Kepler's Formula ... 203
- Basic Positional Solar Astronomy - Part 3: Fourier Derived Formulae ... 207
  - Preamble ... 207
  - The Phase Angle & Cycle ... 208
  - Right Ascension ... 209
  - Appendix ... 210
- Code to do the Calculations ... 213
  - Kepler's Method ... 214
  - Fourier EoT and Declination ... 216
  - Equation Tables ... 218
  - Analemma Plotting ... 221
  - General Service Routines ... 226
- An Unrealised Heliochronometer ... 229
  - Preamble ... 229
  - Pictorial Description ... 229
  - Gear Chains ... 231
- Reading List & Available Videos ... 233
  - Available Videos ... 234

return to "Table of Contents"

return to "Table of Contents"

PART 1 - EQUATION OF TIME - GENERAL

Introduction

PREAMBLE

Even when a sundial has been correctly engraved and correctly installed, it seldom seems to tell the same time as one's watch.

- The sundial tells Local Solar Time - otherwise historically known as True Time or God's Time. The Sextant reads Local Solar Noon.
- The watch tells Civil Time, which is related to the world's legal standard of Universal Time Coordinated (UTC).

The two time scales are different.

The difference between watch and sundial comes from 3 easily-calculated locally-constant human-made factors and 2 more complex and variable astronomical factors.

1 The difference between one's Longitude and that of the Time Zone meridian.

2 The presence or absence of Summer Time or Daylight Saving Hours.

3 The presence or absence of Leap Seconds: see 'Note 1' on page 6..

4 The Ellipticity of the Earth's orbit around the Sun.

5 The Obliquity of the Earth's spinning in relation to the Equatorial plane.

The combined effect of the last two factors is called the Equation of Time. It can be as much as 16 minutes.

THE NAME

The Equation of Time or in its original form - Æquātiō Diērum - is common astronomical terminology.

The 'Equation of xxx' means the difference between the observed value of xxx and its mean value. In astronomy, there are many examples e.g. the equation of time, the equation of centre, the equation of the equinoxes, the equation of origins, the equation of light.

In mediaeval times, Æquātiō Diērum or 'Equation of the Day' was used. Clocks were not accurate enough to differentiate between minutes. Only astronomers predicting lunar movements were concerned with the fact that the length of the solar day varies throughout the year, as discovered by Ptolemy in the 2nd C CE. See 'Early Days & Ptolemy's Invention of the Equation of Time' on page 73.

FORMAL DEFINITIONS AND CAVEAT

Astronomical Definition

From 1965, the Equation of Time was formally defined as:

> 'The correction to be applied to 12hrs + Universal Time to obtain the Greenwich Hour Angle of the Sun or more generally the correction to be applied 12hrs + Local Mean Time to be obtain the Local Hour Angle of the Sun. It is so tabulated in the almanacs for navigators and surveyors.'

Explanatory Supplement to the Astronomical Ephemeris & the American Ephemeris and Nautical Almanac

Or more recently:

> 'the difference apparent solar time minus mean solar time.'

The Astronomical Almanac

Note that the latter definition relates to *local* mean solar time, which is broadly local standard time +/- the longitudinal difference mention above and see below.

Gnomonical Definition

While Astronomers use the definition above, almost all gnomonists - and formalised by Commission des Cadrans Solaires of Société Astronomique de France - use the negative. That is to say the correction to be applied to Solar Time (i.e. the Local Hour Angle of the Sun) to obtain Local Mean Time.

Beware the Difference

see the following page

Some computer applications use one definition and some the other. All of the charts herein use the gnonomical definition.

The Equation of Time (EoT) is almost always quoted in minutes, but sometimes in degrees, where $1^{o} = 4^{mins}$ or $360^{o} = 24^{hrs}$.

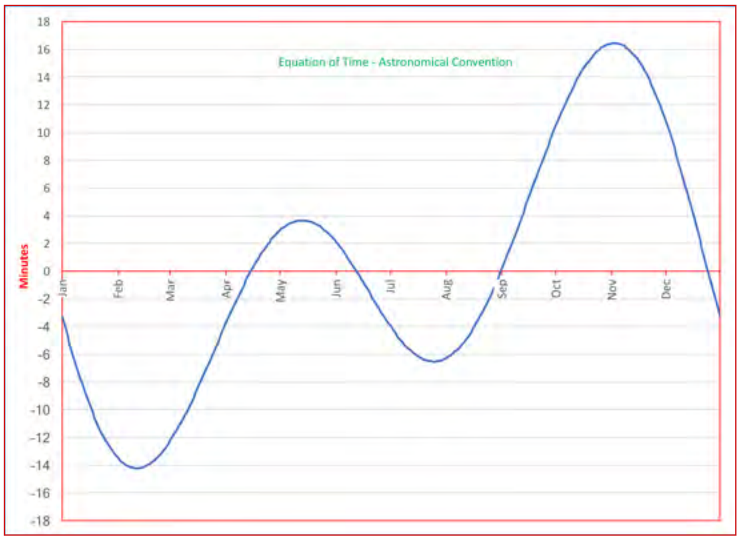
THE LONGITUDINAL EFFECT

For almost all *practical* purposes, the longitudinal difference between one's longitude and that of the time zone meridian (item 1 above) needs to be included in the Equation of Time, as formally defined.

The Earth appears to move in a easterly direction relative to the Sun. Thus, west of one's time zone meridian, the sun is overhead later than civil mean time.

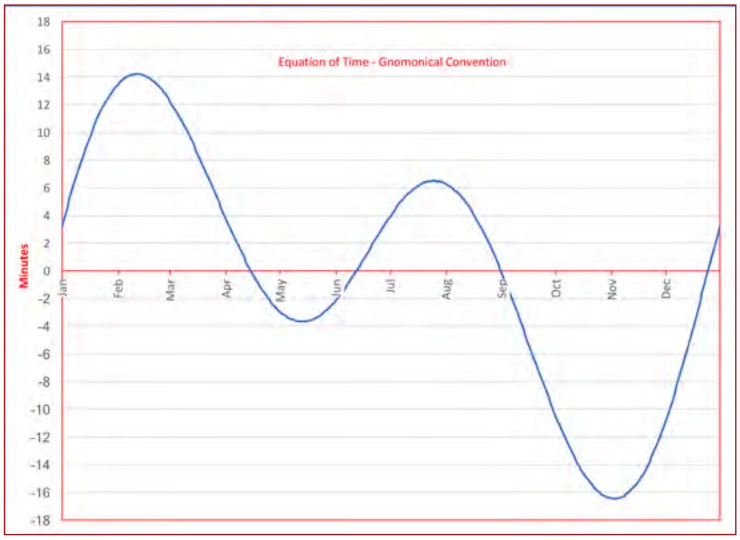
Foe example, if one lives 2^{o} West of your local time zone meridian, 8^{mins} needs to be added to the usually quoted value of the EoT.

$$\text{The Longitude Correction}^{mins} = 4^{mins/°} \times (\text{Time Zone meridian longitude}^{o} - \text{local longitude}^{o})$$



Astronomical Convention.

This definition is sensible for astronomers and solar farmers, who look at their watch and want to know where the Sun is.



Gnomonical & French Convention - as used in this document

This definition is sensible for almost anyone else. They look at the Sun and want to know what the Time is.

Marg Folkard - Mount Annan Sundial, Royal Botanic Garden, Sydney showing the Equation of Time in the gnomonical convention.

Note that the minimum value shown on the EoT graph here is –20 ^{mins}. While, on the standard graph above, the minimum is –17 ^{mins}. Thus, the sundial is longitude corrected and ca. ¾° East of the AEST

Footnote

Throughout this book, 'Solar Time' or 'God's Time' are used interchangeably.

Such time, historically, is told in 'equal', 'common', 'equatorial', 'equinoctal' or 'common' hours.

All of these are essentially the same.

return to "Table of Contents"

The Equation of Time in the History of Time Keeping

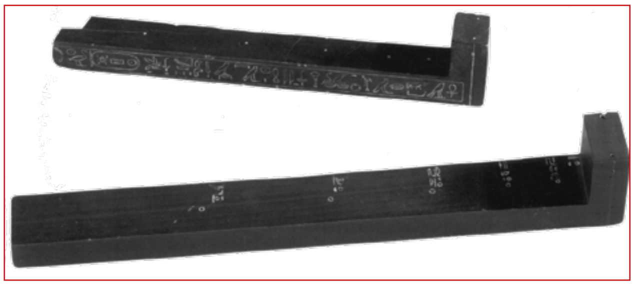
This chapter briefly describes the many stages for timekeeping throughout history and the position of the Equation of Time therein.

EARLY DAYS - SOLAR TIME

From pre-history, everyone used Circadian Time - what one feels in the tummy and where the Sun is in the sky: still used by gardeners.

In Ancient Egypt, there were sundials that marked events rather than counted hours.

Starting in Babylon and developed by the Greeks and Romans, many designs of sundials were made that told 'unequal hours' - 12 hours between sunrise and sunset : shorter hours in the winter and longer hours in summer.



Ca. 1500 BC. Egyptian Event Marker. These would be pointed East/West. The Sun's shadow was read by markings that signified civil events, such as the opening of the city gates.

Around 150 years CE, the astronomer Ptolemy correctly described how and why the length of the solar day varied throughout the year. If the length of the day varies, then so does the length of the hour.

Roman hemicyclum sundial reading unequal hours (12 hours from sunrise to sunset). Thus, the daylight hours were longer in summer than in winter Such dials would typically be in a town market place.

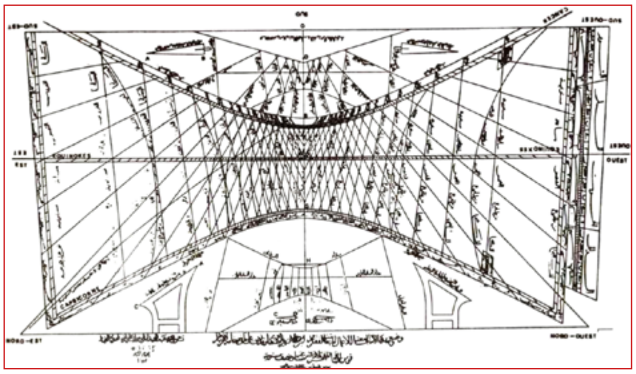
Thus, the sun provides a time scale in which the length of the hour varies (just a little) throughout the year and is thus a non-uniform scale. With no accurate means of measurement, this distinction mattered only to astronomers, who were tracking the moon's movement. However, Ptolemy's published tables of the 'Equation of Days' were used by Arab, Indian and European astronomers until the late Middle Ages.See 'Early Days & Ptolemy's Invention of the Equation of Time' on page 73..

From St Benedict's time in the 6th C, the Church - followed by the Mosque - required that prayers be said on specified times. Many 'mass' sundials, that told unequal hours, were made.

A 8th C Mass Dial from Bewcastle in the UK. This tells the time in Unequal Hours. The Gnomon - the shadow caster - would have been a stick coming horizontally out from the central hole

In the 13th Century, the great Islamic mathematicians and astronomers proved that, by using a gnomon parallel to the earth's axis, they could read the time from a sundial divided into 24 hours of the same length between successive 'high noons'.

The high noon being when the sun is at its highest point in the sky. This defines the Solar, Common, Equinoctial or God's hour.



Ibn al-Shatir's sundial in the Umayyad Mosque in Damascus : 1371

return to "Table of Contents"

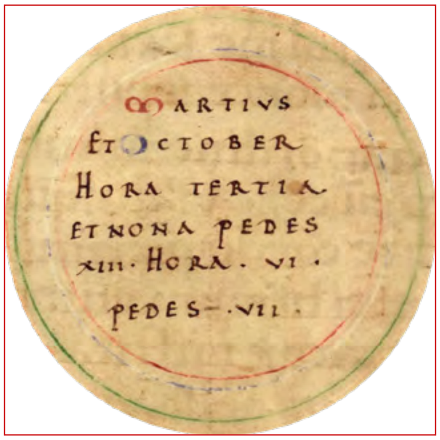
Most sundials are still made following the mathematics of the 13th C Arabic scholars and tell solar time.

A modern vertical sundial by Harriet James

Whether the hours are counted from sunset (Italian hours) or sunrise (Babylonian hours) or noon (Astronomical Time) or midnight (as it now does) varied across the world.

During all this long period of history, various devices were used to tell the time (water clocks, burning candles, burning flax, hour glasses, etc.), but most people told the time, if they needed to tell the time, by the length of their own shadows. This might be called Shadow Time.

> When your shadow is ten times the length of you foot, all you have to do is... ... perfume yourself and come to dinner ....
>
> *Aristophanes' play, 'The Assembly of Women', 391 BC*



MARTIVS
ET OCTOBER
HORA TERTIA
ET NONA PEDES
XIII · HORA · VI ·
PEDES · VII ·

"In March and October, at the 3rd and 9th hour, (your shadow is) 13 feet long, at the 6th hour, 7 feet long"

The 6th Hour is the time of the Christian prayer Nones, which is said at Noon. It was the 6th hour from dawn.

Missal of St Leofric, 9thC Bishop of Exeter.

Up to the 17C, verge and foliate clocks were developed for churches and the wealthy. But they were hopelessly inaccurate. Their accuracy was such that the difference between Solar Time and Mean Time was irrelevant to all but astronomers.

THE GOLDEN AGE OF THE EQUATION OF TIME

Local Mean Time takes over from Solar Time

The golden age of the Equation of Time covers the period when Solar (or God's or True) Time gave way to Mean Time (a man-made invention).

The scientific Enlightenment era was largely influenced by three interconnected factors: the discovery of the Americas, which led to increased commercial needs; Galileo's discovery of the pendulum's uniform beating; and Christian Huygens' assertion that longitude at sea could be accurately found using a clock. These developments led to the invention of the 'seconds' or 'royal' pendulum and anchor escapement in the 1680s, which enabled clock accuracy to within a few seconds, and the ticking of mean time became possible as a result.

1695 - Thomas Tompion's Royal Pendulum Clock- - Clarence House, London.

This was one of the first clocks to read both Solar and Mean Time

See 'Equation Clocks - Cams' on page 167.

Mean time was defined in terms of an imaginary Sun, rotating unoiformly about its axis and moving uniformly around the celestial equator

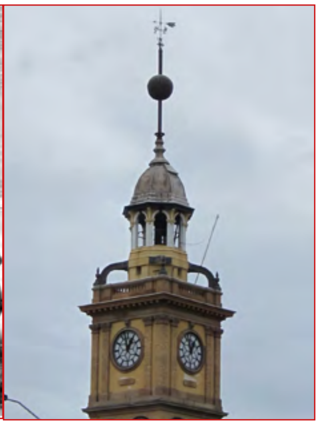
One cannot, of course, measure the position of an imaginary Sun - but one can easily measure the transit of various stars, which do move uniformly around the equator, thus giving so-called Sidereal Time. Mean Time can however be directly and mathematically computed from Sidereal Time. For details, see 'Finding Greenwich Mean Sidereal Time' on page 190..

While astronomers, navigators and the cognoscenti were interested in mean time – most people preferred solar time – something they could directly experience and read from their simple sundials. Over a period of maybe 150 years, until most towns and villages had church or town clocks, the situation gradually reversed and Local Mean Time became the norm.

But clocks had to be set. In large cities, astronomers could read the transit of stars and thus set their watches to Local Mean Time. This time could be broadcast to ship's navigators, clock makers/sellers and the cognoscenti by the noon gun (17th C) or noon ball (1829). Noon balls were set off at 1:00 p.m. to allow the astronomers time to do their sums.

return to "Table of Contents"

Edinburgh Castle Noon Gun : 1861. See Anecdote 1



Noon Ball - half fallen - at 1 minute before 1 pm.

Newcastle - New South Wales

Still - outside significant cities - clocks had to be set using sundials. The sundial's reading of Solar Time could to be converted to Local Mean Time by means of an Equation of Time table.

Although John Harrison created his marine chronometer in 1761 that the needs of navigators would only be finally met many years later - following the advances of other horologists and instrument makers. Solar noon was measured with a sextant. This would have be corrected to Local Mean Time by means of an Equation of Time table. Longitude could then be calculated using the ship's chronometer which had been set in to Local Mean Time in a port of known longitude. See 'Marine Navigation' on page 80.

Harrison's H4 1759 Chronometer, whose invention allowed accurate Longitude computation.

It took more than 50 years from the invention of the chronometer to be commercially available to all mariners. See 'Note 3' on page 6.

Bristol 1753 Corn Exchange Clock, showing Greenwich Mean Time and Bristol Mean Time (the black hand). Bristol local mean time at 2.5°W is 10 minutes behind Greenwich Mean Time

Following the Golden Age

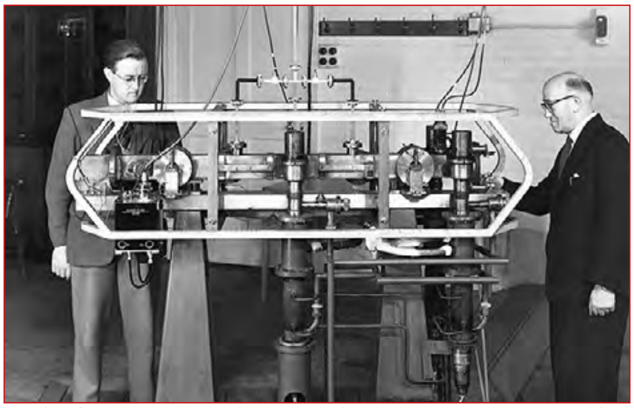
National Mean Time = Standard Time

In the 19th Century, the newly arrived railways (and to a lesser extent, the telegraph offices) needed uniform time-tables across a country. But see 'Note 4' on page 6.. The invention of the telegraph allowed time signals to be broadcast from central observatories across a country (and particularly along telegraph lines strung next to the railway lines).

Railway time tables could now become uniform and the arrival of cheap pocket watches allowed train drivers and signal men to coordinate their work across the country. The switch to national mean time was not initially accepted by all. See 'Anecdote 1' on page 6.

Thus National Mean Time and the international Time Zones were introduced. Greenwich Mean Time (GMT) became the world standard.

Finally, with the arrival of Atomic Clocks, GMT was replaced by UTC (Universal Time Coordinated) in 1961. UTC is the weighted average of many atomic clocks around the world and is managed by a UN department located in Paris.



The first atomic clock in 1955 by Louis Essen in the UK's National Physical Laboratory. Essen was named as 'the man who killed astronomical time'.

However, to account for the slowing of the Earth's rotation and other astronomical irregularities, leap seconds are occasionally added to UTC. These maintains UTC in synchronisation to within ± 0.9^{secs} with observed astronomical mean time (i.e., on average, to keep the sun's highest point at 12 o'clock). 'Note 5' on page 6.

Civil Time

In the early 20th Century, Summer Time or Daylight Saving Time (DST) was introduced in many parts of the world, further altering the time read on one's watch. Civil Time, which is the legal time in all countries of the world, is Standard Time ± DST.

Finally everything to do with time keeping is done electronically....

In Summary

Standard Time $^{\text{hrs}}$ =
UTC $^{\text{hrs}}$ +
Time Zone $^{\text{hrs}}$ *(+ve East of Greenwich)*
and
Civil Time =
Standard Time +
Summer Time (DST) hours, if in operation
and
Civil Time $^{\text{hrs}}$ =
Solar Time $^{\text{hrs}}$ +
Equation of Time $^{\text{mins}}$ */ 60 (Gnomonical Convention)* +
Time Zone $^{\text{hrs}}$ *(+ve East of Greenwich)* +
Longitude Correction $^{\text{hrs}}$ +
Daylight Saving $^{\text{hrs}}$

The illustrations overleaf provide a figurative history of timekeeping and a summary of the various time definitions.

Note 1

There is ample evidence, that the early Greek and possibly Egyptian astronomers were aware that a gnomon parallel to the earth's axis, marked out equal length hours. But this seems to be the preserve of astronomers rather than being in widespread use.

Note 2

The Maliki, Shafi`i, and Hanbali Islamic schools state that Asr prayer should start when the length of your shadow = your height + the length of your shadow at noon. This rule has to be modified depending on one's latitude.

Note 3

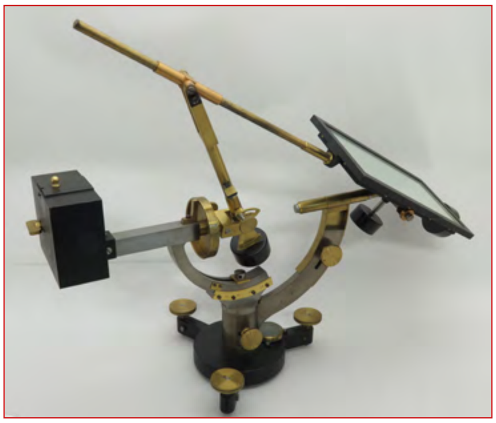
Commercial manufacture of chronometers was made possible by, on one hand, many improvements introduced in Britain by John Arnold and Thomas Earnshaw and in France by Pierre Le Roy and Ferdinand Berthoud. And on the other, by the introduction of chronometer calibration services provided by the national observatories such as Greenwich.

Note 4

The switch from local to national mean time is usually credited to the railways. However, the first country actually to make the change was New Zealand. They had few railway tracks but, when a submarine cable was laid between North and South Islands, it was the telegraph offices that promoted the move to national mean time.

Note 5

At the time of writing, the decision to abandon the leap second (by 2035) had been made, primarily to please computer and satellite programmers, who found it difficult to deal with the irregularities introduced by time's connection with the physical world. The change, however implemented, will add another factor - albeit small - to calculations of the EoT.



1862 - Heliostat, based on clockwork, built on a form devised by Leon Foucault. It required daily EoT correction.

A heliostat tracks the sun, with the mirror constantly adjusting the reflected beam onto the same spot.

2023 - A beautiful Solenica heliostat by Diva Tommei - all driven by electronics, sensors & software.
No need for the EoT!

Anecdote 1

The people of Edinburgh still joke that the noon gun fired each day from the castle is so accurate that it always hits the noon ball at the observatory on the other side of town. It was, of course, the observatory's clocks (set by the stars) that triggered both the noon ball and the firing of the gun. The illustration shows the gun being fired by an 'Electrosymathetic' system, invented in Edinburgh by Frederick James Richie. This sent a battery-driven signal every second down the wires around the 'Edinburgh Ring.'

Anecdote 2

The move from local mean time to national mean time was resisted by many. In Curtis -v- March : Dorchester Assize Court - 1858. A defendant in a civil case was deemed to have arrived late in court. Thus the judge found against him. The defendant appealed that decision by claiming that he was in court by 10 o'clock local mean time, not 10 o'clock GMT. The appeal judge found for the defendant.

"Ten o'clock is 10 o'clock according to the Time of the Place and the Town Council cannot say that it is not, but that it is 10 o'clock by Greenwich time.

Nor can the time be altered by a railway company.… Nor by any person who regulates the clock on the Town Hall."

return to "Table of Contents"

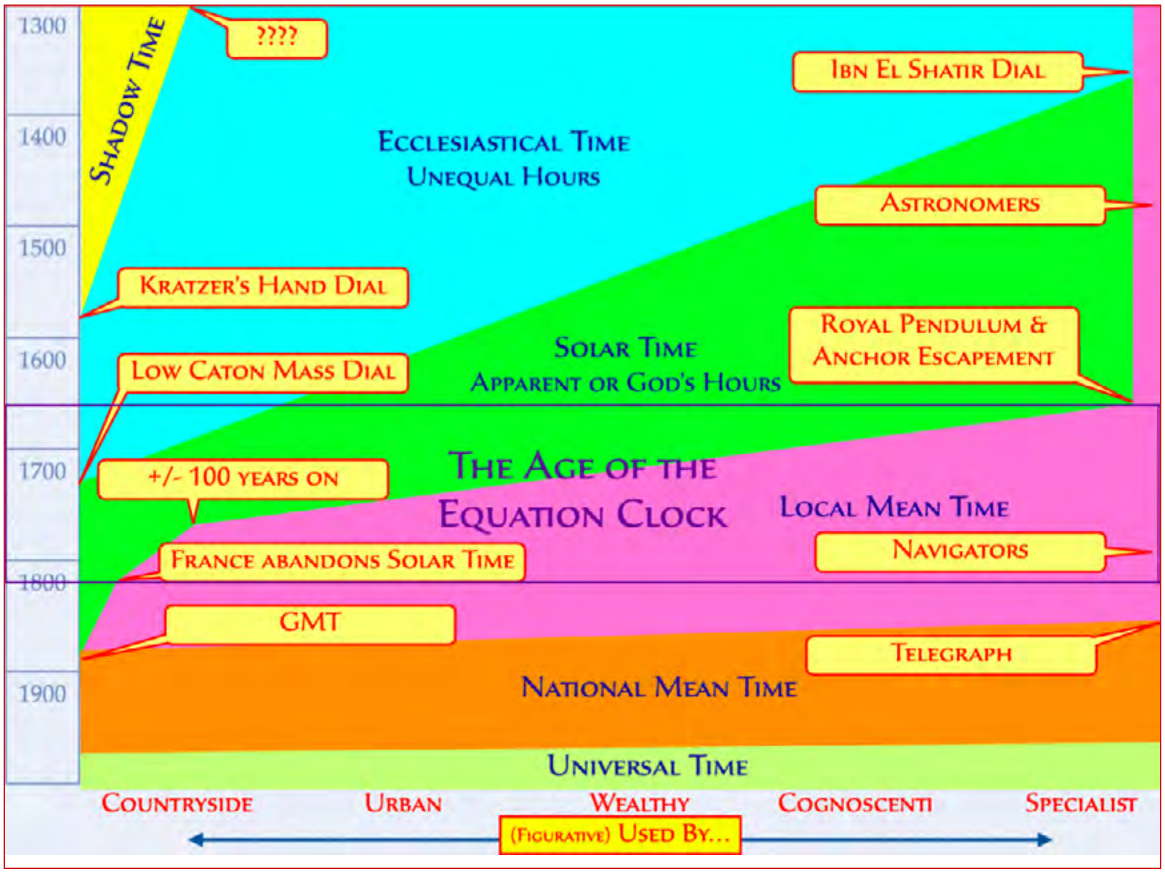
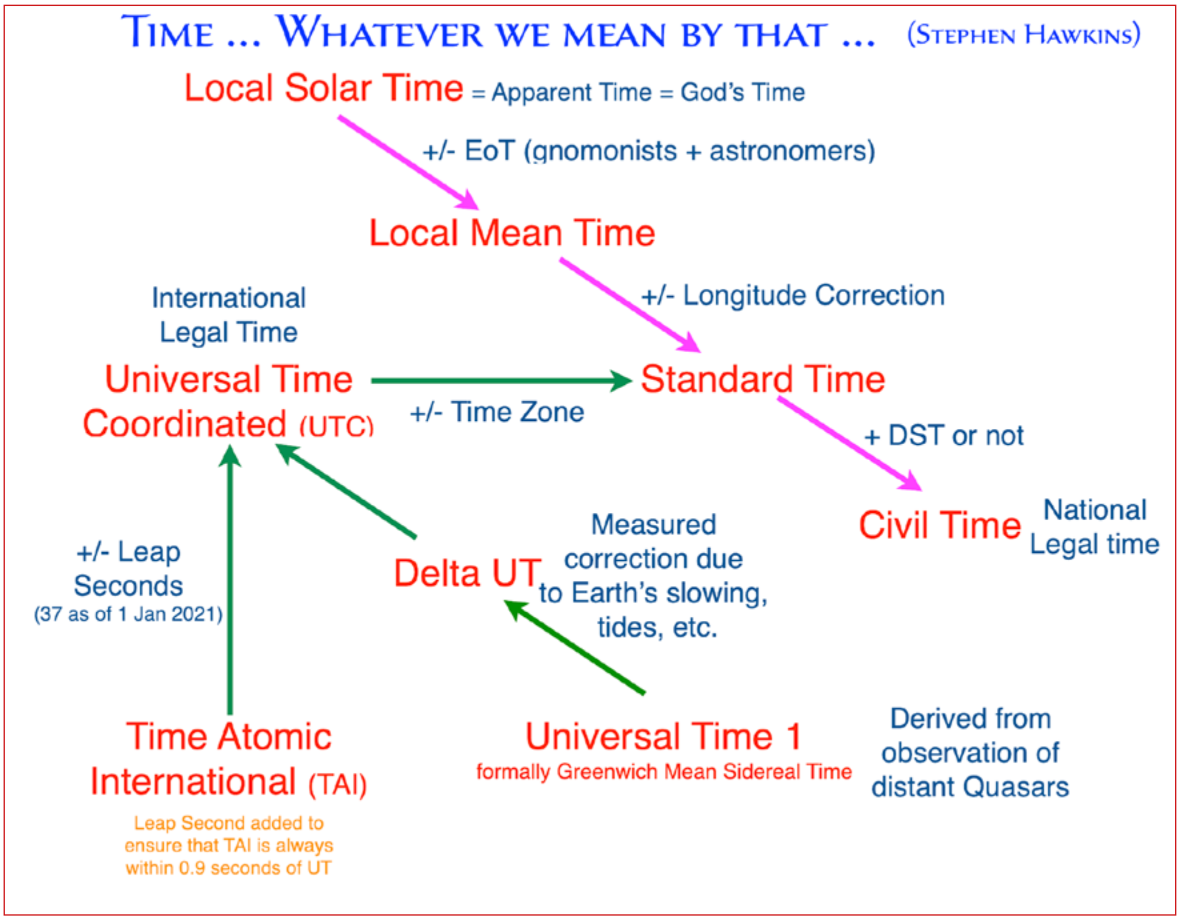


Illustration showing the Golden Age of the Equation in relation to the other prime Hours.
*The x-axis is **figuratively** the amount of use by various populations, showing how technologies percolate slowly from specialist to countryside*



Where the Equation of Time fits into the wider scheme of Timekeeping

return to "Table of Contents"

return to "Table of Contents"

Understanding the Equation of Time

PREAMBLE

Nota Bene.

i) As still used by navigators with sextants, a geocentric view is taken in this discussion. This is just a difference frame of reference than the 'real' situation with the Sun at the centre of the Solar System.

ii) Note that 1° on a celestial equator is equivalent to 4 *mins* of time. When the EoT is greatest at 16 *mins*, that is only 4° of arc. So some illustrations use exaggerated values to clarify matters.

Our time keeping is based on two fundamentals. Firstly the time told should be uniform. Secondly the time told should match the daily rotation of the Earth about its polar axis.

Modern atomic time is the realization of these ideas. But, for our purpose, it is sufficient to return to the older ideas of mean time. For this, one must fall back on a imaginary 'mean sun' travelling uniformly around the Equator. The formal reasoning behind need for a mean sun is given at the end of this chapter.

See 'Footnote - Mean Solar Time' on page 24.

The Equation of Time - the difference between mean time and solar time - arises since the true Sun neither travels around the Equator, nor moves uniformly. This was understood by the ancient Greeks. See 'Early Days & Ptolemy's Invention of the Equation of Time' on page 73.

To find the EoT requires four principle steps...

i) using the 1st Point of Aries (the Spring Equinox) as origin, finding the position of the Mean Sun on the celestial Equator.

The value found is called the 'Mean Longitude'.

ii) using Perihelion as origin - when the Sun is closest to the Earth, around 3rd January - find the position of the Sun on the Ecliptic, which is at 23.4° to the Equator.

The value found is called the 'True Anomaly'.

iii) changing the origin from Perihelion to the 1st Point of Aries. This is a simple angular change.

The value found is called the 'Solar Longitude'.

iv) casting the position of the Sun down its meridian onto the time-keeping Equator.

The value found is called the Sun's Right Ascension'.

The Equation of Time is the difference between Right Ascension and Mean Longitude

This chapter covers each of these steps, firstly by visualizing the problem, and then by how these may be calculated .

For practical & simple means of calculating the values above ... See 'Calculating the Equation of Time' on page 25.

FINDING THE MEAN SUN'S LONGITUDE

Mean Time is based on the concept of the fictitious Mean Sun moving uniformly around the Celestial Equator, completing its circuit in one year. The Mean Sun's longitude is defined as zero at the 1st Point of Aries. By definition, the Earth, rotating once a day, will 'see' that the Mean Sun will transit over Greenwich at $12{:}00^{hrs}$ GMT, i.e. the Mean Sun will be due South.

But it is a fictions body! It is, however, easily derived from astronomical measures of the stars, which do appear to move uniformly around the Earth. However, although the stars move uniformly, they move at $366.25^{days/year}$, rather than our more usual $365.25^{days/year}$, as seen in Fig 1.

The time told from stars is called Greenwich Mean Sidereal Time (GMST). Formally ...

> The position of the Mean Sun is defined by
> the Hour Angle of the Greenwich Meridian (GHA)

Astronomers can measure this when certain well-known stars cross their meridian. They do this by using 'transit' telescopes, that can only move in the plane of the local meridian.

The Hour Angle is zero at noon. Thus the mean sun is due South at Noon (or North in the Antipodes), as one might expect.

Figs 2 - 5 show how Mean Longitude can be found and it can be observed that...

$$\text{Sun's Mean Longitude}^{hrs} = \text{GHA}^{hrs} \text{ (by definition)}$$
$$= \text{GMST}^{hrs} + 12^{hrs} - \text{GMT}^{hrs}$$

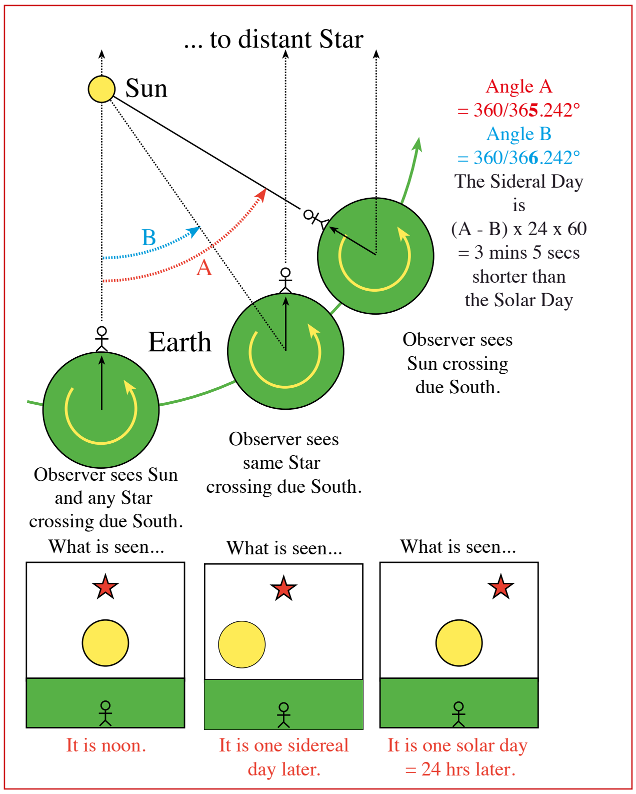


Fig. 1 - Why Sidereal Time differs from Mean Time.
(for South - read North in the Antipodes)

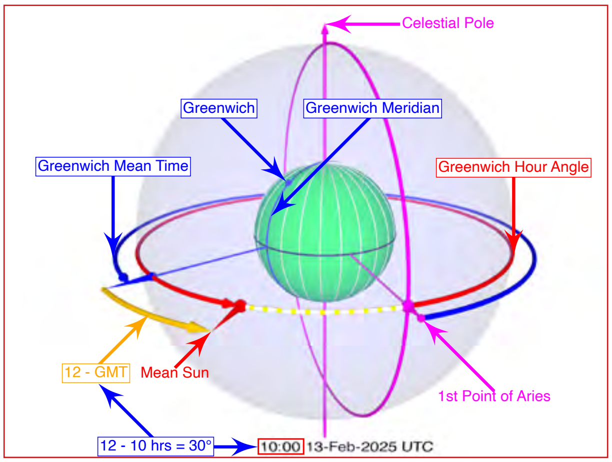


Fig. 2 - The Situation two hours before Midday

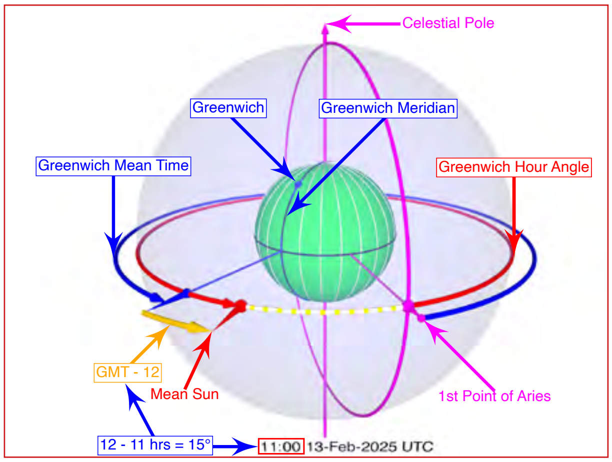


Fig. 3 - The Situation one hours before Midday

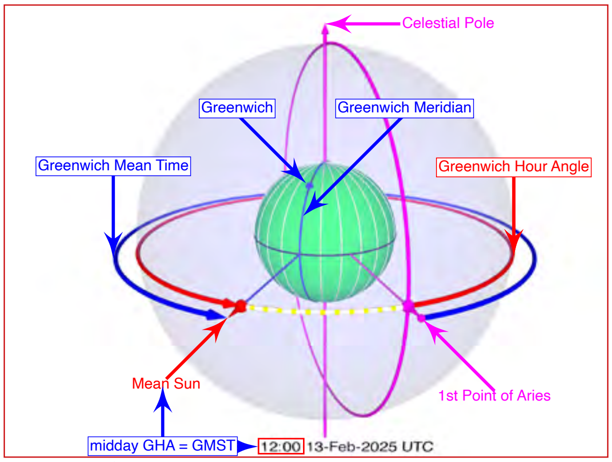


Fig. 4 - The Situation at Midday. Greenwich Mean Time = the Greenwich Hour Angle

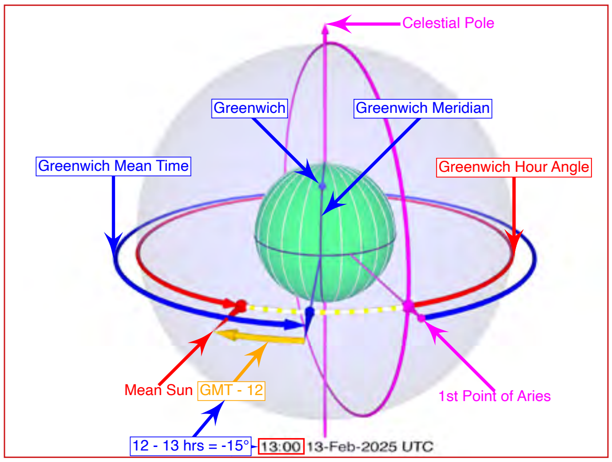


Fig. 5 - The Situation one hour after Midday

return to "Table of Contents"

GMST is explicitly defined and calculated. The definition is owned by the International Astronomical Union. The definition is shown in Fig 6. Epoch$_{2000}$ is noon UTC on 1st January 2000.

$$\begin{aligned} GMST^{\text{hrs}} = & \\ & 6.697\,374\,558^{\text{hrs}} \longleftarrow \text{Greenwich Hour Angle of Sun @ Epoch}_{2000} \\ + & \; 0.065\,709\,824\,419\,08 \times D_0 \longleftarrow \text{Days since Epoch}_{2000} \\ + & \; 1.002\,737\,909\,35 \times UTC^{\text{hrs}} \\ + & \; 0.000\,026 \times T^{\text{centuries}} \longleftarrow \text{Julian Centuries since Epoch}_{2000} \end{aligned}$$

Epoch$_{2000}$ is noon on 1st January 2000

$0.065\,709\,824\,419\,08 = 24^{\text{hrs/day}} \div 365.242\,191^{\text{days / tropical year}}$
this ensures that, in one tropical year, GMST increases by one day, corresponding to the extra sidereal day in a tropical year

$1.002\,737\,909\,35 = 366.242\,191^{\text{sidereal days/year}} / 365.242\,191^{\text{tropical days/year}}$
this converts a tropican hour to a sidereal hour

0.000 026 corrects for the precession of the polar axis

Fig. 6 - Finding the value of GMST as a function of date and time

Note that a similar but more complex definition is used for precision astronomy

In order to calculate GMST on any day, using the definition in Fig. 6, the number of days from Epoch until midnight must be calculated (**Do**) using the routine in Fig 7.

```
Year                 = yyyy
Month                = mmm
Day                  = dd
if    mmm            <= 2
      mmm            = mmm + 12
      yyyy           = yyyy − 1245
a                    = int(yyyy ÷ 4)
b                    = 2 − a + int(a ÷ 4)
Julian Days $^{days}$ = int(365.25 × (yyyy + 4716))
                     + int(30.6001 × (mmm +1))
                     + b + dd − 1524.5
$D_0$                = Julian Days $^{days}$ − 245 1545
```

Fig. 7 - Converting Date to days from Epoch $_{2000}$

VISUALIZING THE ECCENTRICITY EFFECT

Johannes Kepler published his 1st and 2nd Laws of Planetary Motion in Astronoma Nova in 1609, stating

i) The orbit of every planet is an ellipse with the Sun at one of the two foci.

ii) A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

For our purpose, we invert the second law to say that a line from the Earth to the Sun will sweep out equal areas in equal times. Due to gravity, it moves fastest at 'Perihelion', when the Earth is closest to the Sun and slowest at 'Aphelion'.

The 'Mean Dynamical Sun' is defined as a hypothetical body the moves uniformly around the Earth in a circular path.

Using this concept, the 2nd Law can be restated that the area swept out by the Mean Dynamical Sun is proportional to the area swept out be the True Sun, i.e. the area of the yellow wedge is proportional to the area of the red wedge. See Fig. 8 below.

The illustrations overleaf show the development of Eccentricity Effect...

i) on the left, using the true eccentricity - the effect is not very obvious. Bear in mind that 1^{o} of arc is equivalent to 4 ^{mins} of time. So the maximum value of the eccentricity effect of ca. 10 ^{mins} is only 2½ ^{o}

ii) on the right, with a magnified eccentricity to amplify the effect.

There is are animations available of this method.
See 'Available Videos' on page 234.

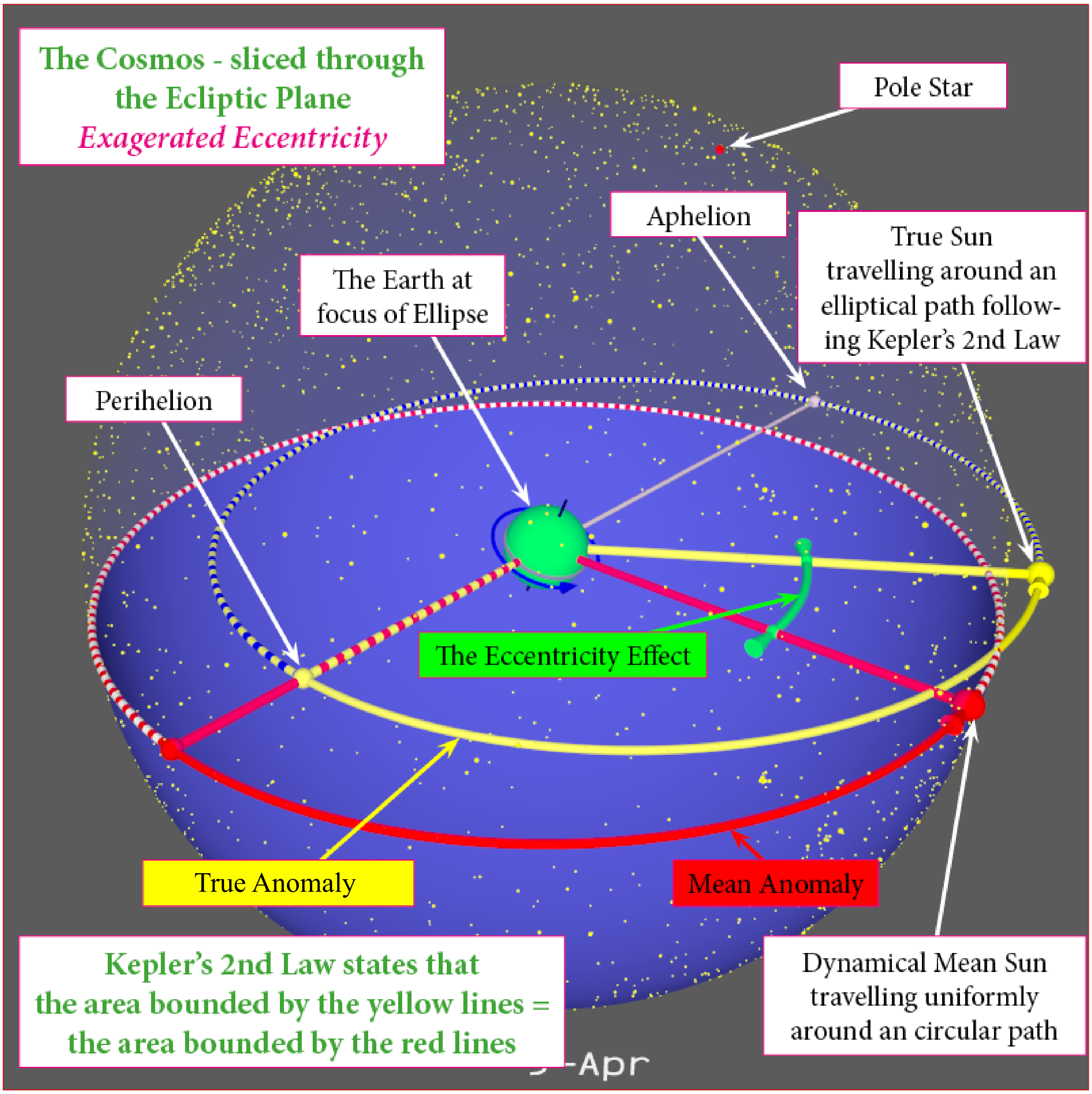
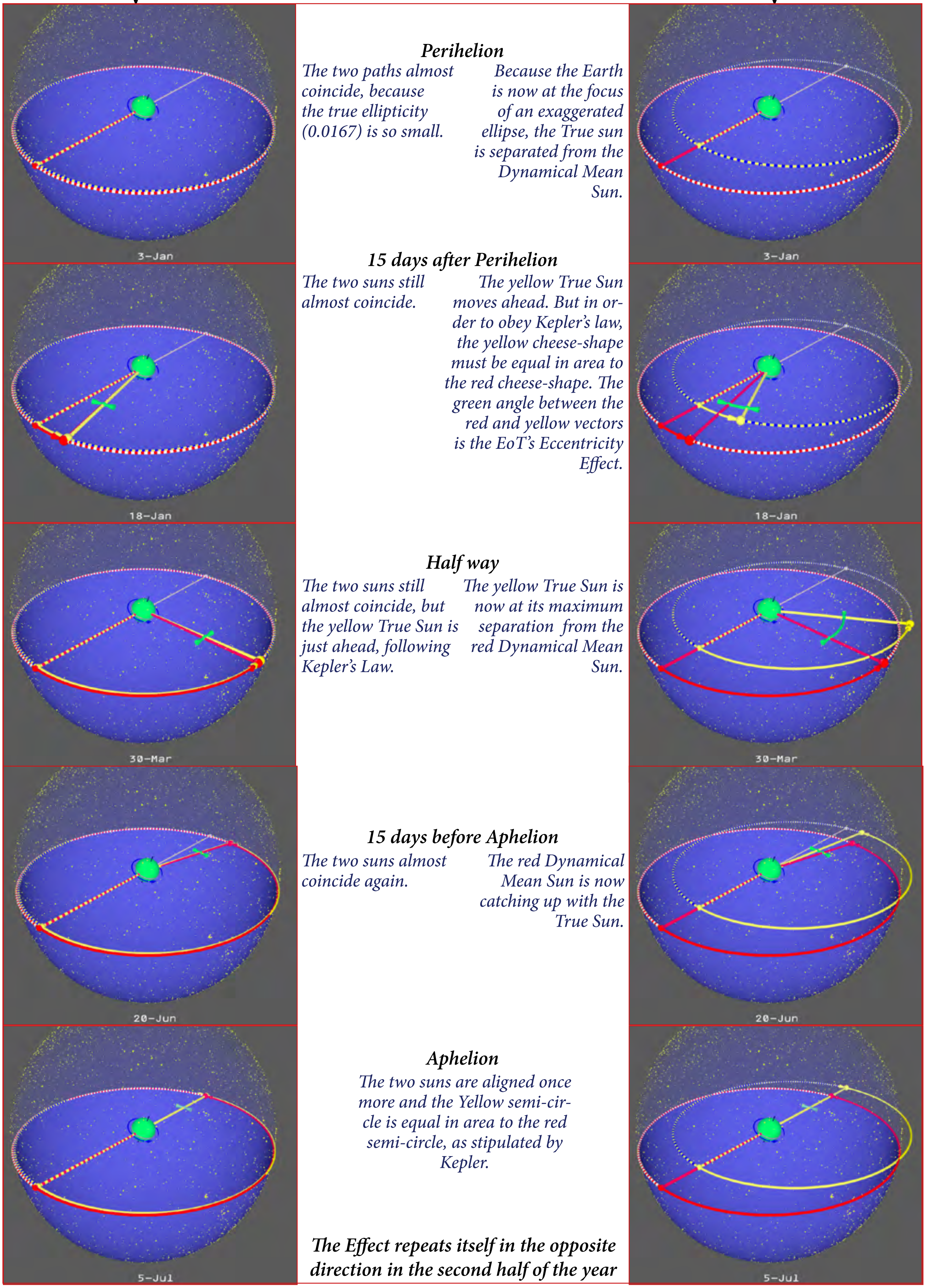


Fig. 8 - Key to the illustrations overleaf. Note that a hugely large value of the Eccentricity is used here.

return to "Table of Contents"

The Eccentricity has its true value of 0.0167.
The red & yellow vectors almost coincide.

In order to clarity the physics,
the Eccentricity is exaggerated to 0.3.



Perihelion

The two paths almost coincide, because the true ellipticity (0.0167) is so small.

Because the Earth is now at the focus of an exaggerated ellipse, the True sun is separated from the Dynamical Mean Sun.

15 days after Perihelion

The two suns still almost coincide.

The yellow True Sun moves ahead. But in order to obey Kepler's law, the yellow cheese-shape must be equal in area to the red cheese-shape. The green angle between the red and yellow vectors is the EoT's Eccentricity Effect.

Half way

The two suns still almost coincide, but the yellow True Sun is just ahead, following Kepler's Law.

The yellow True Sun is now at its maximum separation from the red Dynamical Mean Sun.

15 days before Aphelion

The two suns almost coincide again.

The red Dynamical Mean Sun is now catching up with the True Sun.

Aphelion

The two suns are aligned once more and the Yellow semi-circle is equal in area to the red semi-circle, as stipulated by Kepler.

The Effect repeats itself in the opposite direction in the second half of the year

Fig 9 - The Eccentricity Effect

return to "Table of Contents"

Finding the Sun's True Longitude

This section is not for the faint hearted, and can be ignored at first reading. Simpler approximations are available in the next chapter.

The terminology shown in the key belongs in antiquity...

- the outer red circle is the celestial ecliptic, centered on the Earth. The red Dynamical Mean Sun is arbitrarily set to 35° from perihelion -corresponding to ca. 9th February. It travels *uniformly* around the circle. The angle (**M**) is called the **Mean Anomaly**.
- the orange ellipse is the path of the true sun with the Earth at one of its focii. In order to provide clarity, the eccentricity of the ellipse has been set to 0.4 (rather than its true value of 0.018). On the ellipse is the true sun, whose sun's angle (**ν**)is called the **True Anomaly**.

  It differs from the true longitude of the sun only insofar as it is measured from Perihelion rather than the 1st Point of Aries on March 20th.
- the green circle centered at the middle of the ellipse is the so-call **Eccentric circle**. Its diameter is the same as the major axis of the ellipse. On the circle, immediately above the true Sun, is the green **Eccentric Sun**, at an angle (**E**) called the **Eccentric Anomaly**.

As stated above, Kepler's 2nd law implies that the area swept out by the sun must be equal in equal times....

the area of the yellow segment
as a proportion of the areas of the ellipse
=
the area of the red segment
as a proportion of the area of the circle.

The calculations involved are three-fold.

Steps 1 - 7 finds the Eccentric Anomaly (**E**) in terms of the Mean Anomaly (**M**).

Step 8 finds True Anomaly (**ν**) in terms of the Eccentric Anomaly (**E**).

Step 9 finds the Longitude of the Sun (**λ**) in terms of the True Anomaly (**ν**)

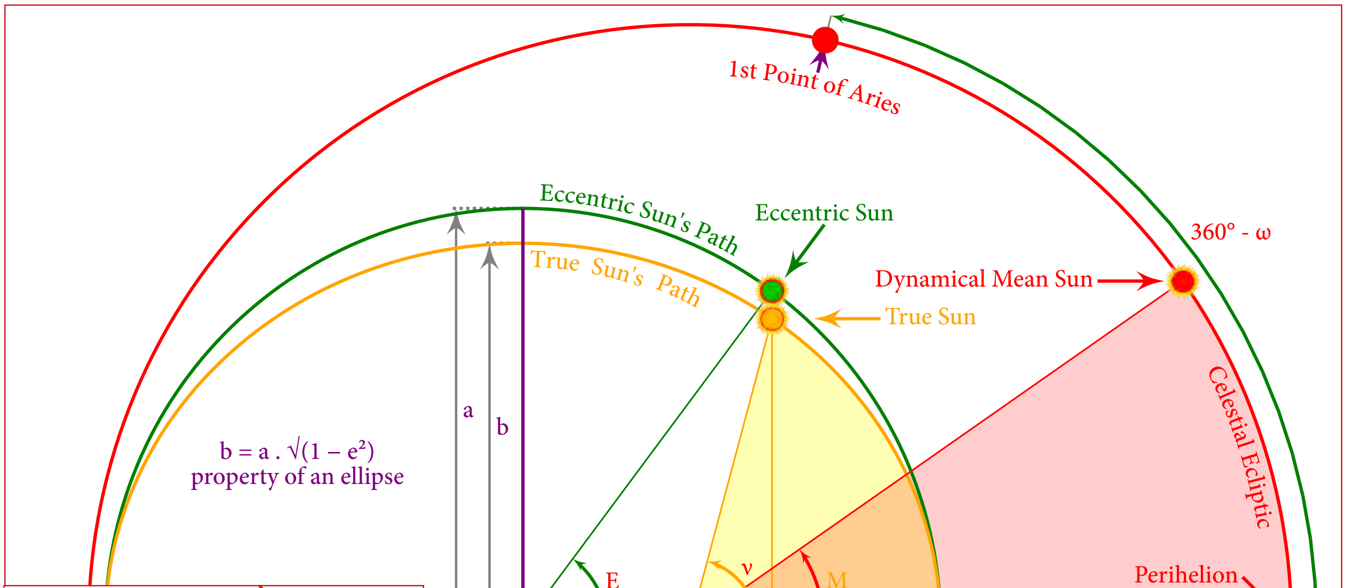


Fig. 10 - showing - in highly exaggerated form - the values that must be calculated.

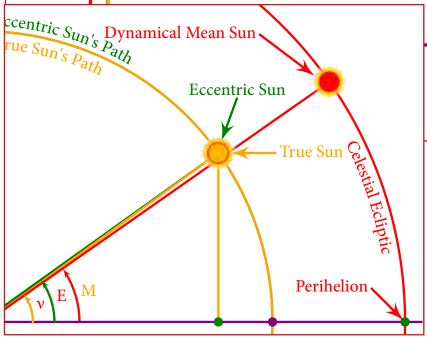
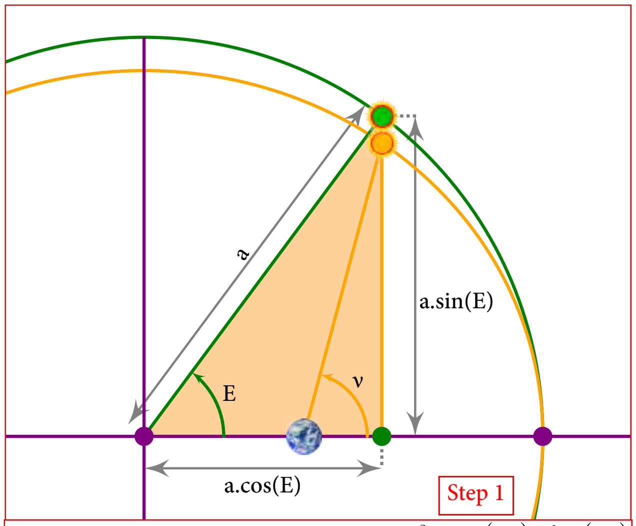
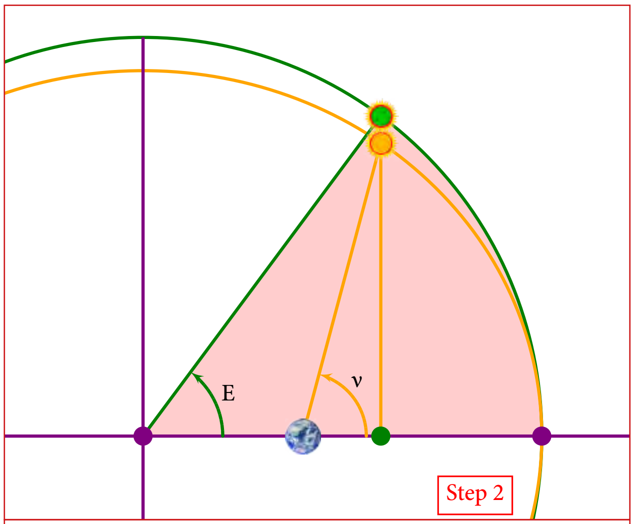


Fig. 11 The same figure as in the Key above, but with the true eccentricity. Note that Eccentric Circle and the True Ellipse almost coincide and the Eccentric Sun is hidden.

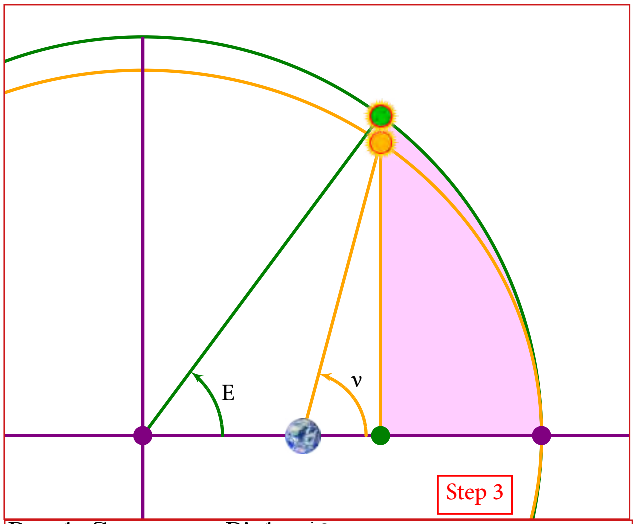
return to "Table of Contents"



Area of Orange Triangle $= 0.5\,a^2\cos(E)\sin(E)$



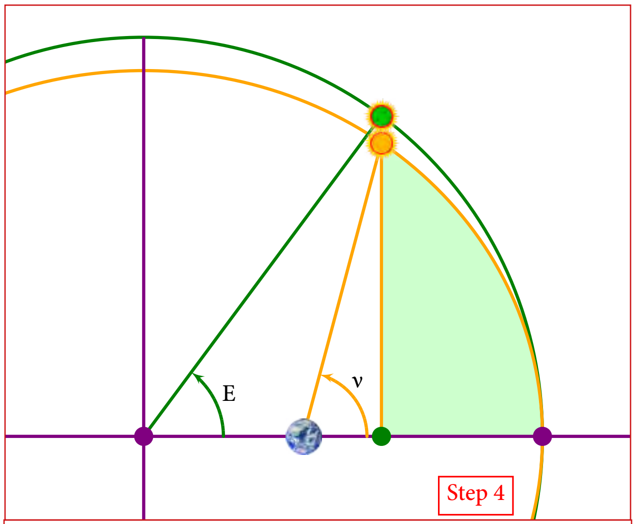
Pink Wedge $= a^2 E^{rad}$



Purple Segment = Pink − Orange

$$= \{E^{rad} a^2\} - \{0.5\,a^2\cos(E)\sin(E)\}$$

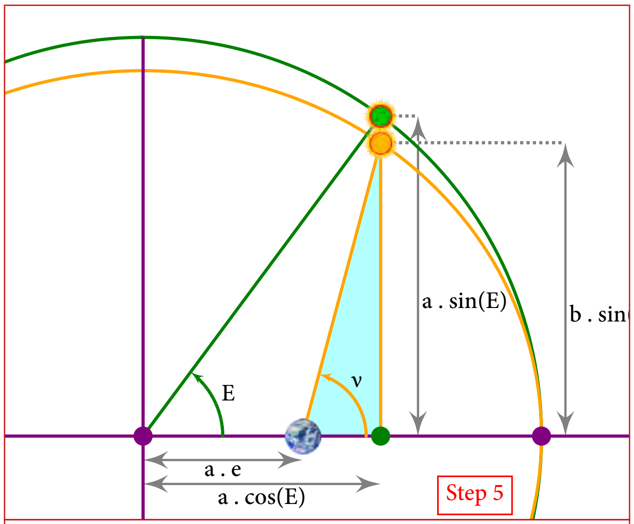
$$= a^2\{E^{rad} - 0.5\cos(E)\sin(E)\}$$



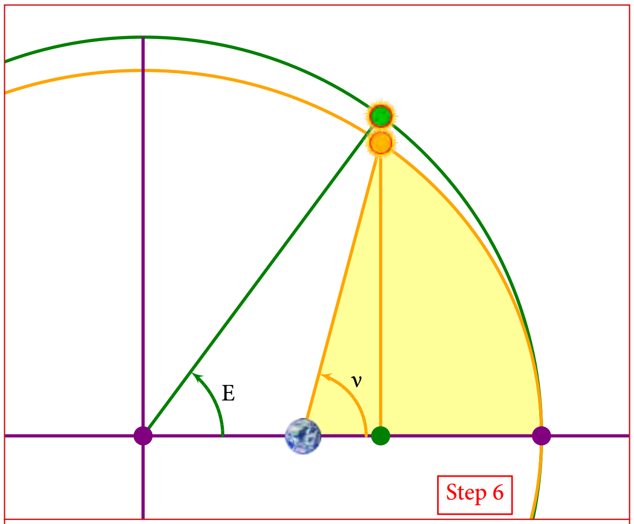
Green Segment $= \frac{b}{a} \times$ Purple Segment

(this is a property of ellipses)

$$= a\,b\{E^{rad} \times \cos(E) \times \sin(E)\}$$



Blue Segment $= 0.5\,a\,b\sin(E) \times \{\cos(E) - e\}$

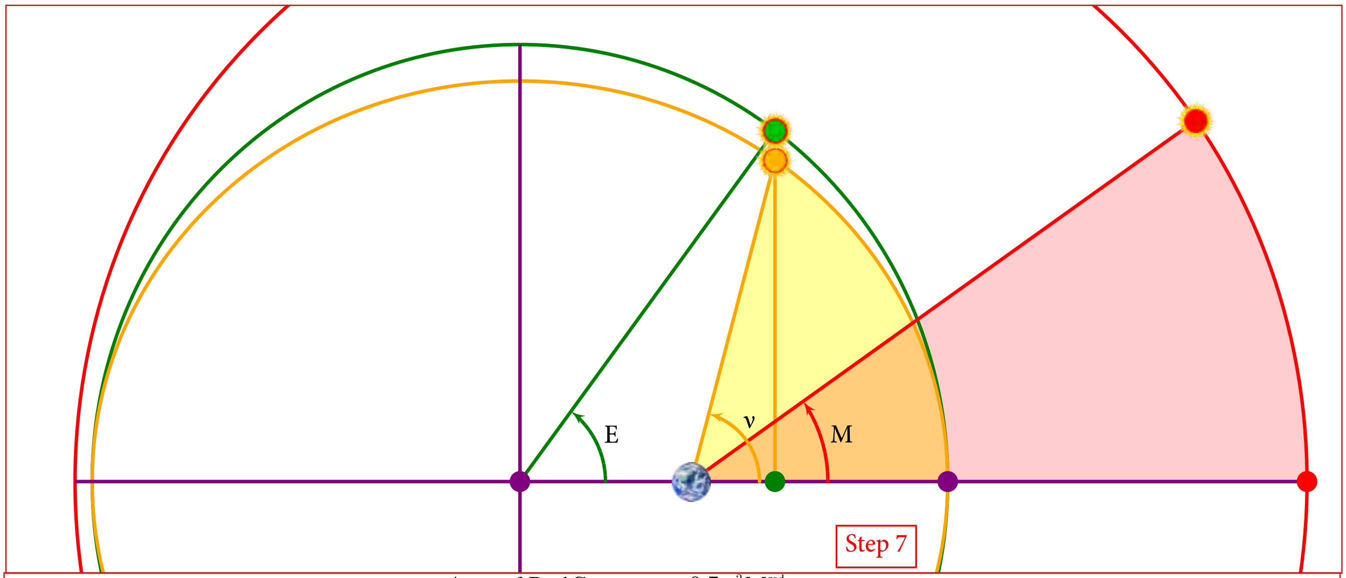


Yellow Segment = Cyan + Green

$$= a\,b\{E^{rad} - \cos(E)\sin(E)\}$$

$$+ 0.5\,a\,b\sin(E)\{\cos(E) - e\}$$

$$= 0.5\,a\,b\{E^{rad} - e \times \sin(E)\}$$



$$\text{Area of Red Segment} = 0.5\,a^2 M^{rad}$$

$$\text{Area of Ecliptic Circle} = \pi a^2$$

$$\text{Area of Yellow Segment} = 0.5\,ab\{E^{rad} - e\sin(E)\}$$

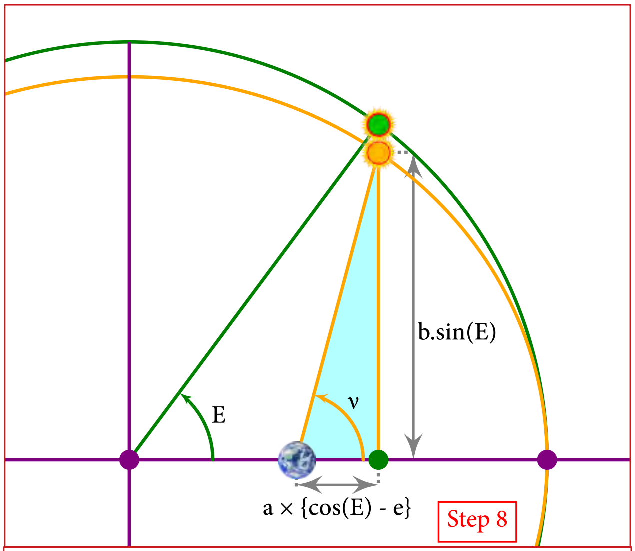
$$\text{Area of Sun's Ellipse} = \pi ab$$

By Kepler's 2nd Law

$$\frac{0.5\,a^2 M^{rad}}{\pi a^2} = \frac{0.5\,ab\{E^{rad} - e\sin(E)\}}{\pi ab}$$

Hence Kepler's equation...

$$M^{rad} = E^{rad} - e \times \sin(E)$$



$$\tan\upsilon = \frac{b\sin(E)}{a(\cos(E) - e)}$$

$$\upsilon = \mathrm{atan2}\{\sqrt{1-e^2}\sin(E), (\cos(E) - e)\}$$

Kepler's equation, as shown in Step 7, cannot be directly solved due to the presence of an angular value both inside and outside a trigonometric function. It necessitates an iterative method, such as that of Newton-Raphson. Because the value of the eccentricity is so small, Newton-Raphson is highly efficient and converges rapidly. In practical applications, only the first iteration is typically needed.

For further information on this method, refer to 'Appendix 2 - Derivation of Newton-Raphson approximation for Kepler's Formula' on page 212.

$$\text{Eccentricity} = e = 0.0167$$

$$\text{Mean Anomaly} = 0.610865^{rad} = 35^{0}$$

$$E_1 = M = 0.610865^{rad} = 35^{0}$$

$$E_2 = E_1 - \frac{E_1 - e.\sin(E_1) - M}{1 - e.\cos(E_1)} = 0.620577^{rad} = 35.556433^{0}$$

$$E_3 = E_2 - \frac{E_2 - e.\sin(E_2) - M}{1 - e.\cos(E_2)} = 0.620576^{rad} = 35.556406^{0}$$

$$E_4 = E_3 - \frac{E_3 - e.\sin(E_3) - M}{1 - e.\cos(E_3)} = 0.620576^{rad} = 35.556380^{0}$$

Fig 12 - Newton Raphson solution to Kepler's Equation

The final step is to convert the True Anomaly (**ν**) - which is zeroed on perihelion to longitude (**λ**), which is based on the 1st Point of Aries. The difference between the two is Longitude of Perihelion (**ω**). This is an astronomical constant of some 283°, measured clockwise from the 1st Point of Aries.

$$\lambda^{\circ} = \mathrm{mod}(\omega^{\circ} + \upsilon^{\circ}, 360^{\circ})$$

the mod function brings the value of λ between 0° and 360°

Fig 13 - Converting from Perihelion to 1st Point of Aries

THE ECCENTRICITY EFFECT

The Eccentricity Effect of the Equation of Time is simply the angular difference between the Sun's True position minus the Dynamical Mean Sun's position.

True Anomaly (ν) - (M) **Mean Anomaly** (M)

The graph below shows the value of the Eccentricity Effect throughout the year. Mathematically, the rise and fall of this angle (from zero to a maximum value, back to zero, then in the second half of the year, falling to a minimum value and finally back to zero) can be shown to be very nearly a sine curve with origin at Perihelion.

The amplitude of the sine curve is some 1.75° or 7 $^{\text{minutes of time.}}$ (n.b. 360° = 24 $^{\textit{hour}}$ >> 4° = 1 $^{\textit{minute of time}}$).

Details of the calculation procedure needed to provide this graph are provided in 'Kepler's Method' on page 25.

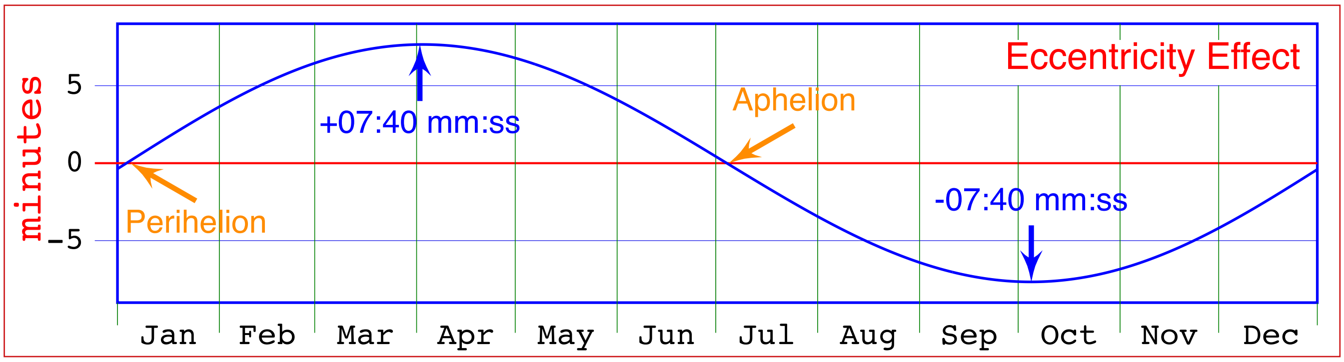


Fig. 14 - The Eccentricity Effect

return to "Table of Contents"

VISUALIZING THE OBLIQUITY EFFECT

The second component of the Equation of Time is the Obliquity Effect.

Since our time keeping is based on the Mean Sun's uniform movement around the Equator, we must project the position of a True Sun from the Ecliptic down its meridian, onto the Celestial Equator. This projection, called the 'RA Sun', must be compared to the 'Mean Sun' to provide the Equation of Time.

The Eccentricity Effect is thus the difference between the Sun's True Longitude and its Right Ascension.

Refer below for the nomenclature used.

On the following page is shown:

i) on the left, how the EoT would develop if the Sun's eccentricity were zero. The rise and fall twice a year evenly between the equinoxes and solstices.

ii) on the right, the true situation where the Obliquity effect overlays the Eccentricity effect.

There is are animations available of this method.
See 'Available Videos' on page 234.

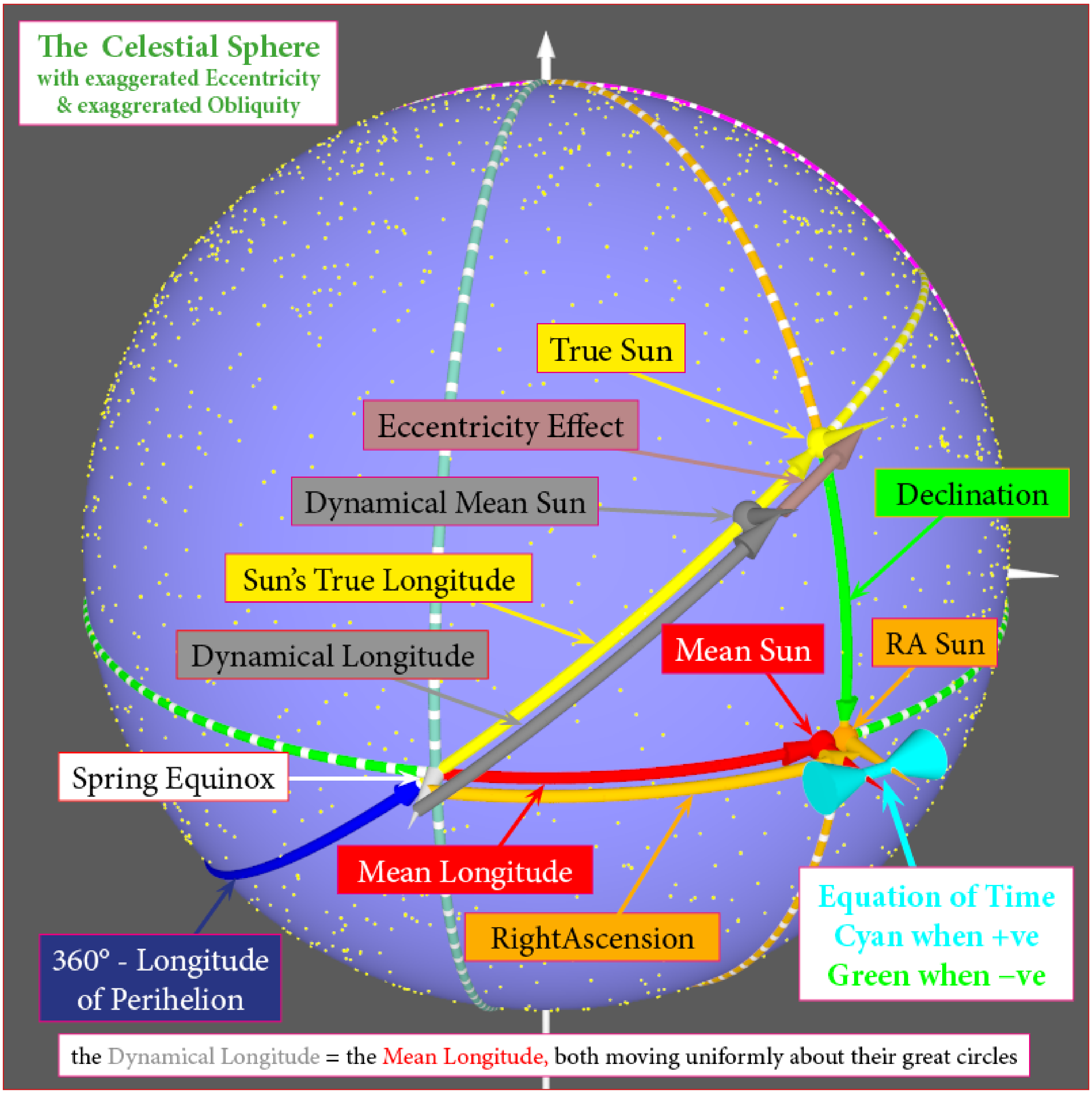


Fig 15 - To visualize various 'components' that are needed to calculate the EoT more clearly, the illustration above is shown with...
the value of the Eccentricity vastly increased from its true value of 0.018 to 0.3
& the value of the Obliquity increased from its true value of 23.4° to 30°.

return to "Table of Contents"

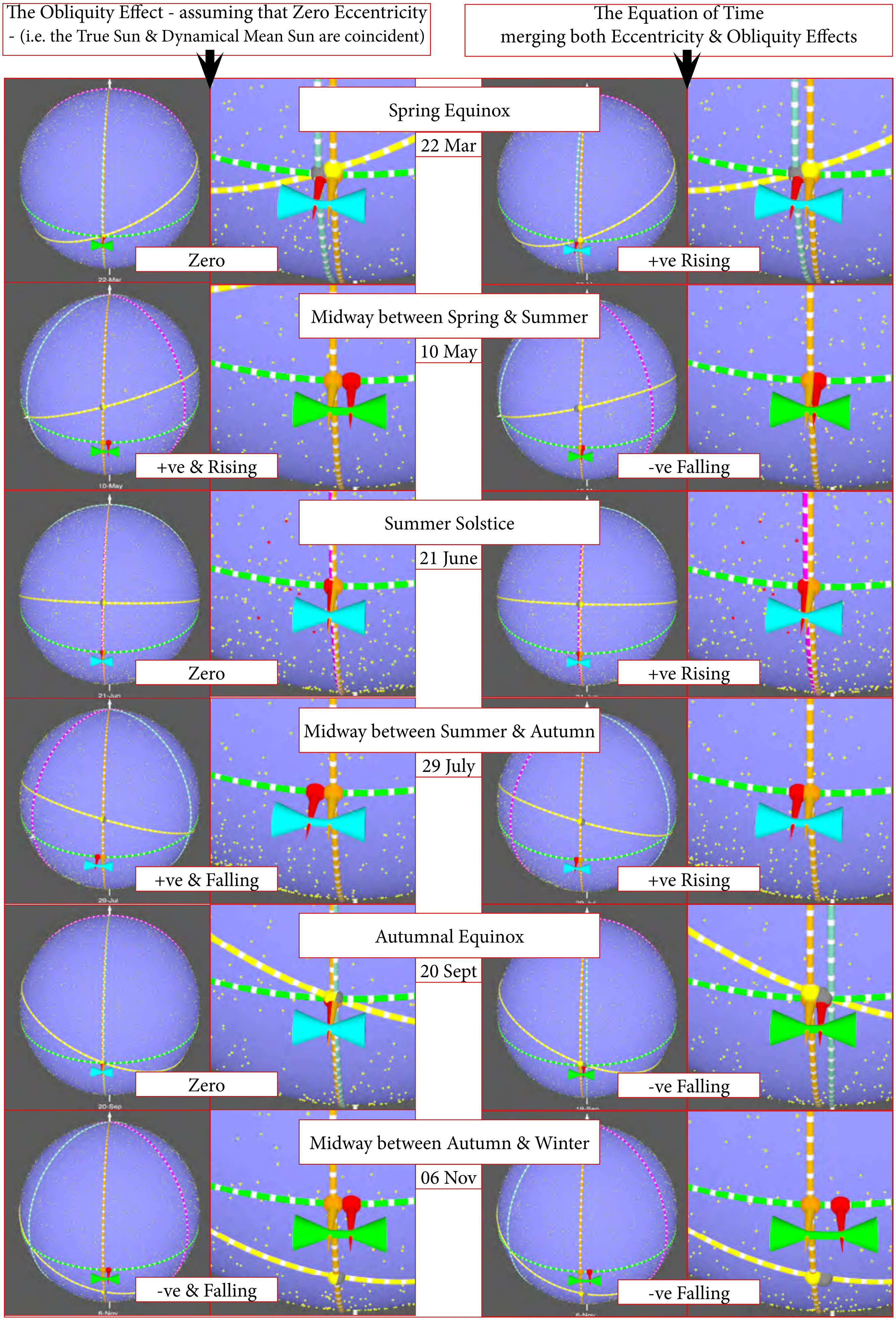


Fig 16 - The Obliquity Effect

return to "Table of Contents"

CALCULATING THE SUN'S RIGHT ASCENSION

The Right Ascension (**α**) is found from right-angled spherical triangle comprising the Sun's Longitude (**λ**), its Declination (**δ**) and the Obliquity (**ε**).

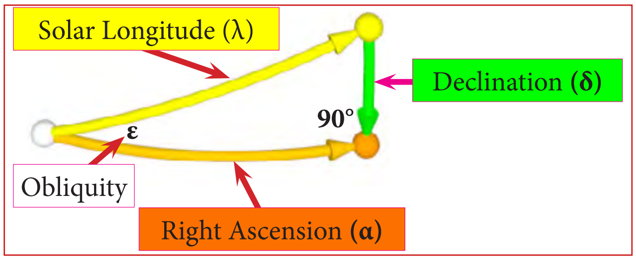


Fig 17 - The spherical right angle triangle needed to find the Right Ascension

This can be solved using standard formula. See Wikipedia, Spherical Triangles. Thus:

$$
\begin{aligned}
\tan(\alpha) &= \cos(\varepsilon) \times \tan(\lambda) \\
\alpha^{\text{radians}} &= \operatorname{atan2}[\cos(\varepsilon) \times \sin(\lambda), \cos(\lambda)] \\
\alpha^{\text{degrees}} &= \alpha^{\text{radians}} \times 180/\pi \\
\alpha^{\text{hours}} &= \alpha^{\text{degrees}} \div 15
\end{aligned}
$$

Fig 18 - The general equation for solving the spherical right angle triangle in Fig. 17.

RECAPITULATION

The steps taken can be summarised :

Using the fixed Stars as reference...

1. Greenwich Mean Sidereal Time (**GMST**) was calculated as a function of Date and UTC;
2. the position of the imaginary Mean Sun (**Mean Longitude**) was calculated simply by definition from GMST.

Using Perihelion (around the back of the figure) as origin around the Ecliptic...

3. the Eccentric Anomaly (**E**) was calculated as a function of the Mean Anomaly (**M**) to produce Kepler's insoluble Equation;

4, Kepler's Equation was solved using a Newton Raphson approximation;

5. the True Anomaly (**ν**) was calculated as a function of the just solved Eccentric Anomaly (**E**);
6. the Longitude of the Sun (**λ**) was calculated by rotating the True Anomaly around the Ecliptic by the Longitude of Perihelion (**ω**).

Using the 1st Point of Aries as origin around the Celestial Equator...

7. the Declination (**δ**) and Right Ascension (**α**) were calculated, using the formulae for right-angled spherical triangles.

Finally...

8. the Equation of Time (**EoT**) was found as the difference between the **Mean Longitude** and Right Ascension (**α**).

$$
\begin{aligned}
\text{EoT}_{\text{Astronomical}}^{\text{mins}} &= 60 \times (\text{Mean Longitude}^{\text{hrs}} - \text{RA}^{\text{hrs}}) \\
\text{EoT}_{\text{Astronomical}}^{\text{mins}} &= 60 \times (\text{GMST}^{\text{hrs}} - \text{GMT}^{\text{hrs}} + 12^{\text{hrs}} - \text{RA}^{\text{hrs}}) \\
\text{EoT}_{\text{Gnomonical}}^{\text{mins}} &= -\,\text{EoT}_{\text{Astronomical}}^{\text{mins}}
\end{aligned}
$$

Fig 19 - The final formula for the Equation of Time

9. If one subtracts the Eccentricity Effect from the EoT, the result is the Obliquity Effect. See the central graph in Fig 20 overleaf.
10. For all practical purposes, EoT derived above must be corrected for the Longitudinal Effect, involving the local Longitude and Time Zone. The effect of Daylight Saving must also be acknowledged.

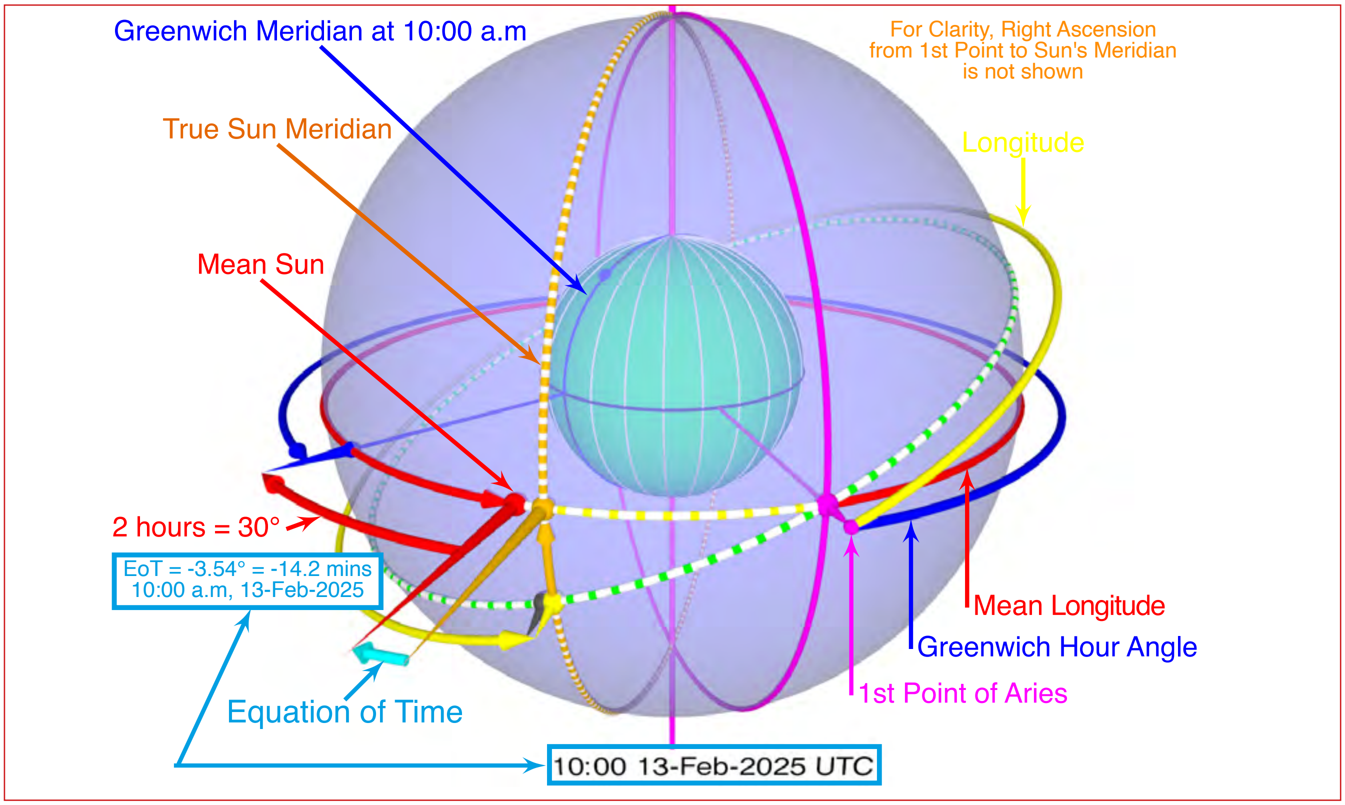


Fig 20 - Graphical Recapitulation of the parameters used in the calculation of EoT.

COMBINING THE EFFECTS

Using the calculation method outlined above and adding together the Eccentricity and Obliquity effects, as shown in Fig 20, provides the familiar shape of the Equation of Time.

The curve is asymmetrical since the two effects do not share the same origin : one at Perihelion, one at the Equinox.

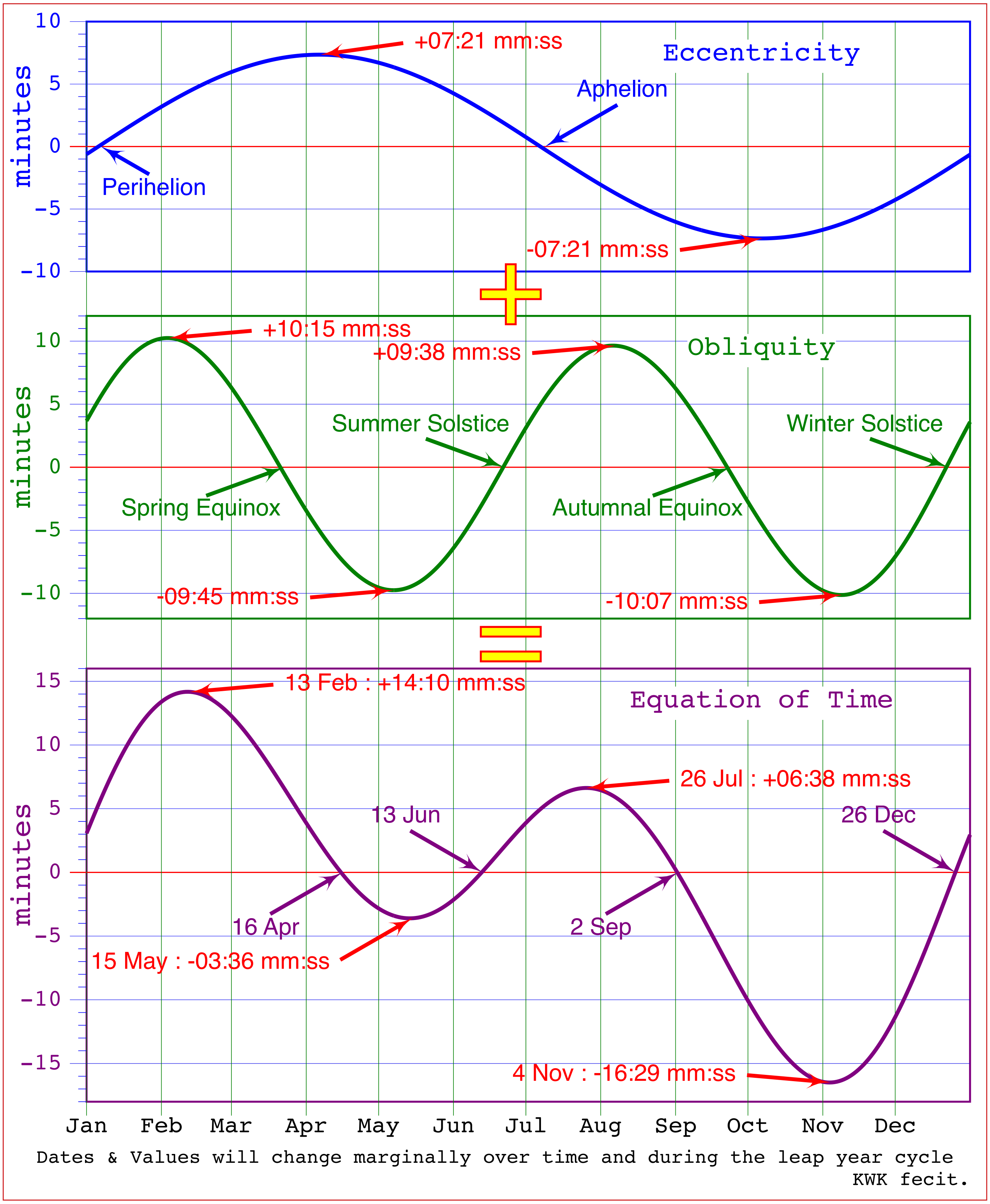


Fig 21 - Generating the EoT by combining Eccentricity and Obliquity Effects

FURTHER ANALYSIS

Fourier Transform can break down any data series into its various sine components and their shift away from any origin. By using such analysis, one can break down the components of the Equation, as shown in Figs. 22 and 23.

Fig 22 shows 4 columns...

i) For the Eccentricity Effect, note that there is a second small overtone in solution of Kepler's equation. Thus, it is more than a single simple sine curve,There are other overtones, but they are totally insignificant.

If the eccentricity was larger than its current small value of 0.0167, those further overtones would become larger.

The origin of the large overtone is Perihelion

ii) This shows the Obliquity Effect, if the Ecliptic path was circular rather than elliptical. It is the Equation if the Dynamical Mean Sun were used for the calculations rather than that for the True Sun.

'A Minimalistic Approach' on page 25.The obliquity effect is thus the solution of the spherical triangle (as shown in Fig. 17) which is atan(.92 x tan λ) : where .92 is the cosine of the Earth's obliquity. This produces just two significant overtones. These are the 2nd and 4th, reflecting the fact that the effect cycles twice in a year.

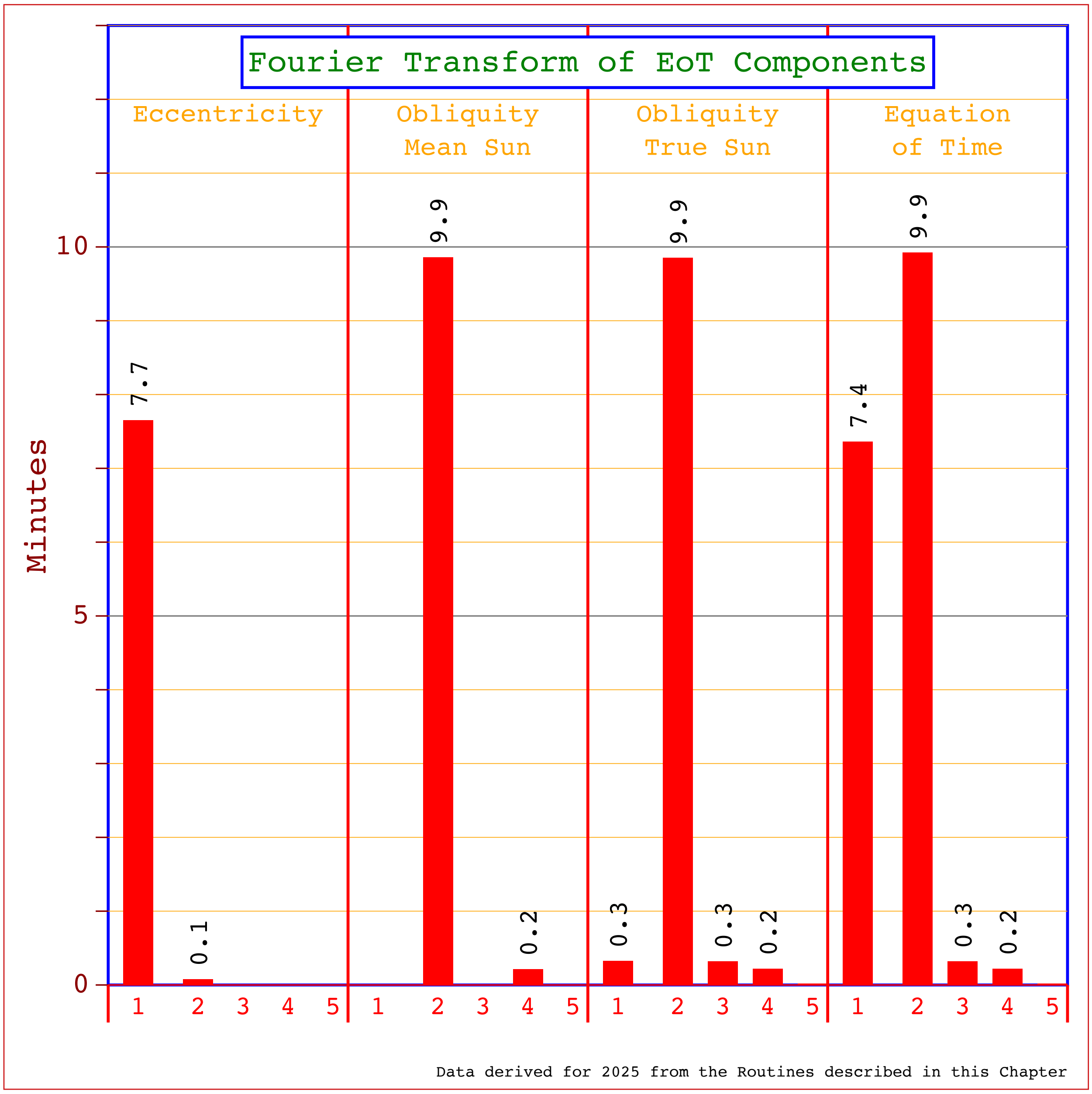
iii) This shows the true Obliquity effect - calculated as the total EoT minus the Eccentricity effect. Here one may see the fact that the obliquity effect is 'built' on top of the Eccentricity effect, and hence has 4 overtones.

The origin of the main overtone is the 1st Point of Aries.

iv) Finally, the overtones of the whole equation of time is shown. The four significant values are those used in one of the many approximate methods of calculating the Equation of Time

For further details concerning Fourier Transforms, in general, see 'Basic Positional Solar Astronomy - Part 3: Fourier Derived Formulae' on page 207.

For their implementation in a simply means of calculating the EoT, see 'Kepler's Method' on page 25.



Data derived for 2025 from the Routines described in this Chapter

Fig 22 - Generating the EoT by combining Eccentricity and Obliquity Effects

return to "Table of Contents"

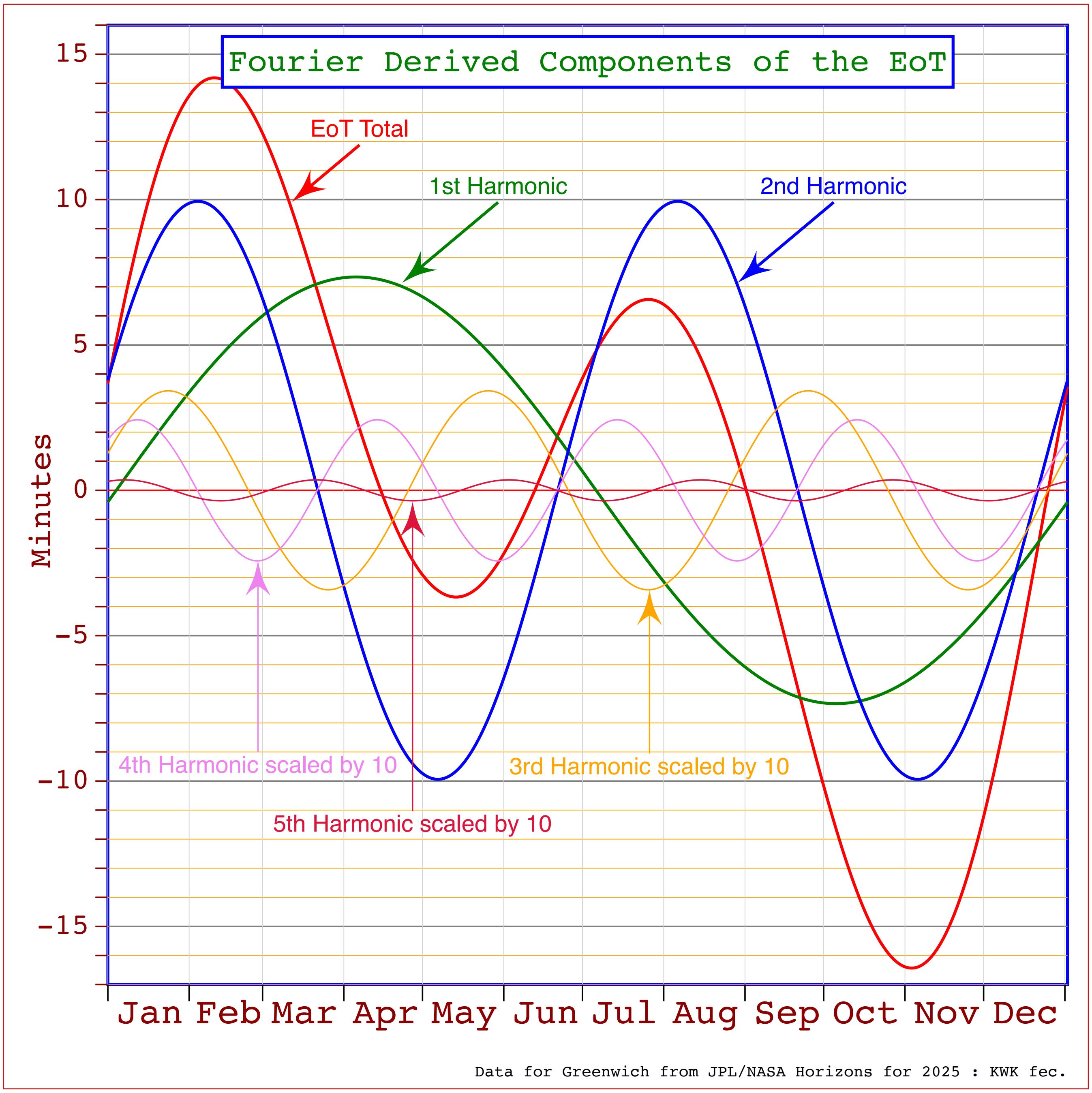


Fig 23 - The EoT being the sum of its four overtones

Footnote - Note on the Complexities of Astronomical Values and Nomenclature

Nothing is stable or constant in the heavens. There are long-term changes, for example:

- Precession, the gyration of the Earth's axis caused by the gravitational forces of the Moon and Sun on Earth's equatorial bulge. This has a period of about 26,000 years. It causes the movement of Perihelion towards the 1st Point of Aries.
- Obliquity, over the next 1,000 years, the obliquity will change from its current value of 23.436° to 23.307°, due to the tidal influence of the moon and gravitational influence of the planets
- Eccentricity of the Earth's obit is slowly reducing at the tiny rate of some 0.00044/1000 years. However note that a small change of eccentricity can have a significant change in the Eccentricity effect

There are also small and short-term changes, for example,

- Nutation, which cause minor changes in the position of the 1st Point of Aries and in the value of the Earth's obliquity. Because the Moon's complex movement around the Earth, these changes are complex, but have a longer term period of some 18.6 years.

Because of these variations, one finds the term 'mean' and 'apparent' used to distinguish between the actual and the average. And, of importance, the term 'Equation of ....' means the difference between these two.

Even the nomenclature changes depending on one's preference. Because its has a historical ring. I have used the term '1st Point of Aries' to be synonymous with the 'Vernal Equinox'. In fact, because of Precession, the Vernal Equinox in no longer in the constellation of Aries at all, but in Pisces. This has led to confusion amongst Astrologers!

return to "Table of Contents"

Footnote - Mean Solar Time

A reckoning of time which conforms more or less closely to the recurrence of daylight and darkness determined by the diurnal motion of the Sun, and which is quickly obtainable with high precision from observation, is a practical necessity.

Because of the variations in the rate of motion of the Sun in hour angle, due to the inequalities in the annual motion along the ecliptic and to the inclination of the ecliptic to the equator, the measure of time that is directly defined by the actual diurnal motion of the Sun, known as apparent solar time, is impracticable for the purpose of precise timekeeping.

Instead, mean solar time was introduced, determined by the apparent diurnal motion of an abstract fiducial point (= taken as standard of reference) at nearly the same hour angle as the Sun, but located on the mean celestial equator of date and characterized by a uniform sidereal motion along the equator at a rate virtually equal to the mean rate of the annual motion of the Sun along the ecliptic. Relative to any meridian of longitude, this point has a diurnal motion in hour angle virtually the same as the average diurnal motion of the Sun, and uniform except for variations of the local meridian; the position in hour angle is never more than 16mins from the Sun.

Explanatory Supplement to the Astronomical Ephemeris & the American Ephemeris and Nautical Almanac

Footnote - GMST

In past years, GMST was found measuring the position of certain stars using transit telescopes. In recent years, long range interferometry of extra galactic quasars is used to deduce GMST, since stars do move against the celestial sphere: their 'proper motion'.

Footnote - Accessing videos

1 go to www.zenodo.org. You do not need to login.
2 pull down the search bar and enter...
3 Kevin AND Karney.
4 download the appropriate video.

return to "Table of Contents"

Calculating the Equation of Time

There are many ways to derive the Equation of Time, depending on one's need for accuracy.

(i) Kepler's procedure, which follows the astronomical process descibed in the previous chapter, is sufficient for most normal use.

(ii) the Almanac procedure is slightly less complex and slightly less accurate

(iii) the Fourier procedure is the simplest and is sufficiently accuate for most gnomonical use.

Also described are a number of other ways and resources

The Wikipedia page on the Equation of Time provides much mathematical analysis, but does not easly show how to incorporate leap years and one's location and time zone..

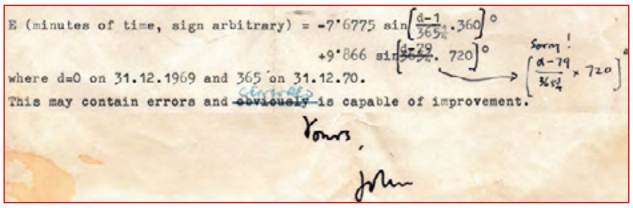
There is a footnote at the end of this chapter concerning quoted accuracies.

Why Bother to Calculate at All...

On September one, trust the Sun.
Come Halloween, subtract sixteen.
On Christmas Day, you're OK.
For your Valentine true, add a dozen and two.
The mid of month four, add no more
At the mid of May, take four away.
On June fourteen, don't add a bean.
When August begins, add seven little mins.
The rest is easy: for any date,
All you do is interpolate.

A Poem by Ted Dunn

A Minimalistic Approach



E (minutes of time, sign arbitrary) = $-7{\cdot}6775 \sin\left[\frac{d-1}{365\frac{1}{4}}.360\right]^{\circ}$

$+9{\cdot}866 \sin\left[\frac{d-79}{365\frac{1}{4}}.720\right]^{\circ}$

where d=0 on 31.12.1969 and 365 on 31.12.70.

This may contain errors and ~~obviously~~ certainly is capable of improvement.

Sorry! $\left[\frac{d-79}{365\frac{1}{4}} \times 720\right]$

Yours,

John

Fig 1 - A 1970 letter from the author's Uncle John Wigham Richardson explaining how to calculate the Equation of Time

Kepler's Method

This 'astronomical' method follows the procedure given in 'Recapitulation' on page 20.

Input Observer's Location				
1	Longitude : +ve East of Greenwich	Λ°	23.71667	The Acropolis, Athens
2	Latitude : +ve N of Equator	φ^{o}	37.96667	
3	Time Zone : +ve East of Greenwich	TZ^{hrs}	2	
Input Observer's Date & Civil Time				
4	Year	Year	2025	Noon, 13th February 2025 UTC_{uncorr}^{hrs}
5	Month	Month	2	
6	Day	Day	13	
7	Hour	Hour	12	
Time related Parameters				
8	Year corrected	$Year_{corr}$	2024	if Month <=2 then Year = Year – 1
9	Month corrected	$Month_{corr}$	14	if Month <=2 then Month = Month + 12
10	UTC	UTC^{hrs}	10	Hour – Zone

Table 1 a - continues overleaf

return to "Table of Contents"

11	Days since Epoch 2000 (1 Jan 2000 12:00)	aaa	20	$\mathrm{int}(\mathrm{Year}_{corr} / 100)$
12		bbb	-13	$2 - \mathrm{aaa} + \mathrm{int}(\mathrm{aaa} / 4)$
13		D_{2000}^{days}	9174.916 67	$\mathrm{bbb} + \mathrm{int}(365.25 \times \mathrm{Year}_{corr})$ $+ \mathrm{int}(30.6001 \times \{\mathrm{Month}_{corr} + 1\})$ $+ \mathrm{Day} - \mathrm{UTC}^{hrs}/24 - 730\,550.5$
14	Julian Centuries since Epoch	T	0.25120	$D_{2000}^{days} \div 365\,25$
15	Greenwich Mean Sideral Time	GMST^{o}	293.69309	$\mathrm{mod}(280.460\,618\,37 + 360.985\,647\,366\,29 \times$ $D_{2000}^{days} + 0.000\,387\,933 \times T^{2}$ $- T^{3}/387\,100\,00 , 360)$
16		GMST^{hrs}	19.57954	$\mathrm{GMST}^{o} \div 15^{deg/hr}$
17	Local Mean Sidereal Time (illustrative only)	LMST^{hrs}	21.16065	$\mathrm{GMST}^{deg} + \Lambda^{o} \div 15^{deg/hr}$
18	Mean Sun Longitude	L^{hrs}	21.57954	$\mathrm{GMST}^{hrs} + 12 - \mathrm{UTC}^{hrs}$
19		L^{0}	323.69309	$L^{hrs} \times 15^{deg/hr}$
Astronomical Facts				
20	Perihelion Longitude	ω^{o}	283.36503	$282.938^{o} + 1.7 \times T$
21	Eccentricity	e	0.01670	$0.0167\,086\,17 - 0.000\,04 \times T$
22	Obliquity	ε^{o}	23.43603	$23.439\,29111 - 0.013 \times T$
23		ε^{rad}	0.40904	$\varepsilon^{o} \times \pi \div 180^{o}$
Solving Kepler				
24	Mean Anomaly	M^{o}	40.32806	$L^{o} - \omega^{o}$
25		M^{rad}	0.70386	$M^{o} \times \pi \div 180^{o}$
26	Eccentric Anomaly (iteration 0)	E_{0}^{rad}	0.70386	M^{rad}
27	Eccentric Anomaly (iteration 1)	E_{1}^{rad}	0.7148	$E_{0}^{rad} + [M^{rad} + e \times \sin(E_{0}^{rad}) - E_{0}^{rad}) \div$ $(1 - e \times \cos(E_{0}^{rad})]$
28	Eccentric Anomaly (iteration 2) (not usually necessary)	E_{2}^{rad}	0.7148	$E1^{rad} + [M^{rad} + e \times \sin(E_{1}^{rad}) - E_{1}^{rad}) \div$ $(1 - e \times \cos(E_{1}^{rad})]$
29	True Anomaly	ν^{rad}	0.72582	$2 \times \mathrm{atan2}$ $\{\mathrm{sqrt}[(1 + e)/(1 - e)] \times \sin(E^{rad} \div 2),$ $\cos(E^{rad} \div 2)\}$
30		ν^{o}	41.58633	$\nu^{rad} \times \pi \div 180^{o}$
31	Sun's True Longitude	λ^{o}	324.95136	$\nu^{o} + \omega^{o}$
32		λ^{rad}	5.67147	$\lambda^{o} \times \pi / 180^{o}$
33	Eccentricity Effect	EE^{o}	1.25827	$\lambda^{o} - L^{0}$
34		EE^{min}	5.03306	$4 \times \mathrm{EE}^{o}$

Table 1 b - continues overleaf

return to "Table of Contents"

Finding Right Ascension, Declination & Equation of Time				
35	Right Ascension	α^{rad}	5.711 32	$\text{mod}\{\text{atan2}[\cos(\varepsilon^{rad}) \times \sin(\lambda^{rad}),\ \cos(\lambda^{rad})], 2\pi\}$
36		α^{o}	327.234 45	$\text{degrees}(\alpha^{rad})$
37		α^{hrs}	21.815 63	$\alpha^{o} \div 15^{hrs/deg}$
38	Declination	δ^{rad}	-0.230 44	$\text{asin}(\sin(\varepsilon^{rad}) \times \sin(\lambda^{rad}))$
39		δ^{o}	-13.203 02	$\text{degrees}(\delta^{rad})$
40	Equation of Time	EoT^{0}	3.541 36	$\alpha^{o} - L^{o}$
41			3.541 36	if $\text{EoT}^{0} > 180^{o} : \text{EoT}^{o} = \text{EoT}^{o} - 360^{0}$ if $\text{EoT}^{0} < -180^{o} : \text{EoT}^{o} = \text{EoT}^{o} + 360^{0}$
42		EoT^{min}	14.165 45	$4 \times \text{EoT}^{0}$
43	Obliquity Effect	OE^{min}	9.132 39	$\text{EoT}^{min} - \text{EE}^{min}$
44	Longitude Correction	σ^{min}	25.133 32	$4 \times (\text{TZ}^{hrs} \times 15^{deg/hr} - \Lambda^{o})$
45	Equation of Time - longitude corrected	$\text{EoT}_{l\text{-}cr}^{\ min}$	39.298 77	$\text{EoT}^{min} + \sigma^{min}$
Additional Relevant Parameters				
46	Solar Noon	sn^{hrs}	12.654 98	$12^{hrs} + (\text{EoT}_{l\text{-}cr}^{\ min} \div 60)$
47	Solar Hour Angle	h^{hrs}	-0.654 98	$\text{GMST}^{hrs} + \Lambda^{o} \div 15^{o/hr} - \alpha^{hrs}$
48		h^{rad}	-0.171 47	$\text{radians}(h^{hrs} \times 15^{o/hr})$
49	Latitude radians	φ^{rad}	0.662 94	$\text{radians}(\varphi^{o})$
50	Solar Altitude	alt^{rad}	0.663 35	$\text{asin}\{\sin(\varphi^{rad}) \times \sin(\delta^{rad}) + \cos(\varphi^{rad}) \times \cos(\delta^{rad}) \times \cos(h^{rad})\}$
51		alt^{o}	38.00715	$\text{zzz}\{\text{degrees}(\text{alt}^{rad}) , 360\}$
52	Solar Azimuth	a	0.130 97	$-\cos(\delta^{rad}) \times \cos(\varphi^{rad}) \times \sin(h^{rad})$
53		b	-0.607 22	$\sin(\delta^{rad}) - \sin(\varphi^{rad}) \times \sin(\text{alt}^{rad})$
54		az^{rad}	2.929 16	$\text{atan2}(a,b)$
55		az^{o}	167.828 72	$\text{degrees}(\text{az}^{rad})$
56	≈ Sunrise see note below	$\text{ha}_{civil}^{\ hrs}$	5.296 75	$(\text{degrees}\{\text{acos}[-\tan(\varphi^{rad}) \times \tan(\delta^{rad})]\}) \div 15$
57		$\text{SR}_{civil}^{\ hrs}$	7.358 23	$\text{sn}^{hrs} - \text{ha}^{hrs}$
58	≈ Sunset	$\text{SS}_{civil}^{\ hrs}$	17.951 73	$\text{sn}^{hrs} + \text{ha}^{hrs}$
59	≈ Sunrise Azimuth	r^{o}	73.159 13	$\text{degrees}\{\text{acos}((-\sin(\delta^{rad}) \div \cos(\varphi^{rad}))\}$
60		SRA^{o}	106.840 88	$180^{o} - r^{o}$
61	≈ Sunset Azimuth	SSA^{o}	253.159 12	$180^{o} + r^{o}$

Table 1 c

return to "Table of Contents"

Notes

Functions used...

mod(x,y) reduces the first parameter to the range of the second parameter. e.g. mod(123,100) = 23 & mod(-123,100) = 77

asin(x), acos(x) & atan(x) are the same as sin^{-1}(x), cos^{-1}(x), tan^{-1}(x)

atan2(y,x) is the same as atan(y/x) but ensures the result is in the correct quadrant. ***Note:*** Microsoft in Excel uses atan2(x,y) which is trigonometrically incorrect.

$\text{degrees}(x^{rad}) = x^{rad} \times 180 / \pi$ and $\text{radians}(x^{deg}) = x^{deg} \times \pi/180$

Accuracies in this example compared to Horizons

Parameter	This Routine	Horizons	Error	*Accuracy of this routine’s calculation of EoT is better that +/- 2 seconds between 2000 and 2200*
Local Mean Siderial Time	21.160 65^{hrs}	21.160 79^{hrs}	-0.5$^{\text{secs of time}}$	
Right Ascension	21.815 63^{hrs}	21.815 47^{hrs}	0.6$^{\text{secs of time}}$	
Declination	-13.203 02^{o}	-13.207 43^{o}	15.8$^{\text{secs of arc}}$	
EoT Longitude Corrected	39.298 77^{min}	39.287 31^{min}	0.7$^{\text{secs of time}}$	
Altitude	38.007 15^{o}	38.003 25^{o}	-.2$^{\text{mins of arc}}$	
Azimuth	167.828 72^{o}	167.832 92^{o}	-0.3$^{\text{mins of arc}}$	

If the value of ‘q’ in step 57 is > 1 or < -1, it is full day or full night in the artic regions, so this step will give an error

The above routines are Geocentric and use *mean* parameters, whereas JPL-NASA Horizons calculation are Topocentric (i.e. for local conditions), use *apparent* parameters (i.e. include corrections for Nutation, Abberation, etc.) and generally use the most sophisticated astronomical methods.These differences are generally irrelevant in the context of this document. However, see the Chapter on Variation in the Equation of Time.

The code, in Python, used to provide the above calculations in provided to view in ‘Kepler’s Method’ on page 214. And may be dowloaded from a file ‘Equation of Time Values.py’, which can be downloaded from https://github.com/kevinkarney/Equation-of-Time

The Astronomical Almanac Method

This differs from the method in the previous section, in its simplifed calculation of the Sun's longitude and the Earth's obliquity

Input Observer’s Location				
1	Longitude : +ve E of Greenwich	Λ°	23.71667	The Acropolis, Athens
2	Latitude : +ve N of Equator	φ^{o}	37.96667	
3	Time Zone :+ve E of Greenwich	TZ^{hrs}	2	
Input Observer’s Date & Civil Time				
4	Year	Year	2025	Noon, 13th February 2025-
5	Month	Month	2	
6	Day	Day	13	
7	Hour	Hour	12	
8	Day Saving Hours	DST^{hrs}	0	

Table 2 a

Calculations				
9	*Follow Steps 9 - 14 from the Previous Chapter. 'Kepler's Method' on page 25.*	JD^{days}	2460719.5	
10	Time from D_{2000} Epoch	n^{days}	9174.91667	$(JD^{days} + UTC^{hrs} \div 24) - 245\,2545.0$
11	Mean Longitude of Sun	L^{o}	323.69276	$\text{mod}(280.460^{o} + 0.985\,6474^{o} \times n^{days}, 360)$
12	Mean Anomaly of Sun	g^{o}	40.32862	$\text{mod}(357.528^{o} + 0.985\,60034^{o} \times n^{days}, 360)$
13		g^{rad}	0.70387	$\text{radians}(g^{o})$
14	Ecliptic Longitude	λ^{o}	324.95182	$\text{mod}\{L^{o} + 1.915^{o} \times \sin(g^{rad}) + 0.020^{o} \times \sin(2 \times g^{rad}), 360^{o}\}$
15		λ^{rad}	5.67148	$\text{radians}(^{\lambda^{\circ}})$
16	Obliquity	ε^{o}	23.43533	$23.439^{o} - 0.000\,0004 \times n^{days}$
17		ε^{rad}	0.40902	$\text{radians}(\varepsilon^{o})$
Finding Right Ascension, Declination & Equation of Time				
35	Right Ascension	α^{rad}	-0.57186	$\text{mod}\{\text{atan2}[\cos(\varepsilon^{rad}) * \sin(\lambda^{rad}), \cos(\lambda^{rad})], 2\pi]$
36		α°	327.23477	$\text{degrees}(\alpha^{rad})$
37		α^{hrs}	21.81565	$\alpha^{0} \div 15$
38	Declination	δ^{rad}	-0.23044	$\text{asin}(\sin(\varepsilon^{rad}) \times \sin(\lambda^{rad}))$
39		δ°	-13.20248	$\text{degrees}(\delta^{rad} \div 15)$
40	Equation of Time	EoT^{0}	-3.54201	$L^{0} - \alpha^{0}$
41			-3.54201	if $EoT^{0} > 180 : EoT^{0} = EoT^{0} - 360^{0}$ if $EoT^{0} < -180 : EoT^{0} = EoT^{0} + 360^{0}$
42		EoT^{min}	14.16804	$4 \times EoT^{\circ}$
43	Obliquity Effect	OE^{min}	9.13239	$EoT^{min} - EE^{min}$
44	Longitude Correction	σ^{min}	25.13332	$4 \times (TZ^{hrs} \times 15 - \Lambda^{\circ})$
45	Equation of Time - longitude corrected	EoT_{corr}^{min}	39.30136	$EoT^{min} + \sigma^{min}$

Notes				
Accuracy of this example compared to results from NASA-JPL Horizons application				
Parameter	This Routine	Horizons	Error	*The routine's accuracy:* *between 2025 and 2075 is +/- 3.25 seconds.* *between 2000 and 2200 is +/- 4 seconds*
EoT Longitude Corrected	39.30136^{mins}	39.28731^{mins}	1.2^{secs}	

Table 2 b

THE FOURIER METHOD

Any repeating value (for example the Eccentricity or Obliquity effects or the Equation of Time itself) can usually be broken down mathematically into a series of sine components. See Fig 2.

In the context of the Equation of Time, it will not be necessary to look at more than a few of the harmonic terms. However in accurate astronomy, many hundreds of harmonic terms may be used to define the position of, for example, a planet.

This method used the sophisticated astronomical package (NASA/JPL Horizons) to make a calculation every 6 hours during the whole of the 21st Century. The data set was split into 100 leap year cycles of 365.25 days. Each set was Fourier analysed to calculate the amplitude and phase of each of the first six EoT harmonics. The error of each individual cycle was typically a few seconds.

The results of the first two harmonics are shown in Fig 3. Observation of these results show...

(i) The extreme 'spiky-ness' of the graphs is a result of the moon's erratic movement around the year, which means that the actual moment of perihelion varies substantially from its mean.

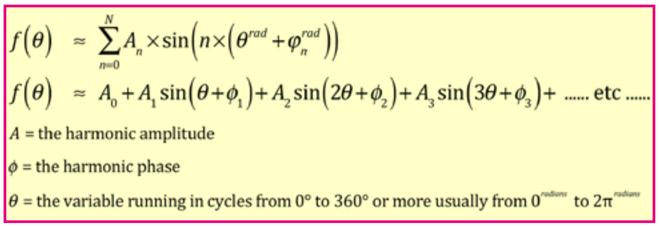
(ii) for the 1st Harmonic,

the reduction in amplitude of the trend line reflects the slowly changing ellipticity of the earth's orbit.

the change in phase indicates the movement of mean perihelion towards the 1st Point of Aries as a result of precession.

(iii) for the 2nd Harmonic,

The reduction in amplitude result from the slow reduction of the Earth's Obliquity.



$$f(\theta) \approx \sum_{n=0}^{N} A_n \times \sin\left(n \times \left(\theta^{rad} + \varphi_n^{rad}\right)\right)$$

$$f(\theta) \approx A_0 + A_1 \sin\left(\theta + \phi_1\right) + A_2 \sin\left(2\theta + \phi_2\right) + A_3 \sin\left(3\theta + \phi_3\right) + \text{...... etc ......}$$

A = the harmonic amplitude

ϕ = the harmonic phase

θ = the variable running in cycles from 0° to 360° or more usually from $0^{radians}$ to $2\pi^{radians}$

Fig 2 - The Basis of Fourier analysis. Any repeating sequence can be broken down with this method

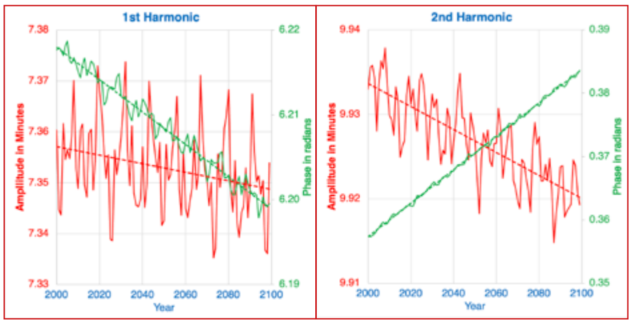


Fig 3 - Fourier results for 100 x 365.25 day cycles of the Equation of Time as predicted by NASA/JPL Horizons program

return to "Table of Contents"

The Fourier Method

1	Longitude : +ve East of Greenwich	Λ^{0}	23.71667	
2	Time Zone +ve East of Greenwich	TZ^{hrs}	2	
3	Year	Year	2025	
4	Month	Month	2	
5	Day	Day	13	
6	Year corrected	$Year_{corr}$	2024	if Month <=2 then Year = Year – 1
7	Month corrected	$Month_{corr}$	14	if Month <=2 then Month = Month + 12
8	UTC	UTC^{hrs}	10	Hour – Zone (Hour assumed local midday)
9	Days since Epoch 2000 (1 Jan 2000 12:00)	aaa	20	$\text{int}(Year_{corr} / 100)$
10		bbb	-13	$2 - aaa + \text{int}(aaa / 4)$
11		D_{2000}^{days}	9174.916 67	$bbb + \text{int}(365.25 \times Year_{corr})$ $+ \text{int}(30.6001 \times \{Month_{corr} + 1\})$ $+ Day + UTC^{hrs}/24 - 730\ 550.5$
12	Days since Start of Leap Cycle (of 365.25 ^{days})	I^{days}	174.666 7	$4 \times D_{2000}^{days}$ mod 1461
13	Phase within Leap Cycle	φ^{rad}	0.7512	$0.004301 \times I^{days}$
14	EoT_1 : 1st Harmonic	EoT_1^{min}	4.6037	$7.3529 \times \sin(1 \times \varphi^{rad} + 6.2085)$
15	EoT_2 : 2ND Harmonic	EoT_2^{min}	9.4774	$9.9269 \times \sin(2 \times \varphi^{rad} + 0.3704)$
16	EoT_3 : 3RD Harmonic	EoT_3^{min}	0.1829	$0.3337 \times \sin(3 \times \varphi^{rad} + 0.3042)$
17	EoT_4 : 4TH Harmonic	EoT_4^{min}	−0.1268	$0.2317 \times \sin(4 \times \varphi^{rad} + 0.7158)$
18	Equation of Time	EoT^{min}	14.1382	$EoT_1^{min} + EoT_2^{min} + EoT_3^{min} + EoT_4^{min}$
19	Longitude Correction	LC^{min}	25.1333	$4 \times (TZ^{hr} \times 15 - \Lambda^{o})$
20	Equation of Time - longitude corrected	EoT_{corr}^{min}	39.2715	$EoT^{min} + \sigma^{min}$

Notes

Accuracy of this example compared to results from NASA-JPL Horizons application

Parameter	This Routine	Horizons	Error	
EoT Longitude Corrected	39.2715 ^{mins}	39.28731 ^{mins}	0.9 ^{secs}	*Accuracy of this routine's calculation of EoT is better than +/- 9 seconds between 2025 and 2075. Thereafter, the accuracy degrades sharply.* *If just 3 Fourier terms are used, accuracy falls to +/- 20 secs.* *If just 2 Fourier terms are used, accuracy falls to +/- 35 secs.*

The code, in Python, used to provide the above calculations in provided for reference : 'Fourier EoT and Declination' on page 216. and also at https://github.com/kevinkarney/Equation-of-Time/

Table 3

return to "Table of Contents"

Meeus' Algorithms

The most complete 'non-professional' algorithms for calculation of the Equation of Time are provided in by the notable astronomical genius, Jan Meeus.

Astronomical Algorithms (1998), 2nd ed-ISBN 0-943396-61-1.

Meeus provides two means to find the position of the sun (and hence the EoT). The simpler one will calculate the EoT to within 4^{secs} over this century, while the more complex algorithms to better than 1^{sec}.

Any reader, who wishes to improve his knowledge of the solar system's dynamics, should obtain a copy of this book. The Python code for Meeus' work is available at

https://github.com/kevinkarney/Equation-of-Time

Other Sources

For more information of the astronomical background of the Equation of Time, see the article which was published in NASS Compendium : Vol 25 Nos 3 & 4, Sept & Dec 2018. (Including some corrections from the published text).

See 'Basic Positional Solar Astronomy : Part 1. Essential Parameters and the Equation of Time' on page 187.

For another useful source see Alan Whitmans 'A Simple Expression for the Equation of Time'

Ref: NASS Compendium Vol 14 Number 2 - Sept 2007

Horizons

JPL (Jet Propulsion Laboratory in the California Institute of Technology), working on behalf of NASA, has produced a freely available program whose routines are used to produce all the significant astronomical publications : for example, the Astronomical Almanac.

It can be found at

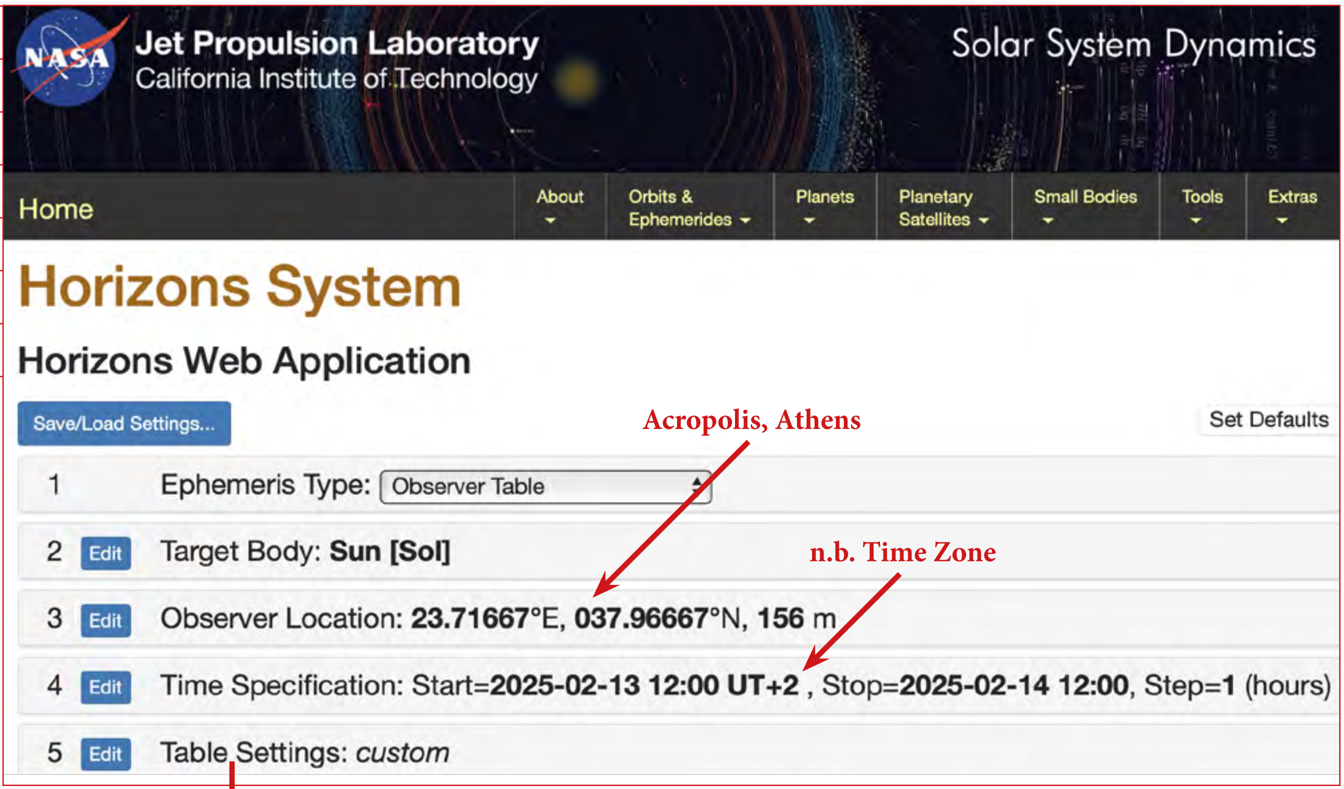
https://ssd.jpl.nasa.gov/horizons/app.html#/

In an early user guide to this program, it states that the user should consult the web-master if using the program for manned planetary landings!

It is a little bit cumbersome to use, at first. But once used a few times, it is quite straight-forward. It is lightening fast, free and the best that there is.

Fig xxx, overleaf, gives an indication of how to set up Horizons is set up. Note the time must be UTC. The quantities, under Table Settings, will output Right Ascension & Declination, Azimuth & Altitude, and Local Solar Time .

return to "Table of Contents"



Horizons System

Horizons Web Application

1 Ephemeris Type: Observer Table

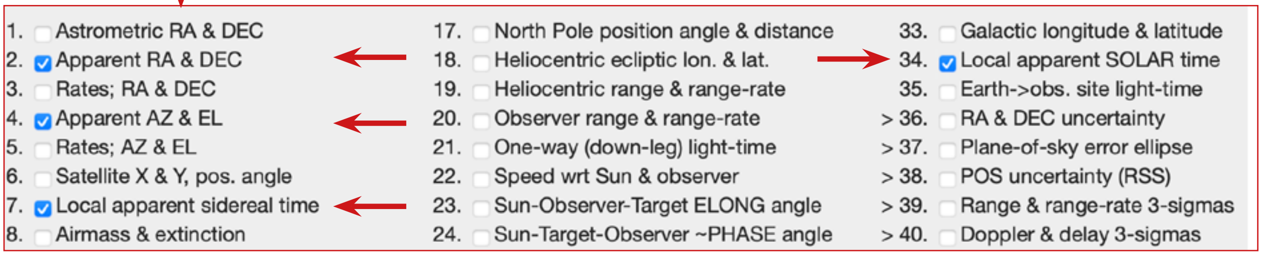
2 Target Body: **Sun [Sol]**

3 Observer Location: **23.71667°E, 037.96667°N, 156** m

4 Time Specification: Start=**2025-02-13 12:00 UT+2** , Stop=**2025-02-14 12:00**, Step=**1** (hours)

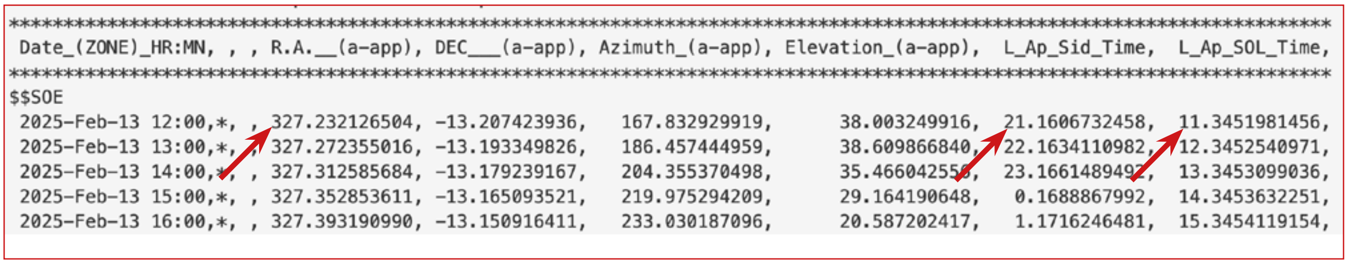
5 Table Settings: *custom*

Fig 4 - Input for the Athens Example used elsewhere in this and the previous chapter



1. ☐ Astrometric RA & DEC
2. ☑ Apparent RA & DEC
3. ☐ Rates; RA & DEC
4. ☑ Apparent AZ & EL
5. ☐ Rates; AZ & EL
6. ☐ Satellite X & Y, pos. angle
7. ☑ Local apparent sidereal time
8. ☐ Airmass & extinction
17. ☐ North Pole position angle & distance
18. ☐ Heliocentric ecliptic lon. & lat.
19. ☐ Heliocentric range & range-rate
20. ☐ Observer range & range-rate
21. ☐ One-way (down-leg) light-time
22. ☐ Speed wrt Sun & observer
23. ☐ Sun-Observer-Target ELONG angle
24. ☐ Sun-Target-Observer ~PHASE angle
33. ☐ Galactic longitude & latitude
34. ☑ Local apparent SOLAR time
35. ☐ Earth->obs. site light-time
\> 36. ☐ RA & DEC uncertainty
\> 37. ☐ Plane-of-sky error ellipse
\> 38. ☐ POS uncertainty (RSS)
\> 39. ☐ Range & range-rate 3-sigmas
\> 40. ☐ Doppler & delay 3-sigmas

Fig 5 - Table Setting required



```
*******************************************************************************************************************
 Date_(ZONE)_HR:MN, , , R.A.__(a-app), DEC___(a-app), Azimuth_(a-app), Elevation_(a-app),  L_Ap_Sid_Time,  L_Ap_SOL_Time,
*******************************************************************************************************************
$$SOE
 2025-Feb-13 12:00,*, , 327.232126504, -13.207423936,   167.832929919,      38.003249916, 21.1606732458,  11.3451981456,
 2025-Feb-13 13:00,*, , 327.272355016, -13.193349826,   186.457444959,      38.609866840, 22.1634110982,  12.3452540971,
 2025-Feb-13 14:00,*, , 327.312585684, -13.179239167,   204.355370498,      35.466042556, 23.1661489492,  13.3453099036,
 2025-Feb-13 15:00,*, , 327.352853611, -13.165093521,   219.975294209,      29.164190648,  0.1688867992,  14.3453632251,
 2025-Feb-13 16:00,*, , 327.393190990, -13.150916411,   233.030187096,      20.587202417,  1.1716246481,  15.3454119154,
```

Fig 6 = Output from the above input

The marked line for 12 noon Athens time = UTC 10:00 ^{hrs} on 13th Feb 2025

gives Longitude Corrected EoT = (Local Time ^{hrs} – L_Ap_SOL_Time ^{hrs}) = (12^{hrs} – 11.3451981456 ^{hrs}) = 39.288^{mins}

One may also calculate the Longitude Corrected EoT = R.A.° ÷ 15 $^{°/hrs}$ – L_Ap_Sid_Time^{hrs} = 39.2873^{mins}

60 × (RA_deg/15 –Local_Apparent_Sidereal_Time_hrs – 12 + Hour)

60 × (Local_Standard Time_hrs –Local_Solar_Time_hrs – 12)

60 × (Local Standard Time_hrs – Local Hour Angle_hrs)

(The asterisk in Col 2 indicates that the refracted upper limb of the sun is above the horizon)

return to "Table of Contents"

Quoted Accuracies

Most quoted EoT calculations (including those in this book) are for geocentric EoT (the sun's position as 'seen' from the centre of the Earth). These are are relatively easy to calculate.

NASA/JPL's Horizons program calculations give topocentric EoT (the sun's actual position as seen from a particular location).The difference between the two is small - a few seconds - see 'Minor Changes : Topographical -v- Geocentric' on page 50.

Accuracies of calculated (geocentric) EoT - in this and the previous chapter - are compared with (topocentric) values for Greenwich observatory. Thus introducing those few seconds uncertainty.

Meaaurement -v- Prediction

As an aside (for pedants, only), Fig 7 compares...

> Horizons results with runs made in 2025 with knowlege of leap seconds, differences between atomic and astronomical time (UT1), and polar movements up to that date
>
> those predicted by the full implementation of Meeus

This suggests that no future predications of astronomical events will ever be better than some half second

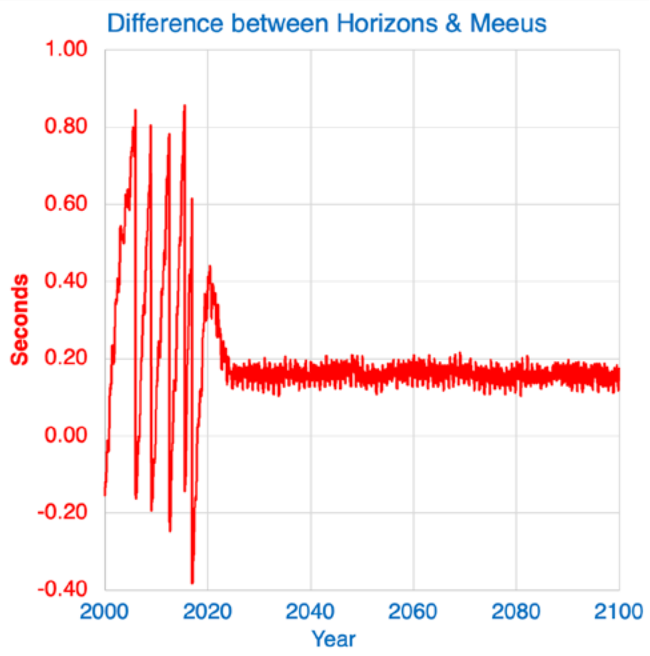


Fig 7 - Difference between Horizons and Meeus predictions. Up to 2025, measured variables are used in Horizons: thereafter fixed values are taken for the unpredictable

For the Pendant.

Precise prediction of the position of the Sun (and hence the EoT) requires consideration of any number of other (mostly minor) variables...

(i) aberration - the time light takes to travel from the Sun, which there is since the Earth travels in an elliptical orbit;

(ii) precession - the slow gyroscopic movement of the Earth's axis, primarily as the result of the gravitational influence of 'the gas giant' planets (Jupiter and Saturn) acting on the Earth's equatorial bulge;

(iii) nutation in both solar longitude and latitude - the wobbling of the Earth's axis under the influence of the Moon and other planets;

(iv) polar movement - probably caused by convectional magma movement in the core of the Earth;

(v) slowing of the Earth's rotational period (the day) - caused by tidal friction;

(vi) long-term changes in the Earth's eccentricity and obliquity.

Definitions for the Pedant.

Formally speaking, UTC should read UT1 and the Epoch that is in terms of UT1. Likewise T is measured the Epoch that is in terms of Terrestrial Time. The difference is insignificant in this context. Thus for the pedantic only the following definition has been adopted by the International Astronomical Union.

Ref. Chapter 2.1 - 2.6 of The IAU Resolutions on Astronomical Reference Systems, Time Scales, and Earth Rotation Models

where D = number of days since Epoch^{UT1}

T = number of Julian Centuries $^{(of\ 36525\ days)}$ since Epoch^{TT}

and Epoch is Noon on 1st January 2000

The difference between UT1 & TT is only relevant at the microsecond level.

The Analemma

History

The word 'analemma' comes from the Greek αναλήμμα meaning a prop or support. It was used in antiquity for graphic diagrams that helped solve all sorts of problems in spherical geometry. The analemma herein only relates to the EoT -v- Declination rendition and not to earlier works of e.g. Vituvius who were discussing projections onto a plane surface,

Reference 'The Equation of Time : The Invention of the Analemma' : Christopher St J H Daniel : BSS Monograph No 1

The analemma in its usual form was first described either by Johann Philipp von Wurzelbau ca: 1715 or by the French astronomer Jean-Paul Grandjean de Fouchy, some 15 years later. The first published work on the analemma was by the mathematician, Antoine Deparcieux. He describes Fouchy's analemma, which appears to be an 'add-on' to the many meridian lines that astronomers loved.

Analemma Basics

The Analemma is another way to display the Equation of Time. However, instead of plotting EoT -v- Date, the Analemma is plotted EoT -v- Solar Declination.

Fig 1. Jean-Paul Grandjean de Fouchy (1707-1788)

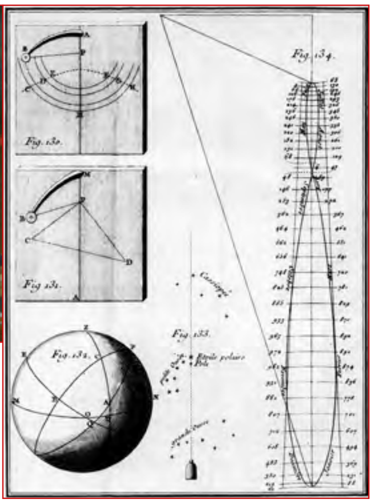
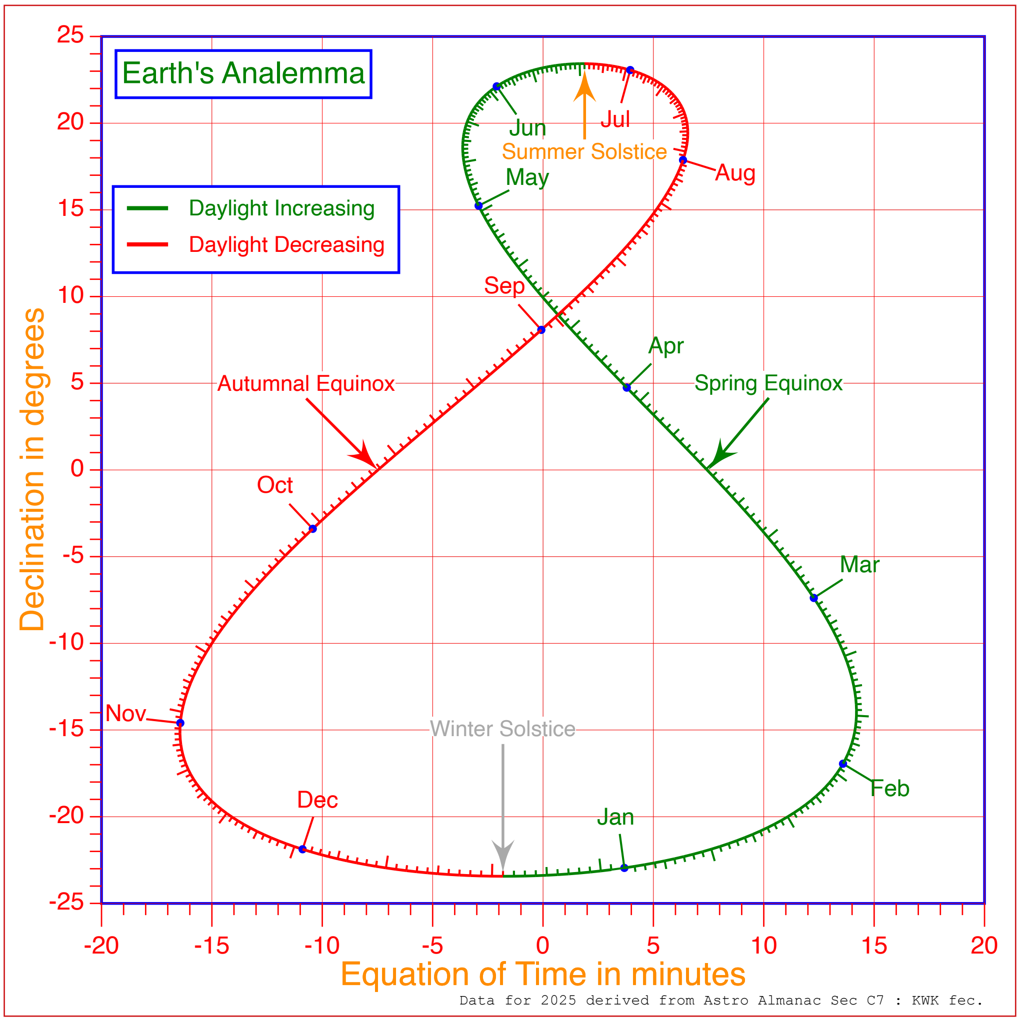


Fig 2 - Deparcieux 1741 Nouveaux Traités de Trigonometrie

Its prime use is either as the marking on a sundial, converting solar to mean time or as a noon marker

Since the Declination is tied to the Date, the analemma can be used to indicate the date (as in the declination lines on any sundial).



Data for 2025 derived from Astro Almanac Sec C7 : KWK fec.

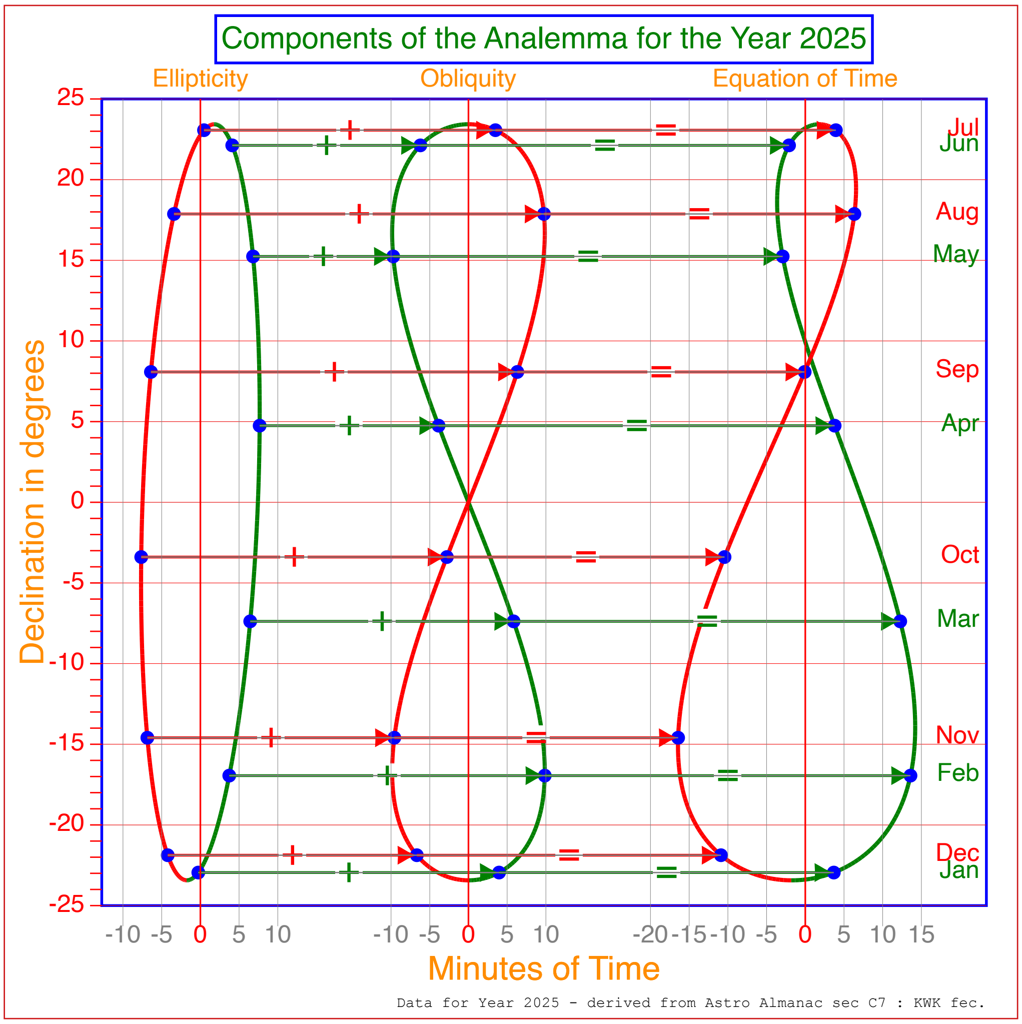
Fig 3 - The Analemma

return to "Table of Contents"

As with the conventional Equation of Time, the Analemma can be constructed the two components of Ellipticity and Obliquity.

Notice that the ellipticity 'oval' is slightly 'bent' - since its position is related to the date of perihelion rather than the 1st Point of Aries.

The shape of the Obliquity figure-of-eight is more-or-less symmetrical and changes very slowly over time. On the other hand, the ellipticity oval changes more rapidly, as a result of precession.



Data for Year 2025 - derived from Astro Almanac sec C7 : KWK fec.

Fig 4 - The two components of the Analemma

I am the Equation-of-Time,
Well known for my 8-shaped line.
Though not very much,
The correction is such
There's no need for a clock with a chime.

Fig 5 - Second prize in the Limerick Competition
during the BSS field trip to the Loire Region.
Author forgotten...

return to "Table of Contents"

VARIATION IN THE ANALEMMA OVER TIME

Fig 6 shows the continuous variability of the Analemma over 20,000 years. This is virtually unreadable since the processional variation covers almost one cycle of 26,000 years.

For clarity, Fig 7 & 8 show it split into BCE and CE.

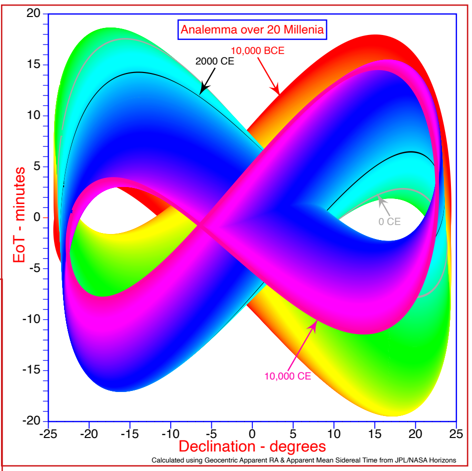


Fig 6 - The Analemma over 20,000 years

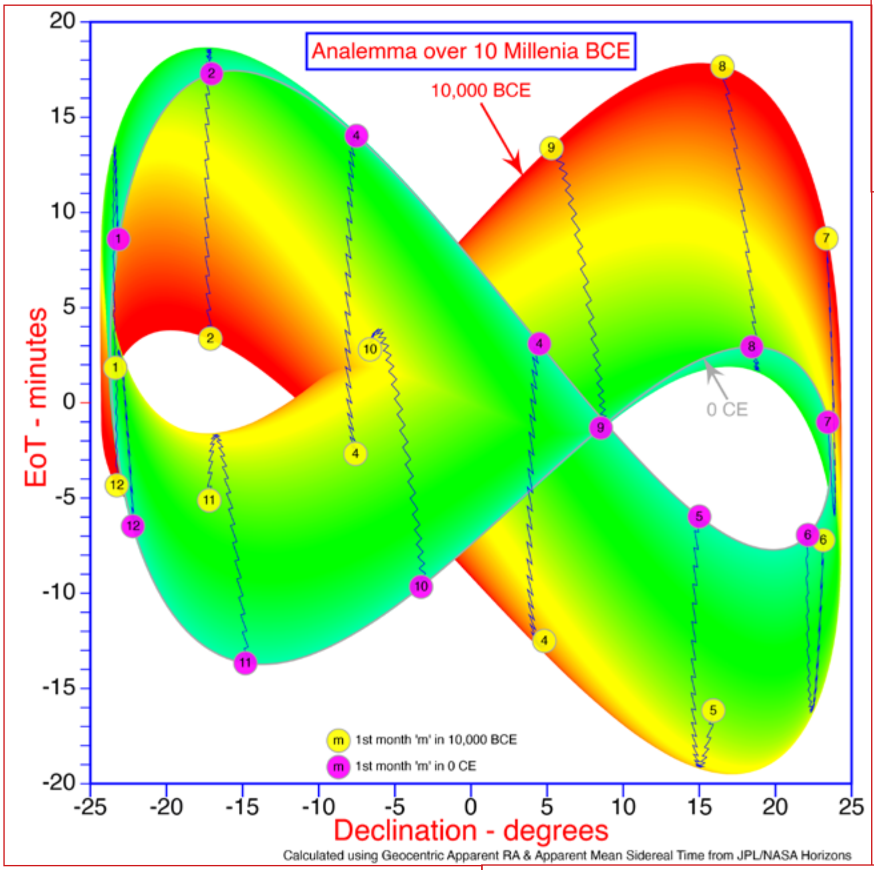


Fig 7 - The Analemma over 10,000 years BCE

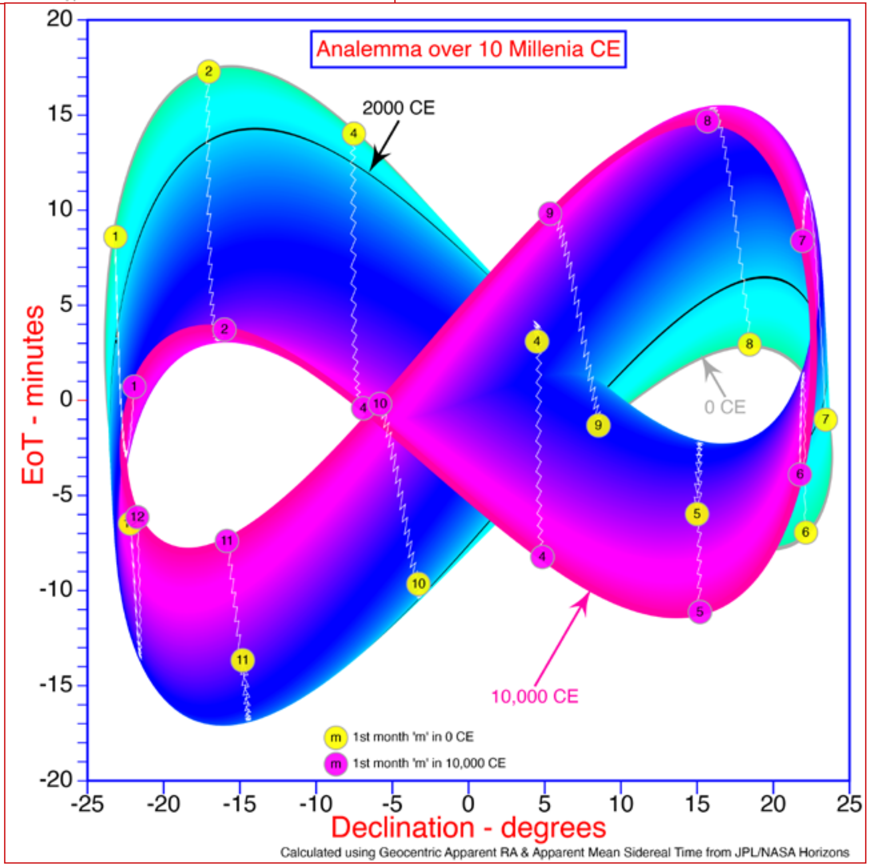


Fig 8 - The Analemma over 10,000 years CE

NOON MARKS

The Analemma Noon Mark became common as-soon-as churches had clocks. The only way to set a clock without reference to time signals or astronomers was to use a noon mark (or any other sundial).

Fig 9 - St Martins Church, Colmar.
Noon Mark (bottom left) used to set the clock just above it

Fig 10 - Urbain Adam - 19C - St Martins, Colmar.
The black line marks solar noon, the yellow mean noon

The extraordinary Noon Mark - in Figs 11 & 12 - was designed by Dr Frank King. It has daily 366 daily strips. Over the four-yearly leap cycle, the noon light spot will follow each strip in four minutes. 29th Feb has a thin strip, as it is only covered ever 4 years. This is probably the only sundial in the world which can correctly pick out noon on 29th February.

Fig 11 - Noon on 29th February

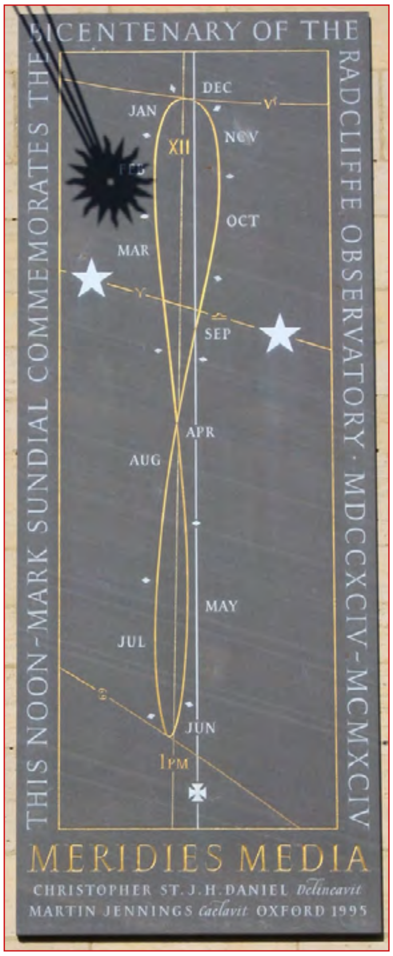


Fig 12 -C. St J H Daniel -1995
Radcliffe Observatory, Oxford

Fig 13 -Frank King & Kristi Shea- 2003
- Stock Exchange London
(photo by Alfred Yeung)

The gnomon support was designed by the Engineering Faculty in Cambridge University and is designed to be temperature stable and withstand the most violent winds.

return to "Table of Contents"

The Analemma on Sundials

'Part 2 - A Picture Book of Equation Corrected Sundials' on page 117. for many, many other examples of the analemma on sundials.

Fig 14 -Erich Pollähne's improved Benoy-Type sundial.
Note how, instead of a shadow,
the time is told by the cusp of light

Fig 15 - Cartoonist Tim Hunkin on Sundials

The Analemma on Objects

Fig 16 - Programmable ceiling light Analemma
by Makendo.
The bright LED indicates the date.

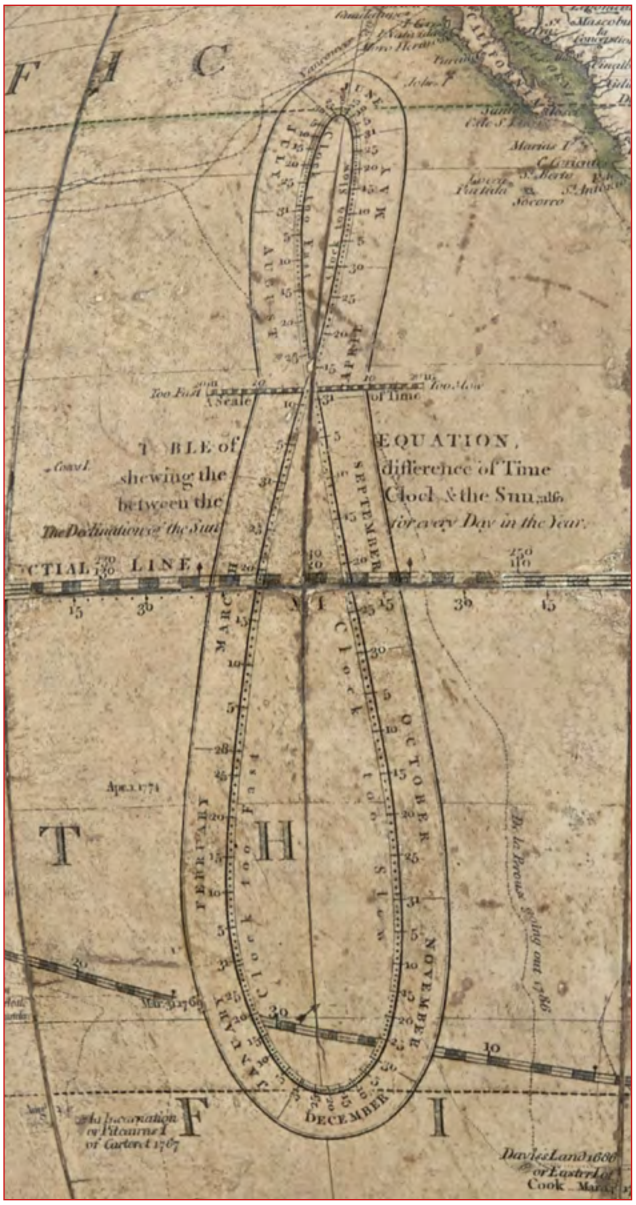


Fig 17 - Cary's New Terrestrial Globe - after 1791

University of Southern Maine
www.oshermaps.org/map/2329.0001

Ref. 'Analemmas' on Globes' - Matthew H. Edney

return to "Table of Contents"

THE ANALEMMA IN THE SKY

By taking a series of photographs throughout the year, with the camera in a fixed position, and at the same ***mean*** time each day, an analemma is drawn in the sky....

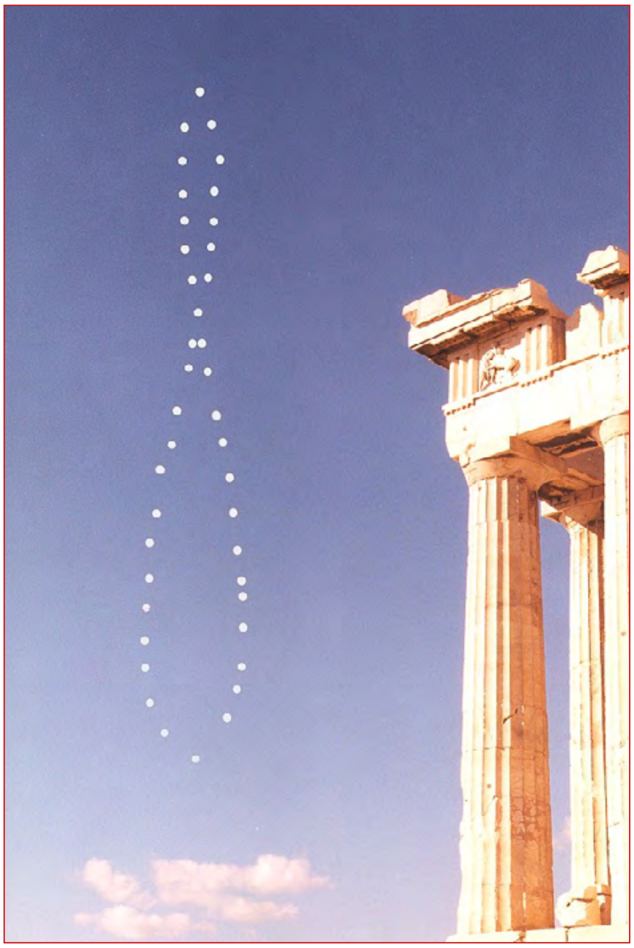


Fig 18 - Mean Noon Analemma in Athens
by Anthony Ayiomamitis

Fig 19 - Analemma over Austria
by Robert Pölzl

Fig 20 - Half Analemma in the Summer at the South Pole
by Adrianos Golemis

Fig 21 - A Pseudo Analemma of the Moon over Turkey
by Betul Turksoy

return to "Table of Contents"

THE MARS ANALEMMA

In 2005, the a Rover named Opportunity landed on Mars. For 14 years, it traversed the planet's surface covering 28 miles and taking more than 200,000 images. At least once a day, the image was of its own sundial and its camera's colour calibration swatches.

The sundial, designed by Prof. Woody Sullivan, provided a means of orienting the Rover should its other navigation computers fail. The surface of the dial showed evidence of both ice and wind-blown sand.

Thousands of the sundial images were processed by Jashandeep Sohi to produce the first measured planetary analemma!

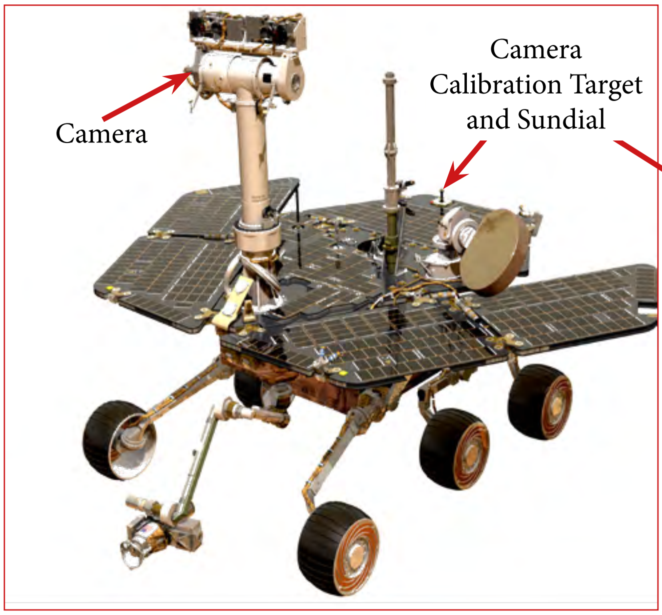


Fig 22 - The Mars Rover

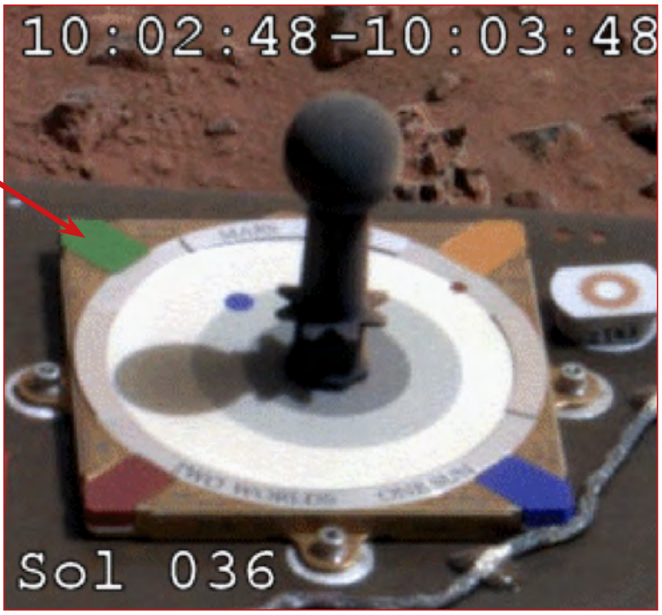


Fig 23 - The Sundial and the Camera's Colour Calibration Swatches

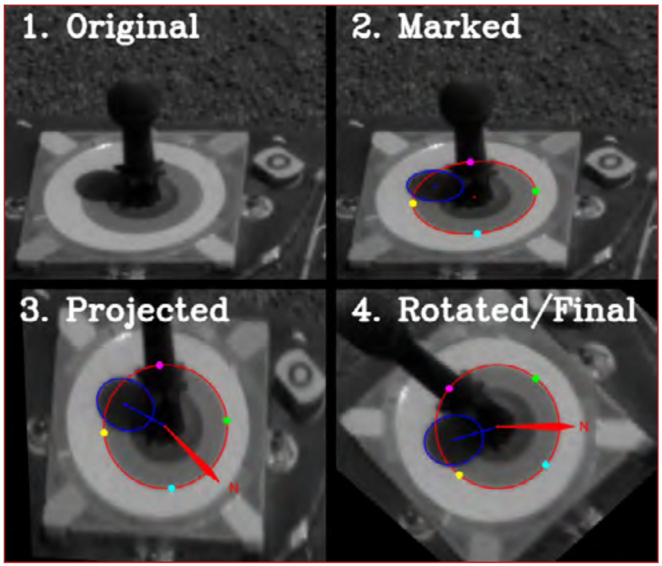


Fig 24 - Images of the Sundial were processed to provide the single spot in the middle of the blue circle

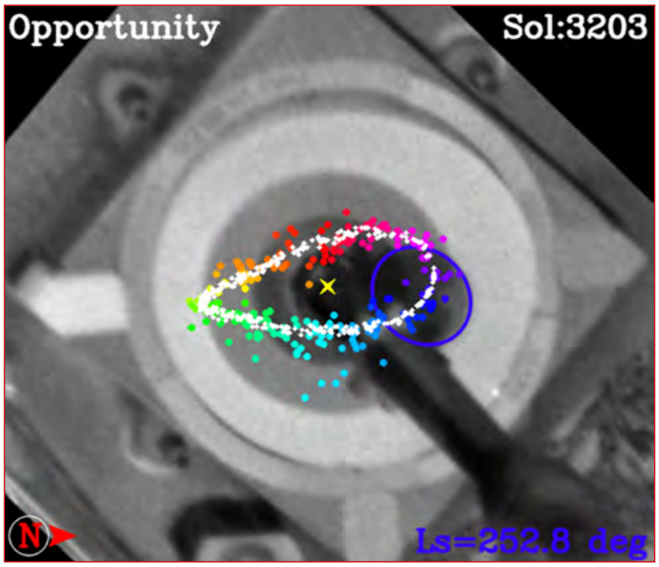


Fig 25 - Thousands of these were combined to provide the coloured-dot analemma. The white spots there those expected. The difference attributed mainly to the uneven terrain in which the Rover was operating

Fig 26 - Nasa's pseudo image of Mars' analemma

return to "Table of Contents"

ANALEMMAS ON OTHER PLANETS

Since all the planets have some degree of Obliquity and Eccentricity, each has its equinoxes and its own analemma.

Figures 28 to 30 show the analemmas of:

- the majority of planets, all of which have similar parameters;
- Uranus and Pluto, each of which has extreme obliquities and eccentricities;
- Venus with its tiny obliquity and almost circular orbit.

Note that these figures are independent of the time of the planetary day, but simply time measured in angular change from the planet's equinox.

Only 4 parameters are required to calculate an analemma.

- the eccentricity which is broadly see in the width of the curve;
- the obliquity of orbit, which broadly effects the height of the curve;
- the longitude of perihelion (measured from 1st Point of Aries)
- the 'longitude of the ascending node' (measured from 1st Point of Aries) - see Fig 27..

Whether an analemma has a 'twist' in it - as in Earth, Saturn and Neptune - depends of the relative position of perihelion to the equinox.

The illustrated analemmas were all computed using the same algorithm shown in 'Kepler's Method' on page 25.. But with the one conversion: the Perihelion Longitude ω^{o,} used in Earthly calculations was replaced by Ω° - π°, see Fig 27. The values of these parameters were obtained from NASA's data sheets for the Epoch.

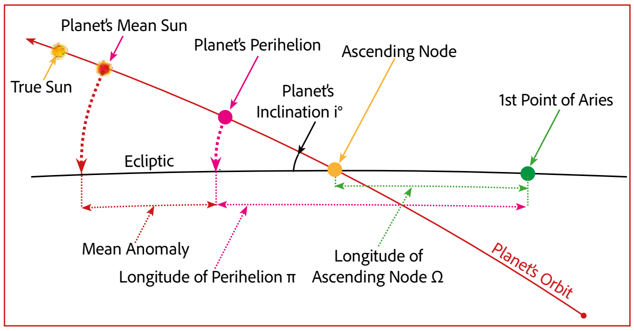


Fig 27 - Planet's Orbital Parameters

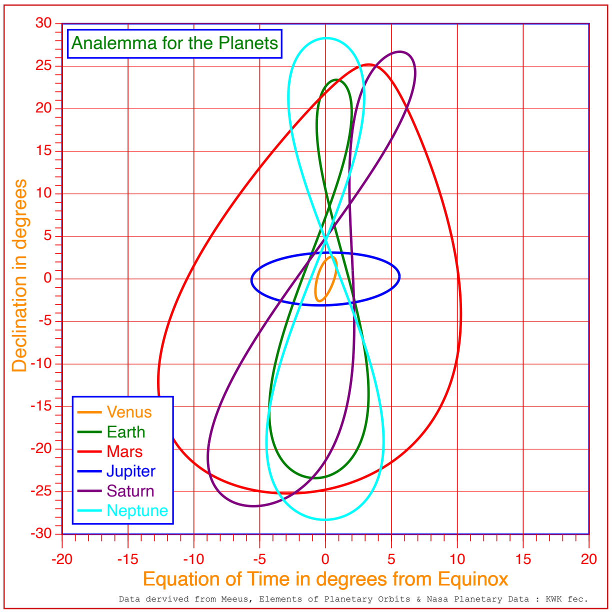


Fig 28 - Analemmas for the Planets. Note that, for the Earth, 1^{o} is equivalent to 4 ^{minutes}; while for Pluto, 1^{o} is 8 ^{months}.

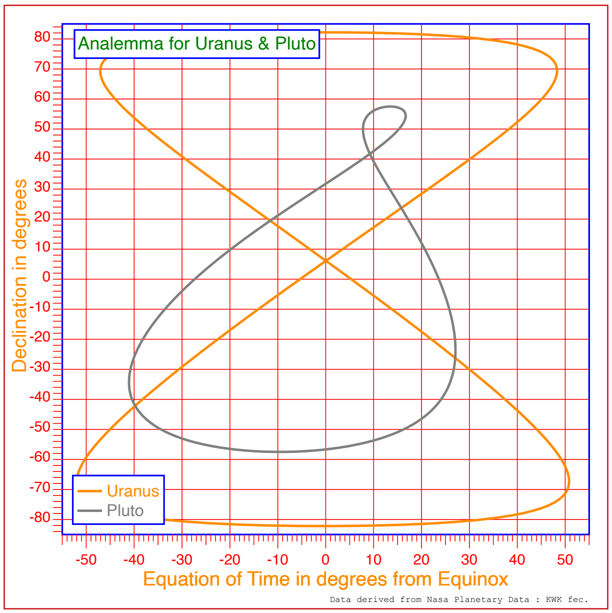


Fig 29 - Analemmas for Uranus and Pluto

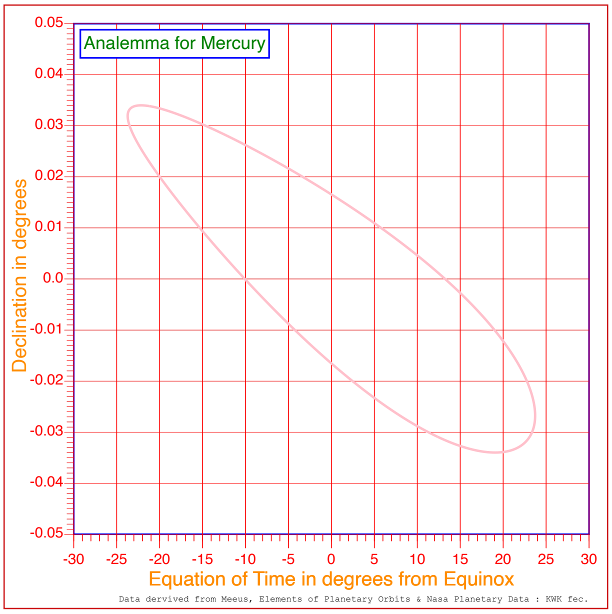


Fig 30 - Analemma for Venus

return to "Table of Contents"

DELINEATING AN ANALEMMA

To delineate an Analemma on vertical south/north facing wall is relatively easy. One just has to be able to calculate the Equation of Time and the Declination on any day.

On any other plane surface, it is more complicated. This can be done using the method created by Denis Savoie and Robert Sagot of Commission des Cadrans Solaire.

$\varphi^{radians}$	Latitude of the place.
$D^{radians}$	Plate Declination from the southern meridian (0° south facing, 90° west, 180° north, 270° east).
$z^{radians}$	Azimuth Distance from vertical (0° horizontal, 90° vertical).
a	Length of a vertical style (perpendicular to the plate)
x,y	Position of the tip of the style's shadow in an orthogonal coordinate system with origin at the foot of the style. (x-axis +ve right horizontal, y-axis +ve up along line of greatest slope).
$H^{radians}$	Hour angle from solar noon (-15° at 11 a.m. solar time, 0° at noon, +15 at 1 p.m. etc.). For an analemma, the Hour angle calculations must include the Equation of Time together with the Longitude correction. If not, the calculations will provide a standard straight line solar sundial.
$\delta^{radians}$	Sun's declination.

From these, calculate...

$$\begin{aligned}
P &= \sin\varphi \times \cos z - \cos\varphi \times \sin z \times \cos D\\
Q &= \sin D \times \sin z \times \sin H\\
&\quad + (\cos\varphi \times \cos z + \sin\varphi \times \sin z \times \cos D) \times \cos H\\
&\quad + P \times \tan\delta\\
N_x &= \cos D \times \sin H\\
&\quad - \sin D \times (\sin\varphi \times \cos H - \cos\varphi \times \tan\delta)\\
N_y &= \cos z \times \sin D \times \sin H\\
&\quad - (\cos\varphi \times \sin z - \sin\varphi \times \cos z \times \cos D) \times \cos H\\
&\quad - (\sin\varphi \times \sin z + \cos\varphi \times \cos z \times \cos D) \times \tan\delta\\
x &= a \times N_x / Q\\
y &= a \times N_y / Q
\end{aligned}$$

There are two additional requirements that ***must*** be met.

1 if Q is negative the sun is behind the plate so will not illuminate the dial, Note that on any given date, Q can be positive, then negative and become positive again.

2 the sun must (of course!) be above the horizon to illuminate the plate, thus one must calculate the value of H at sunrise/set...

$$H_{sunrise/set} = \pm\cos^{-1}(\tan\varphi \times \tan\delta)$$

and check that H is during daylight hours.

The geometry above provides additional information. The coordinates of the foot of a polar stylus passing through the nodus (the dial centre) are ...

$$\begin{aligned}
x_0 &= + a \times \cos\varphi \times \sin D \div P\\
y_0 &= - a \times (\sin\varphi \times \sin z + \cos\varphi \times \cos z \times \cos D) \div P
\end{aligned}$$

The length of the style, u, from centre to nodus is...

$$u = a \div |P|$$

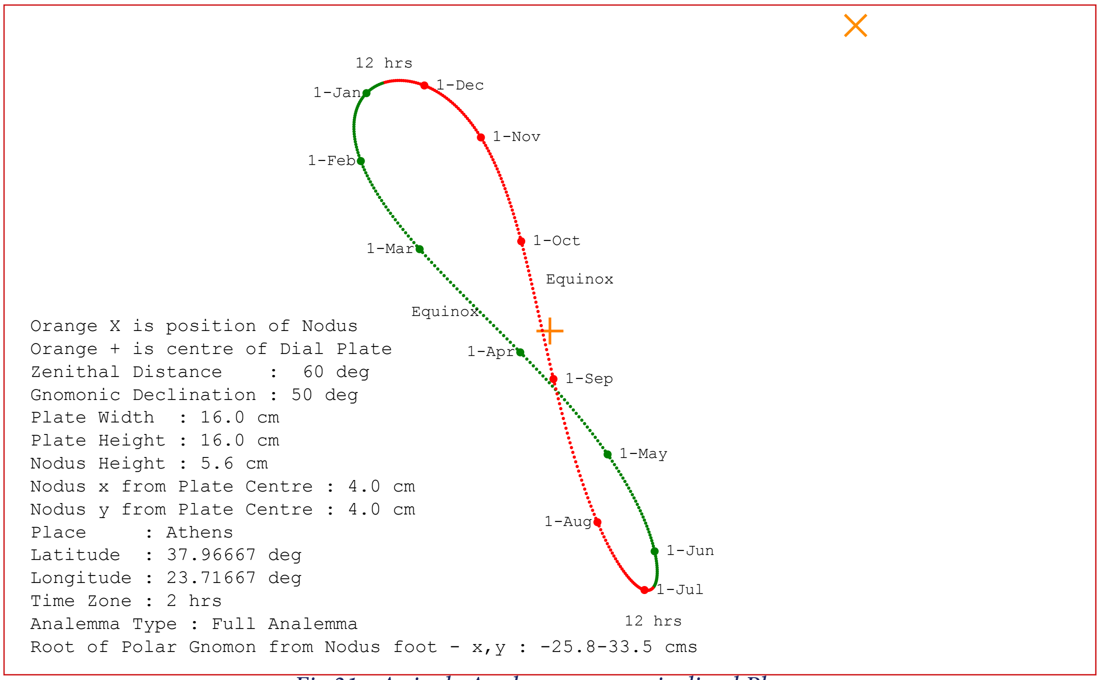
The angle ψ, which a polar stylus makes with the plane is ...

$$\psi = \sin^{-1}|P|$$

To create a usable sundial, one must iterate the above calculations across the year (changing δ) for each hour line required.

If you wants declination lines, one must iterate across each hour (i.e. changing H) for each date required.

The code to perform these calculation in Python is provided in 'Analemma Plotting' on page 221.



```
Orange X is position of Nodus
Orange + is centre of Dial Plate
Zenithal Distance     :  60 deg
Gnomonic Declination : 50 deg
Plate Width   : 16.0 cm
Plate Height : 16.0 cm
Nodus Height : 5.6 cm
Nodus x from Plate Centre : 4.0 cm
Nodus y from Plate Centre : 4.0 cm
Place       : Athens
Latitude   : 37.96667 deg
Longitude : 23.71667 deg
Time Zone : 2 hrs
Analemma Type : Full Analemma
Root of Polar Gnomon from Nodus foot - x,y : -25.8-33.5 cms
```

Fig 31 - A single Analemma on an inclined Plane

ANALEMMAS AS SUNDIALS.

The noon mark is a single analemma drawn proidr thr time of mean noon. These were widely used to set clocks to mean time. For examples, 'Curved Hour Lines Sundials' on page 137.

The problem in using analemmas for other sundials is shown in Figs 32 & 33. The lines hour-lines must either be sparse or they will overlap. In which case, the dial is hardly readable.

The solution to this problem is to split the analemma into to two halves, as shown in Fig 33 below and Figs 35 and 36 overleaf. One half covers the 'days-lengthening' period (winter solstice to summer solstice) : the other to the 'days shortening' period (summer solstice to winter solstice).

Many other examples of the analemma used in sundials are given 'Curved Hour Lines Sundials' on page 137.

Fig 32 - Rafel Soler i Gayà - 1988 - Tarragona

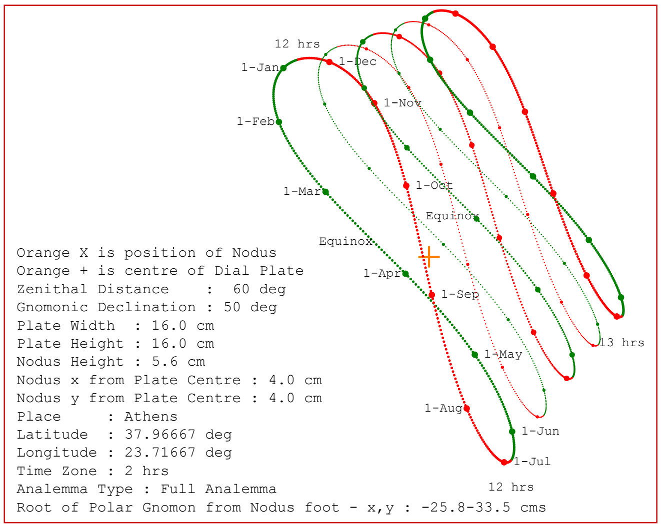


Fig 33 - Even 15 minute hour-lines will overlap.

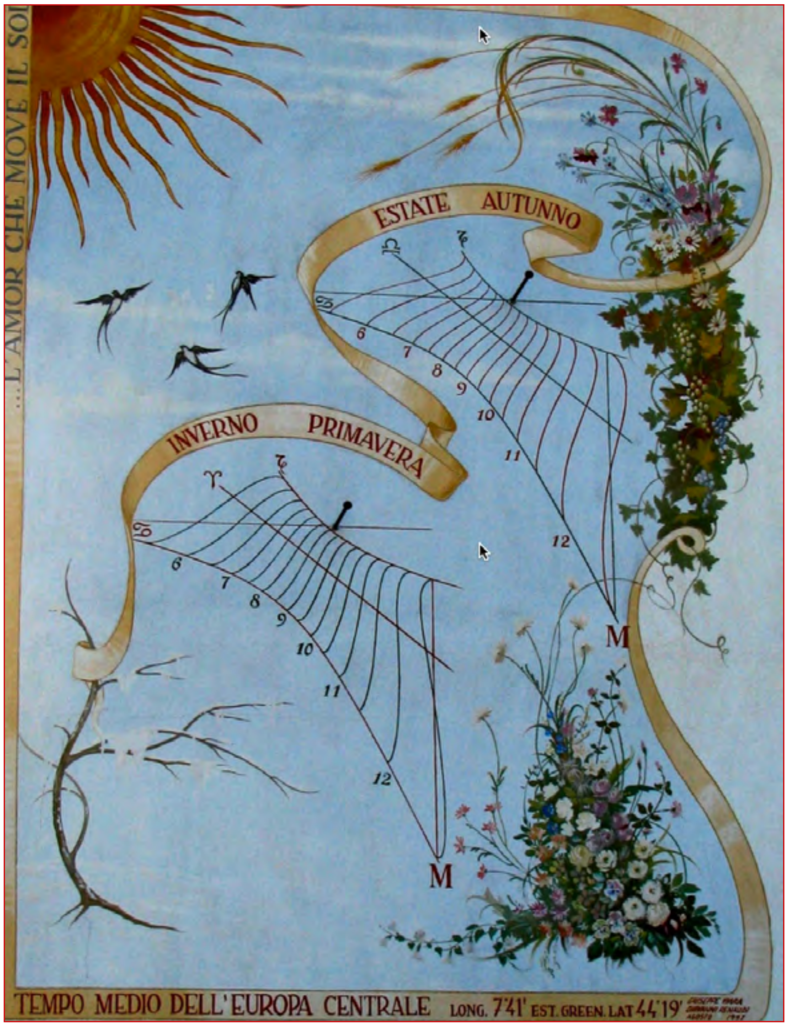
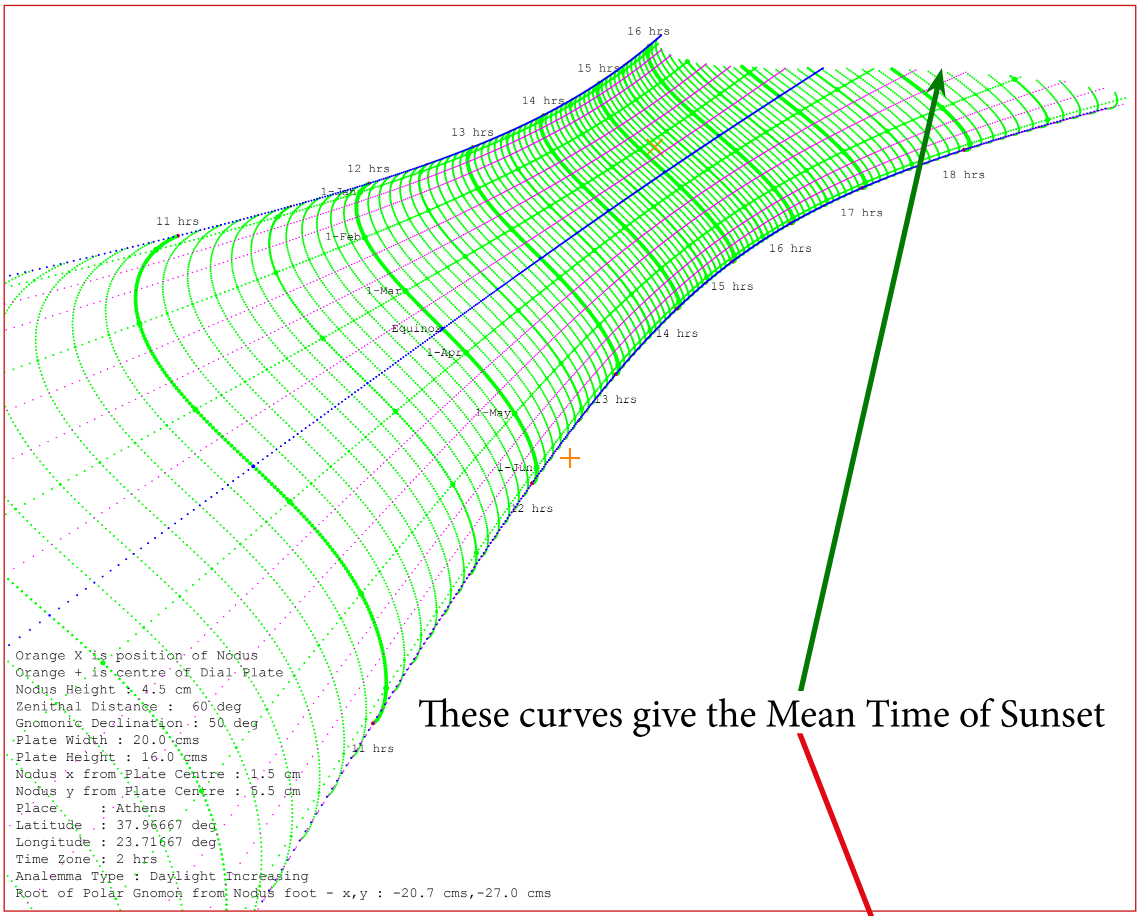


Fig 34 - The exquisite dial of Giuseppe Viara & Giovani Renaudi
Chiusa di Pesio Nr Turin - 1997

return to "Table of Contents"



These curves give the Mean Time of Sunset

Fig 35 - Daylight Increasing - Winter and Spring

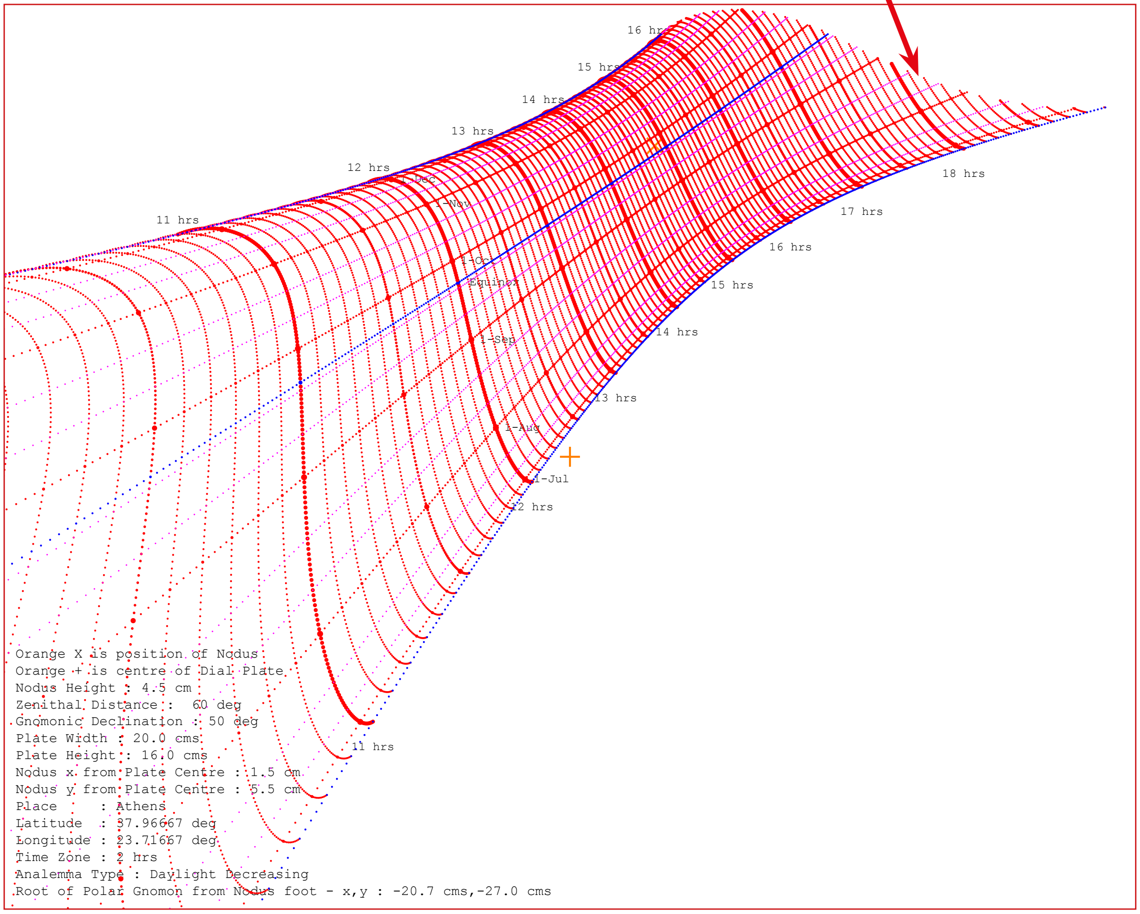


Fig 36 - Daylight Decreasing - Summer and Autumn

Calculating the coordinates of a planar analemma dial is complicated. The computer code used to produce Figs 30, 32, 34 & 35 is provided in 'Analemma Plotting' on page 221.

return to "Table of Contents"

SYMMETRICAL ANALEMMAS

There have been many times when the analemma has been symmetrical in shape. The most recent has been in 1246 CE when Perihelion coincided with the winter solstice, see Fig 37.

In the much longer term, there are many other symmetrical shapes that the analemma can take when the eccentricity of orbit changes, or when perihelion coincides with the equinoxes or solstices.

Werner Riegler - The Shape & Topology of the Analemma
NASS Sep 2022 29(3): 40-51

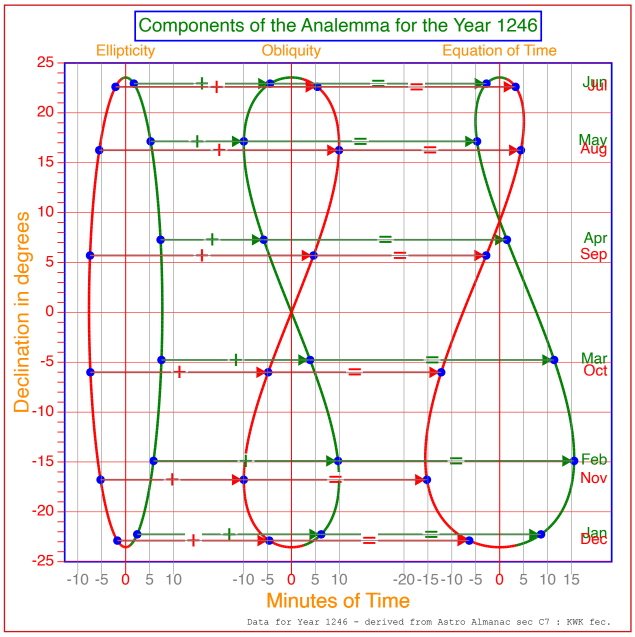


Fig 37- The symmetrical analemma of 1246 CE

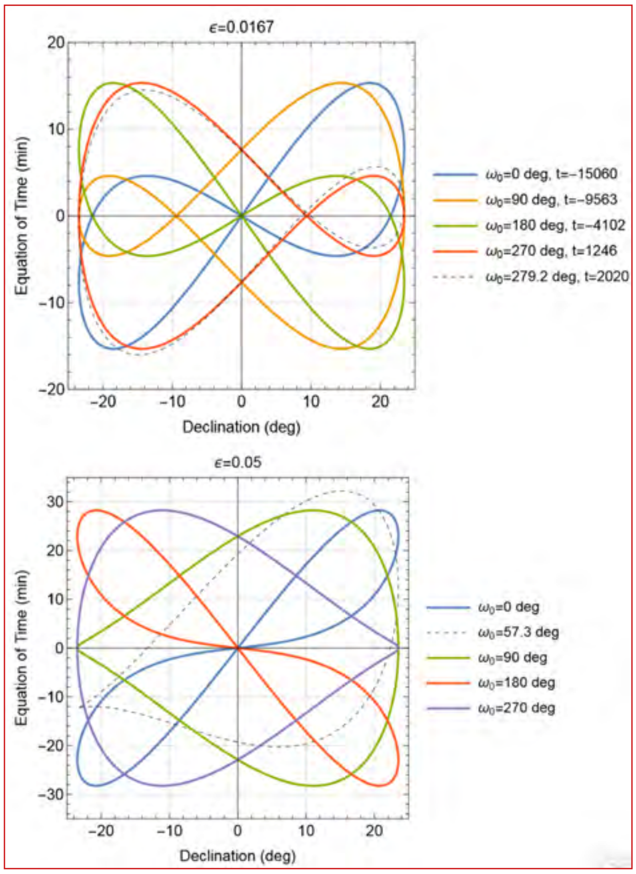


Fig 38 - Symetrical Analemmas for the Earth

The general expression from this work - see Fig 39 - allows the EoT and declination to be easily approximated for the earth or any planet.

Fig 38 show examples for the Earth at its current eccentricity and at a much elevated level. When such conditions are actually achieved and be deduced from

A Berger - Long-Term Variations of the
Earth's Orbital Elements - 1975

A graph from which can be found in the next chapter.

See 'Fig 17 - Over 2 million years, the parameters that govern the shape of the EoT change in a somewhat irregular pattern but see Footnote.' on page 52.

if...

t = time since Perihelion in the year in question

T = Year (+ve CE, -ve BCE)

ε = Eccentricity of orbit

$\varphi^{radians}$ = Obliquity of orbit

$\omega_o^{radians}$ = Ecliptic longitude of Perihelion

then...

$M^{radians} = 2 \times \pi \times t \div T$

$A_{ecc} = 2 \times \varepsilon$

$A_{obl} = \varphi^2 \div 4$

and...

$EoT^{radians} \approx -A_{obl} \times \sin\{2 \times (M + \omega_o)\} + A_{ecc} \times \sin(M)$

$Decl^{radians} = \varphi \times \sin(M + \omega_o)$

and...

$EoT^{mins} = 4 \times 360 \div (2\pi)$

$Decl^{deg} = 360 \div (2\pi)$

Fig 39 - Riegler's Method to estimate EoT and Declination for any Planet

return to "Table of Contents"

Variation in the Equation of Time

PREAMBLE

The Longitude Effect - the difference between one's longitude and that of the time zone meridian - is not, sensu stricto, a variation in the EoT, but it should always be considered. See 'The Longitudinal Effect' on page 1.

In 1683, the renowned clockmaker Thomas Tompion created an Equation table for his clocks. However, 12 years later, the first Astronomer Royal, John Flamsteed, wrote to Sir Isaac Newton that "Tompion's true table of equations, made for a particular year perhaps, fits not the present!". No one wishes to face such critism.

The Equation of Time reflects the Earth's position relative to the Sun, but the heavens are never static. While the Sun's gravitational pull is the primarily influence in the Earth's dynamics, other factors come into play. These include the uneven shape of the Earth's sphere, the position of the Moon, and other planets, especially the massive Jupiter. In the short term, such perturbations have minimal effect on the Equation of Time, at least within a century. Thus, the subject of EoT variation is of little practical interest to makers of clocks or sundials, except for those evaluating the design of instruments made centuries ago.

Fig 1 - shows this small difference over 50 years at the times when the EoT is at its maximum and minimum.

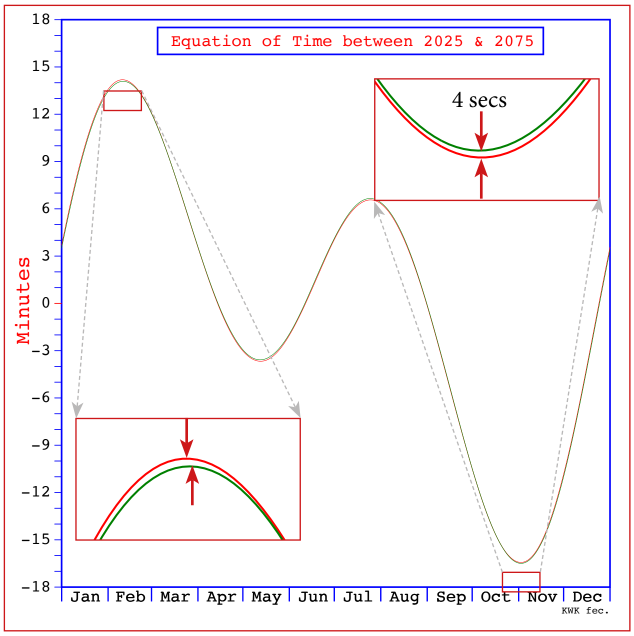


Fig 1 - Chart showing the minor difference in the Equation of Time over the expected life of most clocks and sundials

However, the Equation of Time matters to makers of astronomical clocks and orreries, especially when it comes to the "Clock of the Long Now." This remarkable piece of engineering is covered in some detail on page 159 of "Coming Full Circle: The Clock of the Long Now: Gears & Cam."

MAJOR COMPONENTS OF CHANGE

The calculation of the equation of time involves four primary variables that determine its shape in any given year. These are, in order of importance:

- Date/time of Perihelion - changing by 1.7° towards the mean Vernal Equinox during this century. The earth rotational axis is moving against the celestial background like a gyroscope that is slowing down. This called Precession and has a period of some 26,000 years. As a result, the First of Aries – the moment of the Vernal Equinox and the starting point for astronomical angular measurement – has shifted from its original historical location in the constellation of Aries to that of Pisces.
- Eccentricity of Earth's orbit - changing slowly from 0.016 700 64 to 0.016 744 47 during this century.
- Earth's obliquity ranges between 22.1° & 24.5° in a 41,000 year cycle, mainly under the influence of Jupiter. During this century, it is changing slowly from 24.436 808° to 24.449 957°.

These long term changes over many millennia are shown in the figure below, which is almost unreadable, since later half (during the Common Era) the overlaps the previous half (BCE).

Figs 3 & 4 split these graph into two halves : BCE and CE. In the first, the trend with time in downward. In the second, it is upwards.

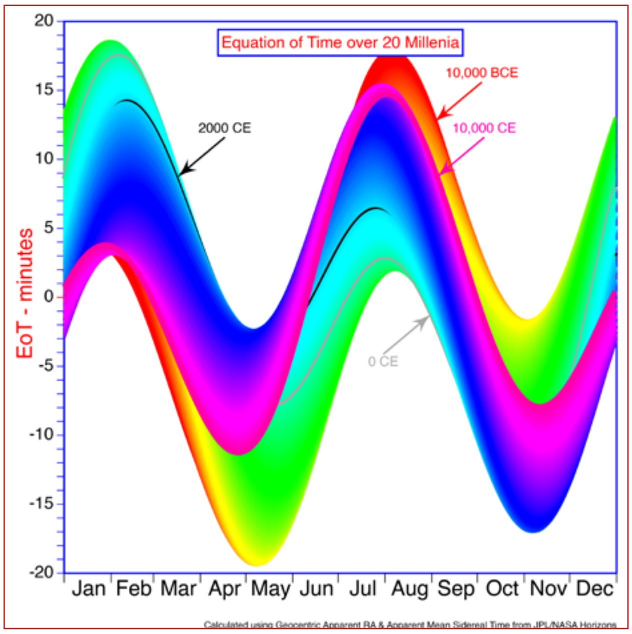
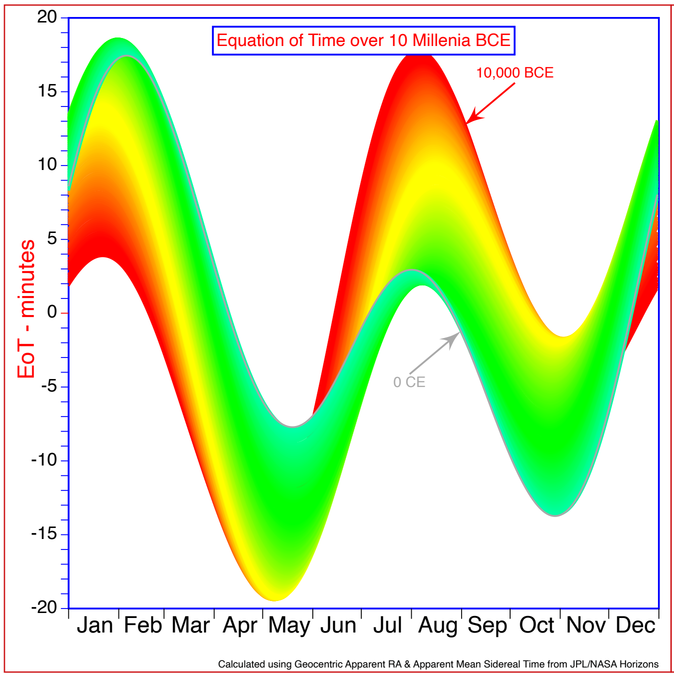
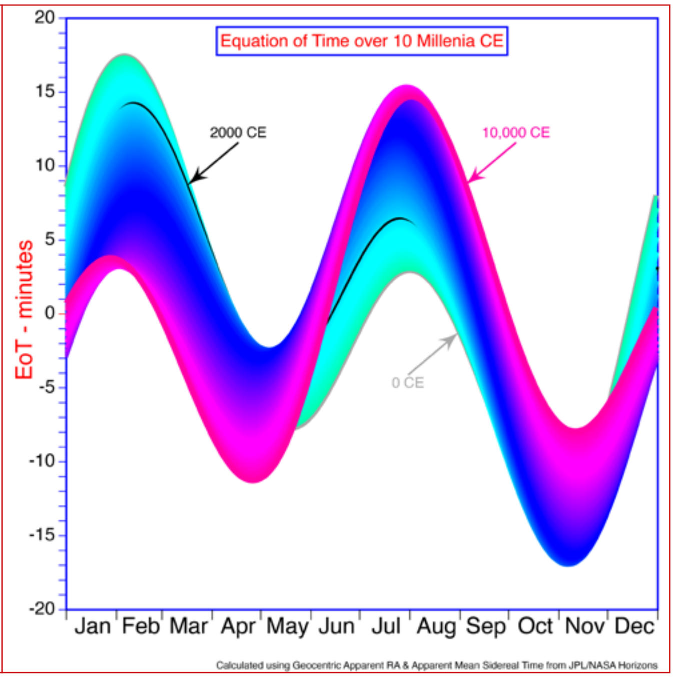


Fig 2 - The Equation of Time over many millennia. Gregorian centuries are used to mask the effect of the calendar change from Julian to Gregorian in 1582.T



Calculated using Geocentric Apparent RA & Apparent Mean Sidereal Time from JPL/NASA Horizons

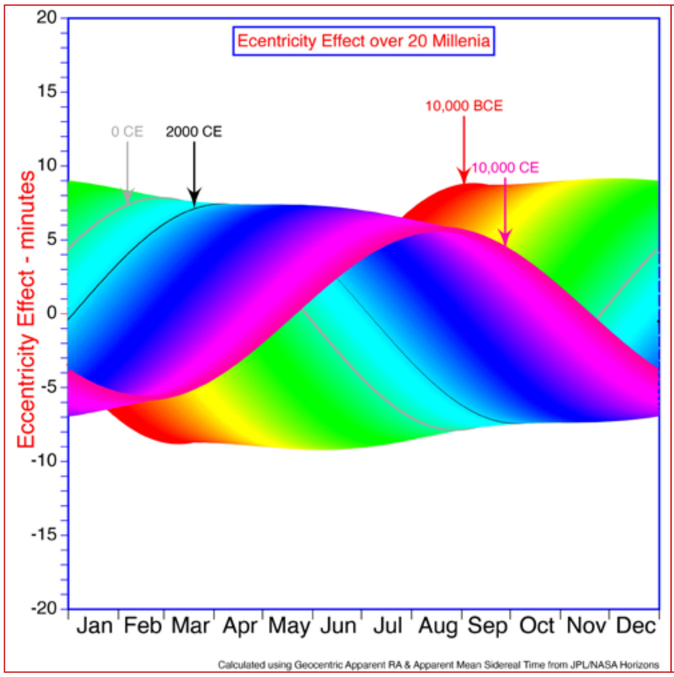
Fig 3 - Changes to the Equation of Time ***before*** *the Common Era.*
The data seems to trend ***downwards*** *with increasing time*



Calculated using Geocentric Apparent RA & Apparent Mean Sidereal Time from JPL/NASA Horizons

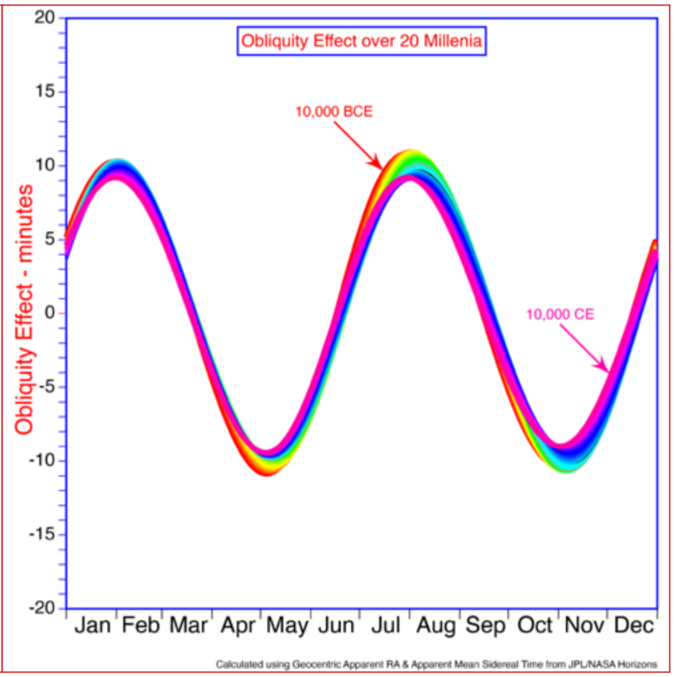
Fig 4 - Changes to the Equation of Time ***during*** *the Common Era.*
The data seems to trend ***upwards*** *with increasing time*

Fig 5 & 6 show the same information, but now split into the eccentricity and obliquity effects. Bearing in mind that the cycle of precession is some 26,000 years and the graphs below represent 20,000 years, it is clear that movement of Perihelion around the vernal equinox is the dominant component of EoT long-term variation.



Calculated using Geocentric Apparent RA & Apparent Mean Sidereal Time from JPL/NASA Horizons

Fig 5 - Changes to the Eccentricity Effect due, primarily, to the movement of Perihelion towards the vernal equinox. To a lesser degree, changing eccentricity



Calculated using Geocentric Apparent RA & Apparent Mean Sidereal Time from JPL/NASA Horizons

Fig 6 - Changes to the Obliquity Effect primarily due to the reducing Obliquity of the Earth's axis

return to "Table of Contents"

MINOR CHANGES - THE CALENDER & LEAP YEARS

The calendar's leap cycle was specifically designed to keep our seasons aligned with the months. Thus, there is a leap year every four years, except when a century is reached that is not divisible by 400; i.e. a leap in 2000 CE, but not in 2100, 2200, 2300 CE.

Fig 6 show the actual time of the equinox, changing by 1/4 of a day per year, until pulled back by the leap year. The Green line shows the trend during a century. The Red line show the trend (or lack thereof) over 400 years. Thus - on average - the Spring Equinox is 78.68 days past 1st Jan.

Fig 7. shows actual time of Perihelion. The Green trend line shows the gradual movement of Perihelion towards the Vernal Equinox, caused by Precession. The 'spikiness' of the graph is caused by the very irregular movement of the Moon. From the Sun's perspective, the Earth/Moon combination rotates is like an unbalanced dumbbell. The centre of gravity of the dumbbell (its 'barycentre' varies between some 4550 and 4800 km from the Earth's centre - that is nearly 3/4 of the way to its surface).

Fig 7. shows the effect of EoT over a leap year cycle covering the days in December when it is at changing fastest.

This effect can 'interfere' with the production of Victorian Equation Tables, 'The Problem with Equation Tables' on page 51.

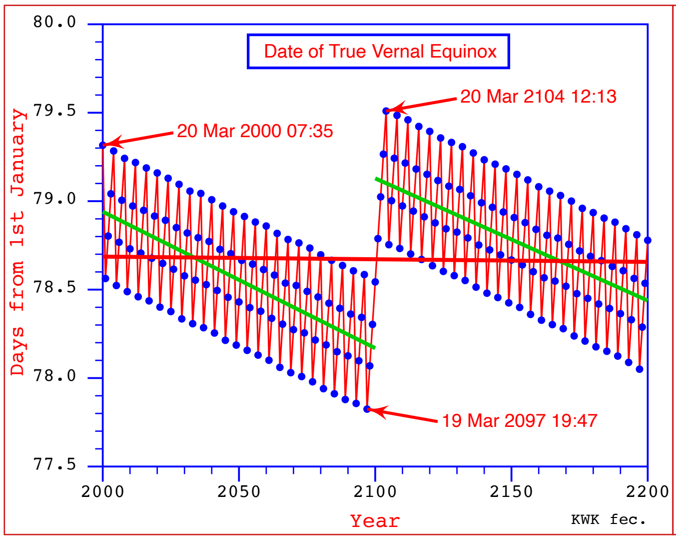


Fig 7 - The actual time of Vernal Equinox (in days after Jan 1st 0:00hrs)

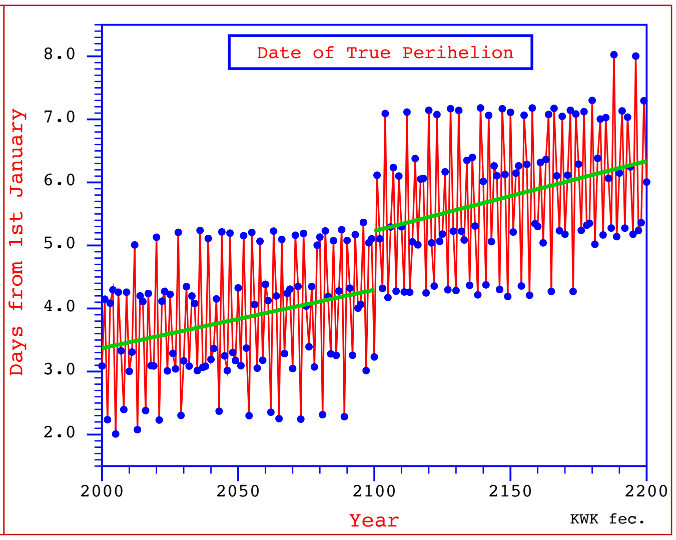


Fig 8 -The actual time of Perihelion (in days after Jan 1st 0:00hrs)

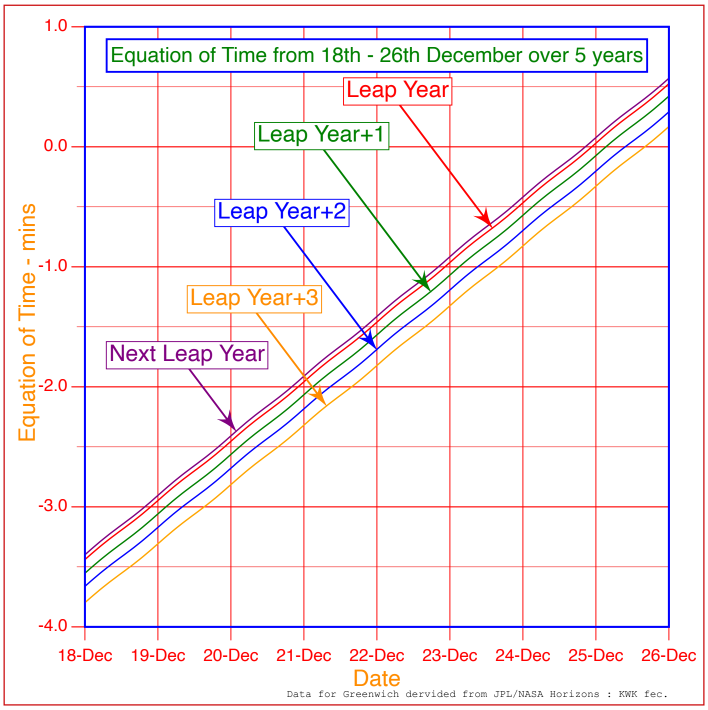


Fig 9 - The EoT over the Winter Solstice covering 5 years. The lines for a leap year and the following leap year nearly overlap, but those of the intervening years differ by up to half a minute. This effect is greatest during this period when the EoT's diurnal change is greatest.

Minor Changes : Topographical -v- Geocentric

Most calculations of the Equation of Time (EoT) are geocentric, meaning they view the sun from the centre of the Earth. However, from a practical perspective, one should consider the sun's position from their actual location, which continually changes throughout the day due to the Earth's rotation. This change is slight and varies with latitude, remaining constant at the Poles on any given day. However, at the Equator, it reaches its maximum, as the difference between a geocentric vector and a topocentric vector toward the sun changes the most. These two vectors align at noon and are most distinct at 6 o'clock.

Figure 10 illustrates the variation by latitude on a specific day, such as the Spring Equinox. The amplitude of the curves undergoes marginal changes throughout the year. Please note that the position of Solar Noon reflects the EoT for that time of year.

Note that the EoT calculated from Nasa/JPL's Horizons application (See 'Horizons' on page 50.) will provide the Topographic value corrected for the longitudinal difference between the locations and the time zone meridian.

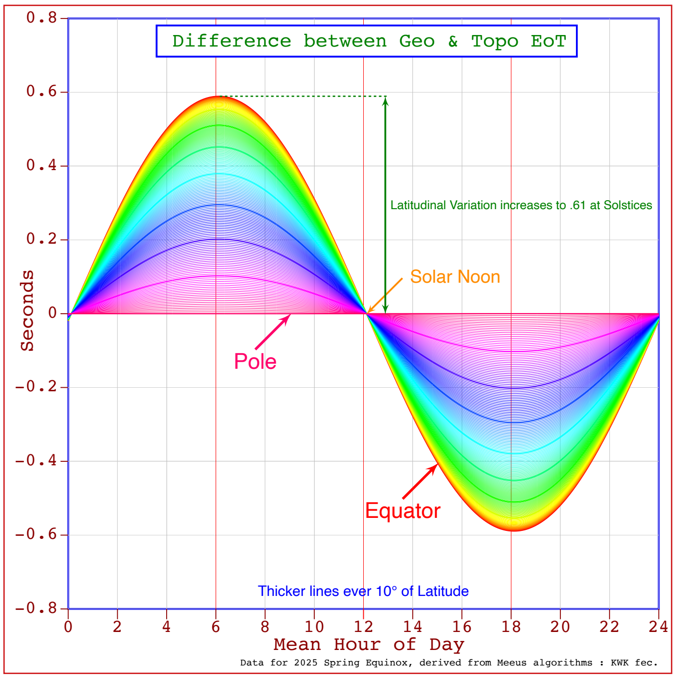


Fig 10 - The minor Difference between Topocentric and Geocentric EoT for varying Latitudes on a specific day of the year

Minor Changes : The Time Zone Error

One should be aware of the Time Zone for which the EoT is calculated. For example, if a table of the equation is calculated for Eastern Standard Time, but is being used in Hawaii, a time zone different by 5 hours, there will an error of about 6 seconds around Christmas, when the value is changing fastest. If a UTC table is being used, the variation can be as much as 16 seconds.

Fig. 11 shows this variation over the year over all time Zones.

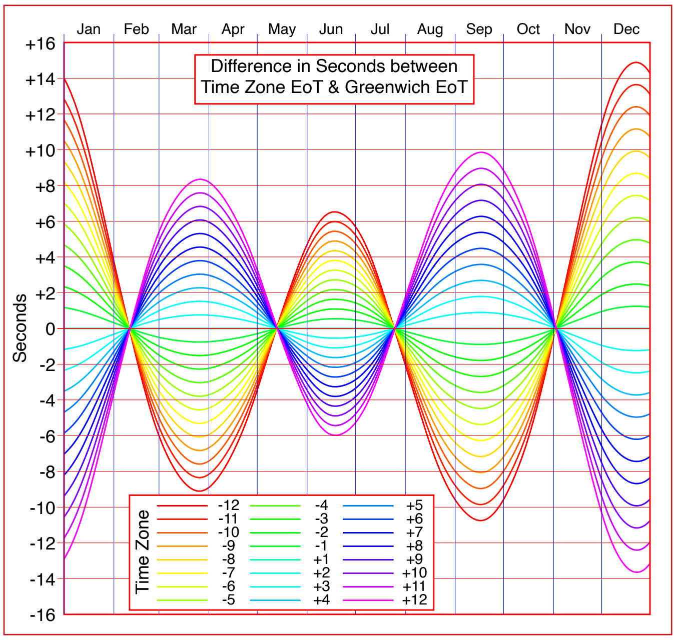


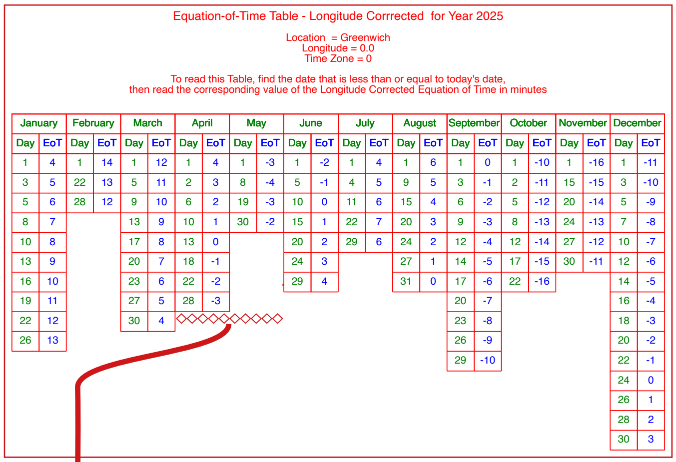
Fig 11 - Time Zone differences throughout the Year

THE PROBLEM WITH EQUATION TABLES

Equation Tables give the dates on which the EoT changes from one value to the next. These where frequently used as compact form of showing the Equation. Sometimes, they are made by one minute increments as a complete table - as shown in Fig 12. Equally they can be shown piecemeal around a sundial with 1 or 2 minute increments - as shown in Fig 13.

However, since any actual year is approximately 365.25 days long, errors are introduced.

Fig 14 shows a Equation Table correctly draw for 2025. Fig 15 shown a fragment of this table covering April and May together with a similar fragment for 2026. The differences are apparent.



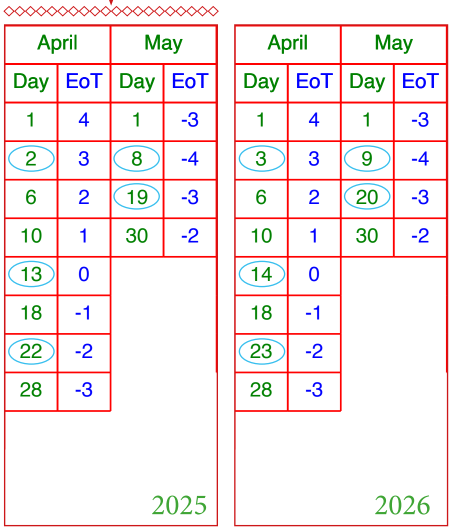
Equation-of-Time Table - Longitude Corrrected for Year 2025

Location = Greenwich
Longitude = 0.0
Time Zone = 0

To read this Table, find the date that is less than or equal to today's date,
then read the corresponding value of the Longitude Corrected Equation of Time in minutes

January		February		March		April		May		June		July		August		September		October		November		December	
Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT	Day	EoT
1	4	1	14	1	12	1	4	1	-3	1	-2	1	4	1	6	1	0	1	-10	1	-16	1	-11
3	5	22	13	5	11	2	3	8	-4	5	-1	4	5	9	5	3	-1	2	-11	15	-15	3	-10
5	6	28	12	9	10	6	2	19	-3	10	0	11	6	15	4	6	-2	5	-12	20	-14	5	-9
8	7			13	9	10	1	30	-2	15	1	22	7	20	3	9	-3	8	-13	24	-13	7	-8
10	8			17	8	13	0			20	2	29	6	24	2	12	-4	12	-14	27	-12	10	-7
13	9			20	7	18	-1			24	3			27	1	14	-5	17	-15	30	-11	12	-6
16	10			23	6	22	-2			29	4			31	0	17	-6	22	-16			14	-5
19	11			27	5	28	-3									20	-7					16	-4
22	12			30	4											23	-8					18	-3
26	13															26	-9					20	-2
																29	-10					22	-1
																						24	0
																						26	1
																						28	2
																						30	3

Fig 14 - Equation Table for 2025 at Greenwich



April		May	
Day	EoT	Day	EoT
1	4	1	-3
2	3	8	-4
6	2	19	-3
10	1	30	-2
13	0		
18	-1		
22	-2		
28	-3		

2025

April		May	
Day	EoT	Day	EoT
1	4	1	-3
3	3	9	-4
6	2	20	-3
10	1	30	-2
14	0		
18	-1		
23	-2		
28	-3		

2026

Fig 15 - Fragments of tables for two successive years showing the differences

Fig 16 shows the errors resulting from using a single year to form the table together the errors incurred in the following 3 years of the leap cycle: there is an error on 8% of the days.

The lower half shows a slight improvement made by using the averaging method over the leap cycle of 4 years : there is an error on 6% of the days.

The same exercise was repeated for the following two leap cycles. It yielded the same 8% and 6% results, indicating the slow change of EoT over a leap cycle.

Fig 12 -Single day Table, Wheddon Cross, Exmoor - 1850

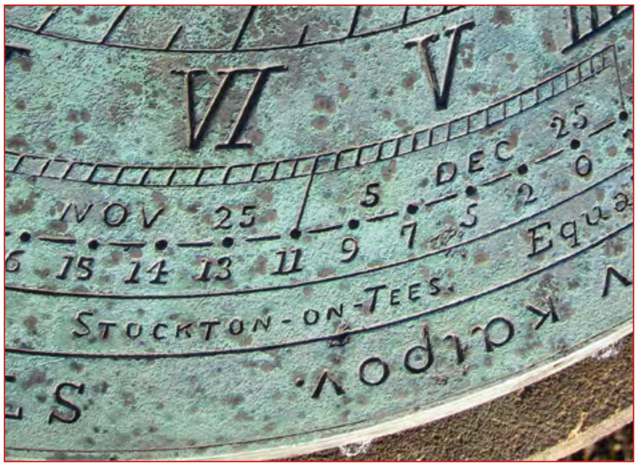


Fig 13 - Part of a 2-day Table at Wheelbirks, Northumberland - 1880

Days in Year when Table will be wrong by one minute

Comparing each year with 2024	2024	0	
	2025	25	123 errors over leap cycle = 8% of days
	2026	41	
	2027	57	
	2028	2	
Comparing each year with 2024-2027 Average	2024	28	
	2025	15	89 errors over leap cycle = 6% of days
	2026	13	
	2027	33	
	2028	31	

Small change between one Leap Year and the next Leap.

Fig 16 - Comparing a single year EoT table with a leap year average Table

The leap cycle averaging is therefore recommended.

Does it matter? Hardly at all, unless you are a Victoria clergyman setting the church clock – using a sundial.

return to "Table of Contents"

A Really Long Term Look

Nothing in the heavens is constant. The combined dynamics of the planets - most particularly the massive gas giants Jupiter and Saturn - indicate that the simple Keplarian view is not valid over long periods. Fig 17, covering 2 million years, shows the parameters relevant to the calculation of the EoT, viz. the eccentricity, the obliquity and the position of perihelion with respect to the vernal equinox (omega in the graph).

For this a few observations can be made.

i) around 900,000 years ago, the eccentricity was zero, the analemma was exactly symmetrical

ii) with the eccentricity ranging between 0 and 0.055, the eccentricity effect can be between 0 and 25.2 mins

ii) with the earth's obliquity ranging from 21.9° to 24.5°, the obliquity effect can be between 8.6 - 10.8 mins

iv) the maximum value of EoT, at the right moment in the processional cycle , the EoT will be around 36 mins

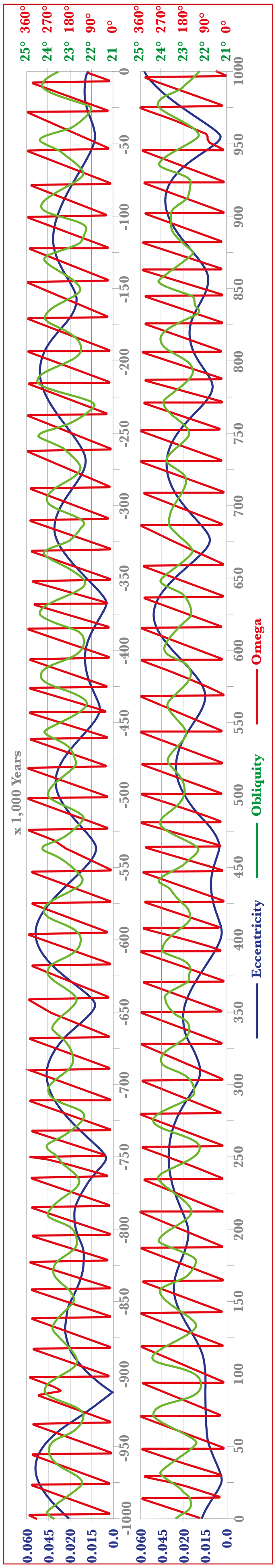


Fig 17 - Over 2 million years, the parameters that govern the shape of the EoT change in a somewhat irregular pattern but see Footnote.

Graphs derived from A Berger Long - Term Variations of the Earth's Orbital Elements - 1975

Footnote on Calculations

The calculations used to make the charts in this chapter have used the geocentric apparent right ascension and apparent sidereal time {EoT = 60 x (GAST - Appt RA - 12 + Hour)} provided by NASA JPL Horizons application - generally accepted as the best-in-class generally available solar system modelling. The application allows calculations from -9999 BCE to 9999 CE (9999 BC/AD was used for the 10,000 year curves). However all long term astronomical calculation carry uncertainty - not least from the unforcastable slowing of the year - coming from, iter alia, changing magma movements, tidal friction and melting ice.

Footnote on Berger's calculations in Fig 17

Berger - writing in 1975 - was building on many previous calculations of a similar nature, and used 1950 AD as his zero point in time.

return to "Table of Contents"

The Equation of Time as Shown on Sundials

John Davis

This piece was first published in BSS Bulletin Volume 16 (iv), December 2003. Reproduced by kind permission.

INTRODUCTION

The Equation of Time (EoT) has been a frequent subject for papers in the *Bulletin*[1-5] and elsewhere[23]. This paper will focus on the way in which it has been represented on sundials with particular emphasis on the period from the first publication of accurate tables in 1672 through to the end of the 19th century. There are two aspects to consider. The first is the actual values of EoT which are calculated by the astronomers and which form the source data for the dial-makers. The second is the format adopted to present the information in engraved bronze, stone, or other material.

The value of the EoT on any particular date can be calculated by a number of methods[6-7], to different degrees of accuracy. The underlying inputs to the calculation are the eccentricity of the Earth's orbit, the obliquity of the ecliptic, the solar longitude at perihelion and the solar longitude at the date in question. The general shape of the EoT, with its two primary and two secondary maxima/minima, is so well known to *Bulletin* readers that it will not be drawn here again. It is widely known[8] that the dates on which the EoT is zero can be used to date a table (and hence an English dial) to before or after the 1752 calendar reform when England finally relinquished the Julian calendar in favour of the Gregorian one. For convenience, the dates when the EoT is either zero or at its maxima/minima are shown in Table 1 for both calendar systems as an aid to dating dials.

The slight fluctuations in the EoT value on any particular day of the year due to the leap year cycle are also well known. It is less widely appreciated that the general shape of the curve changes over the millennia as the Earth's orbital parameters change, the main influence being the longitude of perihelion. Over a period of five centuries, these changes are most noticeable by small but significant changes in the values at the four maxima/minima and, as shown in Fig 1, these can be treated as varying linearly with time. The rates of change of the two maxima (-12.0 and 12.8 seconds per century for the February and July maxima respectively) are larger than those for the minima (9.4 and -5.4 seconds per century for May and October respectively). There are also changes in the dates of the maxima/minima and the date when the EoT has a zero value but these are less easy to document because of the leap year fluctuations which are superimposed on them. It could be used to date dials with EoT data precisely but two factors prevent a simple application of this theory. First, the values of the Earth's orbital parameters, and their variations with time, were not known accurately by 17th century astronomers. Secondly, the dial-makers did not always use up-to-date tables when engraving dials, as will be shown later. Nevertheless, EoT tables on dials can provide much information on the development of both astronomy and dial-making.

HISTORICAL EOT TABLES

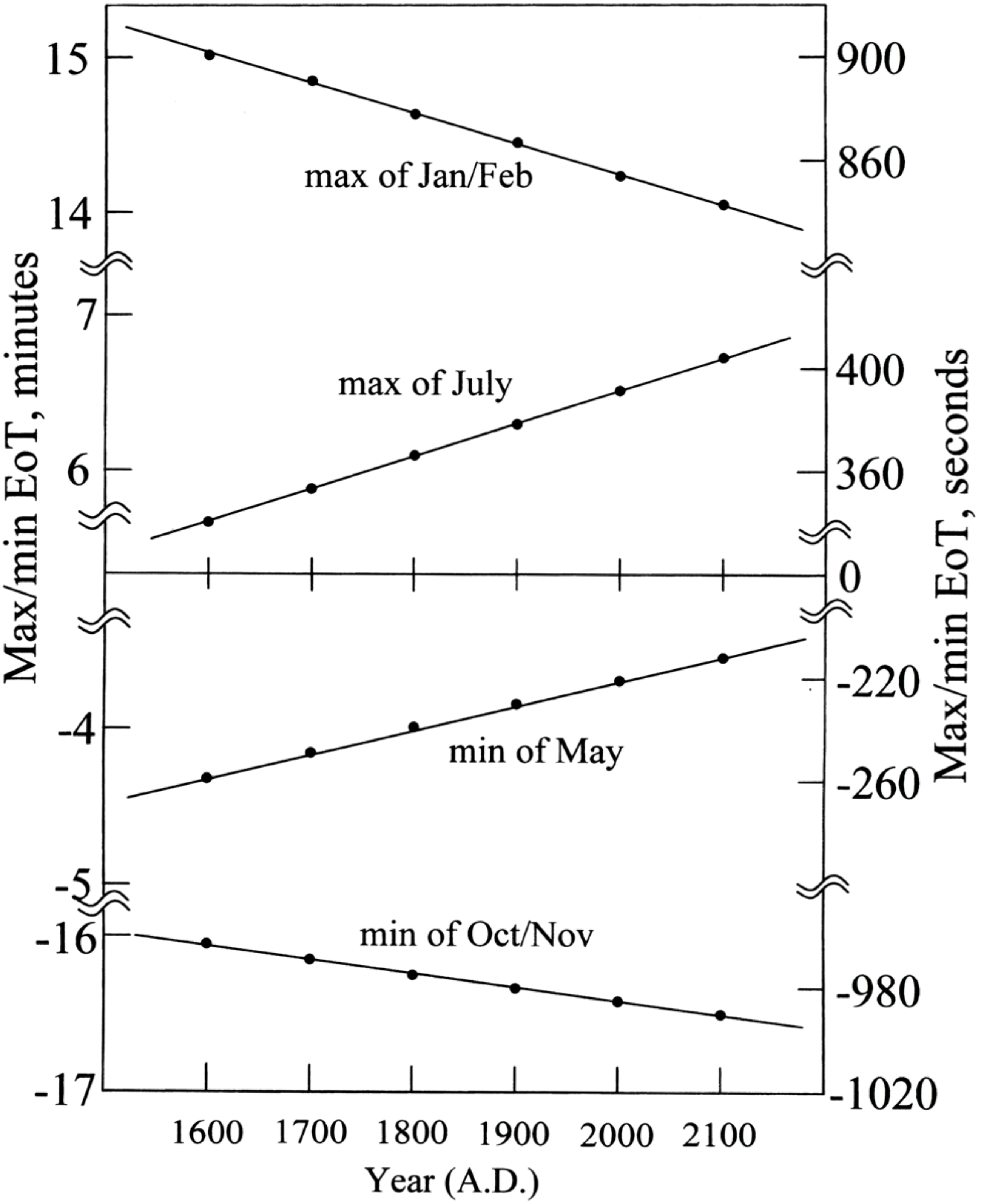
The earliest published EoT tables, from John Flamsteed, Christian Huygens and other astronomers, presented their results in a number of different ways. To allow direct comparisons of the tables with each other and with those on dials, a spreadsheet has been compiled putting the data into a common format. (This spreadsheet table is far too large to be shown here but could be made available to other researchers.) This table has a line for each day of the year, using the Julian calendar as appropriate for the earliest English tables, with tables originally published in the Gregorian calendar being converted by subtracting 10 or 11 days, depending on whether the date is before or after 1700. The EoT value is expressed in seconds using the definition

EoT = mean solar time - local apparent time

as preferred by the BSS Glossary[7] and resulting in positive values of EoT in January and February. Transcribing the tables in this fashion makes it clear that there are often printer's errors in them, such as the number of seconds in excess of 60 or the values not increasing/decreasing monotonically. Wherever possible, these errors have been corrected.

It is worthwhile commenting on some of the earliest English EoT tables, in chronological order:

i) Christian Huygens' 1665 tables were originally printed in *Kort Ondewys...* (The Hague) and so used the Gregorian calendar. They were communicated to the Royal Society and reprinted in English in *Philosophical Transactions* in 1669. He did not fix the origin of his calculation at the perihelion but instead set the minimum value on 9/10 February to zero with a maximum value of 31m 55s from 30 Oct to 2 Nov (Gregorian). In order to fit this table into the common format, the published values have been subtracted from 902 seconds (a value chosen to optimise the fit to other tables) in the spreadsheet.



Figue 1: Theoretical changes in the values of the four EoT maxima/minima over the period from 1600 AD to 2100. Calculated using the modern understanding of the Earth's orbital parameters, according to Meeus[6].

return to "Table of Contents"

ii) John Flamsteed’s 1672 tables are widely regarded as the first accurate tables of the true EoT and have been reprinted in the *Bulletin*[1]. Although they give the EoT in minutes and seconds, the table has 360 values, one for each degree of solar longitude. This is convenient for astronomers and it avoids the fluctuations of the leap year cycle but it is not easy to use on a sundial or for setting a longcase clock. Flamsteed was probably using Jeremiah Horrock’s values for the Earth’s orbital parameters at this time: these were more accurate than Huygens’ but some way from perfect. Flamsteed’s tables were also printed in Nicholas Stephenson’s 1676 *The Royal Almanack...* This is believed[9] to use the early table that Flamsteed communicated to Sir Jonas Moore before his appointment as the Astronomer Royal, not the later one published by John Smart. Later, in 1680, Flamsteed published his *Doctrine of the Sphere* giving a detailed description of the motion of the earth and moon. It was republished the following year as part of Sir Jonas Moore’s *A New System of the Mathematicks....* Although the *Doctrine* contains numerous tables it does not have a simple EoT laid out as a function of date so a dialmaker would have needed substantial help from an astronomer to use the data.

iii) John Smith, a clockmaker, published several pamphlets with EoT tables[9] (1678, 1679, 1686, 1694). These used the Julian calendar and usually took the form of the number of seconds that each day varied from 24 hours, they thus represent the first derivative of the EoT. The version used for the spreadsheet comes from Smith’s 1694 *Horological Disquisitions* but the values are actually the same as the earlier pamphlets. The arithmetic sum of the 365 values comes to just 1 second showing that there is no cumulative rounding error in the calculation. Assuming an EoT value of 532 seconds on 1 January, the values can be integrated to give a standard EoT table which proves to be identical to that of Huygens. It is assumed, therefore, that Smith derived his tables from the Huygens ones which had appeared in 1669. No sundial incorporating an EoT table based on the Smith/Huygens values has yet been identified. Smith also promoted a system of ten “rectifying days” at non-uniformly spaced dates throughout the year. At each of these a clock was to be set with a prescribed offset to solar time, with the result that the clock would remain accurate to within “a sixteenth part of an hour”.

iv) Thomas Tompion (1683) printed a table of EoT to be pasted inside the door of his famous longcase clocks. He also used the table on his sundials (see below). It is believed that this table was originally supplied by the famous scientist Robert Hooke, who was the Royal Society’s Curator of Experiments. Hooke[10] had invented a spring watch which he was having made by Tompion, and his diary for 13 December 1674 says[1] he “gave Tompion a description of Æquating of time for Sir J Moore’s Clock”. Although it seems likely that the table Hooke gave Tompion derived from Flamsteed’s work, it has not so far been possible to locate the exact source.

v) William Molyneux FRS (1686) lived in Dublin where he was a member of the Irish Parliament and had other public offices, as well as being a respected amateur astronomer and mathematician. He is known to have had the elaborate dial with telescopic sights that he had invented, made by Richard Whitehead, one of Henry Wynne’s apprentices, so there was ample scope for his tables to have been used by the London mathematical instrument makers. His EoT table was published as part of *Sciothericum Telescopicum* and its values were close to those of Tompion’s 1683 one although there are distinct differences.

vi) Thomas Tompion printed a second EoT table in 1690, this one in Latin and using the Gregorian calendar, presumably for his European customers as England was still using the Julian calendar at this time. This table, reprinted in the *Bulletin*[3], had some small but significant 3 differences to his 1683 one, not accounted for by the change in calendar. Analysis has show[27] that this table probably derives from the data in Flamsteed’s 1680 *Doctrine of the Sphere*, although it is not known whether the necessary calculations were performed by Flamsteed himself or, for example, by Hooke. Certainly, in *The Mathematical Practitioners*, Taylor[9] says “the famous Tompion being supposed to follow Flamsteed”.

EoT	Julian c.1700	Gregorian c.1785
Max	30-Jan	10-Feb
Min	04-May	15-May
Max	18-Jul	28-Jul
Min	22-Oct	02-Nov
Zero	04-Apr	15-Apr
Zero	06-Jun	15-Jun
Zero	20-Aug	31-Aug
Zero	13-Dec	24-Dec

Table 1. The dates of zero and max/min EoT for both the Julian and Gregorian calendars. Note that the dates are for the 17th/18th centuries and may vary by ±1 day due to the leap year cycle.

vii) John Smart (1710) was Town Clerk of London and published a table which was derived from John Flamsteed's papers and is believed to be a re-print of his 1702 table[9]. Unlike Flamsteed's first table of 1672, this one was tabulated against dates rather than solar longitudes. The values also had distinct changes, showing the results of his major investigations into the Earth's orbit during the period 1689/90[11]. This table was widely copied in various publications over the next half-century: see, for example, the ornate but anonymous table published in the recent book by Christianson[12]. It was also used on many dials.

viii) George Neale (1733) was another clockmaker who had his own table printed for pasting inside his clocks, such as that illustrated in Ref 13. The values are the same as those published by Smart.

ix) James Atkinson Senior (1735) was a teacher of mathematics in Dublin and published an EoT table in the Supplement to his 1735 book *Epitome of the Art of Navigation*. Once again, his values are the same as Smart's although his printed version contains numerous obvious printers' errors.

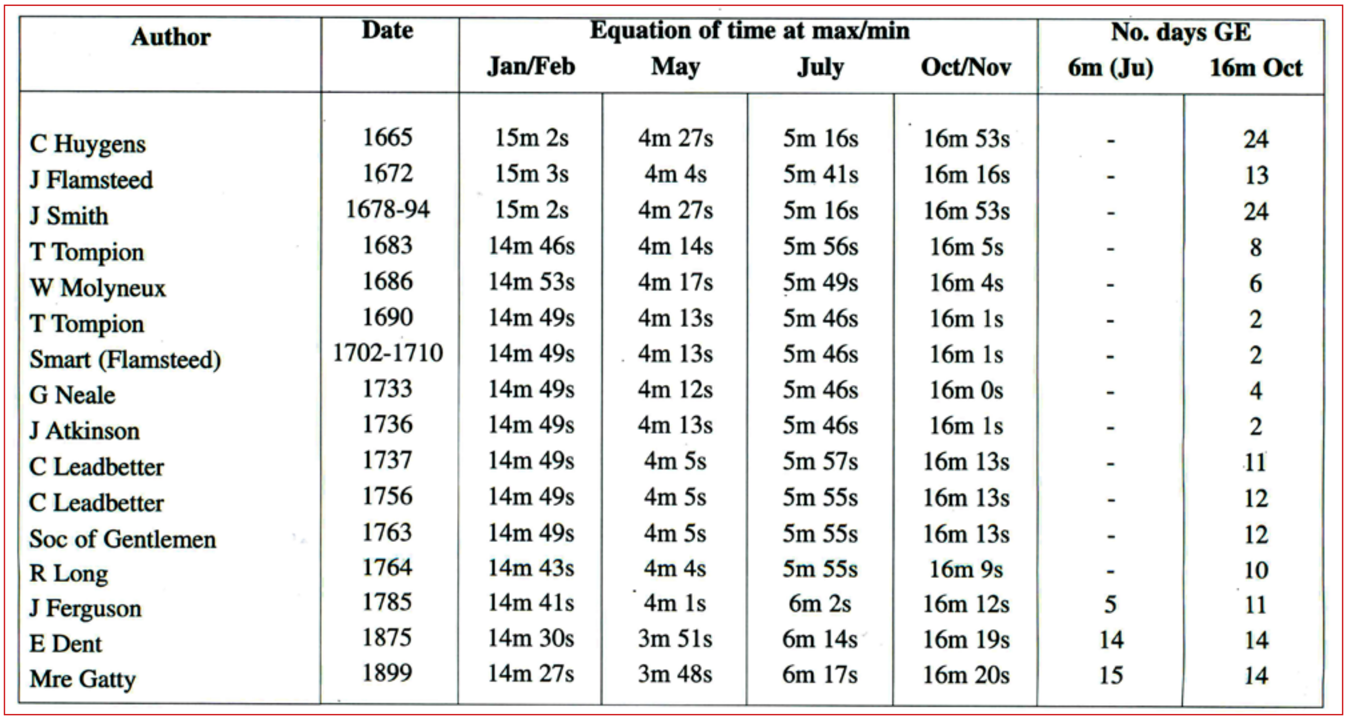
x) Charles Leadbetter (1737) was a London mathematics teacher. The table which he published in his *Mechanick Dialling or the New Art of Shadows* was the first for some years to show a significant recalculation of the basic orbital parameters. He republished the same table in 1739 in his book *The Young Mathematician's Companion...*

xi) Charles Leadbetter (1756). By the time *Mechanick Dialling or the New Art of Shadows* was re-issued in 1756, England had finally adopted the Gregorian calendar and so the dates of all the EoT maxima/minima values had changed by 11 days. Very small changes to the values of the maxima/minima show that Leadbetter had adjusted his parameters for the intervening 19 years.

xii) Society of Gentleman (1763). The second edition of the *Dictionary of Arts and Sciences...* contains an EoT table which is clearly the same as Leadbetter's 1756 one.

xiii) Roger Long FRS (1764) published a New Style (Gregorian) table in his Astronomy in Five Books. The values came from the French table for 1752 in Connaissance des Temps which are close to, but not the same as, contemporary English tables.

xiv) James Ferguson FRS (1785). By the time of the 7th edition of *Astronomy...* was published (including an account of the 1761 transit of Venus), the values of the orbital parameters were well defined and only minor updates to earlier EoT tables were necessary. The treatment here was comprehensive, four separate tables covered each year of the leap year cycle and the (mean) time of noon was given as well as the actual value of the EoT.



Author	Date	Equation of time at max/min				No. days GE	
		Jan/Feb	May	July	Oct/Nov	6m (Ju)	16m Oct
C Huygens	1665	15m 2s	4m 27s	5m 16s	16m 53s	-	24
J Flamsteed	1672	15m 3s	4m 4s	5m 41s	16m 16s	-	13
J Smith	1678-94	15m 2s	4m 27s	5m 16s	16m 53s	-	24
T Tompion	1683	14m 46s	4m 14s	5m 56s	16m 5s	-	8
W Molyneux	1686	14m 53s	4m 17s	5m 49s	16m 4s	-	6
T Tompion	1690	14m 49s	4m 13s	5m 46s	16m 1s	-	2
Smart (Flamsteed)	1702-1710	14m 49s	4m 13s	5m 46s	16m 1s	-	2
G Neale	1733	14m 49s	4m 12s	5m 46s	16m 0s	-	4
J Atkinson	1736	14m 49s	4m 13s	5m 46s	16m 1s	-	2
C Leadbetter	1737	14m 49s	4m 5s	5m 57s	16m 13s	-	11
C Leadbetter	1756	14m 49s	4m 5s	5m 55s	16m 13s	-	12
Soc of Gentlemen	1763	14m 49s	4m 5s	5m 55s	16m 13s	-	12
R Long	1764	14m 43s	4m 4s	5m 55s	16m 9s	-	10
J Ferguson	1785	14m 41s	4m 1s	6m 2s	16m 12s	5	11
E Dent	1875	14m 30s	3m 51s	6m 14s	16m 19s	14	14
Mre Gatty	1899	14m 27s	3m 48s	6m 17s	16m 20s	15	14

Table 2. Key values from early published EoT tables. The signs of the EoT have been ignored. The table also gives the number of days that the EoT is greater than or equal to ("GE") a given number of minutes at the maxima.
Note that the values of Huygens and J. Smith depend on the choice of an arbitrary constant. The values of Mrs Gatty and E. Dent come from the Nautical Almanac.

From this point, the number of published EoT tables proliferated and it becomes increasingly difficult to determine which were new calculations and which were merely derivative. The annual Nautical Almanac, first published in 1767 by Neville Maskelyne, the fifth Astronomer Royal, became the authoritative source of data. Mrs Gatty acknowledged the Nautical Almanac as the source of the tables in her famous books, as did Edward Dent for the tables which accompanied his dipleidoscopes at the end of the 19th century. There were also a number of simplified tables published, giving the EoT to the nearest minute, or just to one value per week. Because the differences in values between tables are small, it is difficult to identify sources with confidence although the format and choice of dates sometimes gives a clue.

Identifying Sources

Table 2 gives the maxima/minima values from the above tables. Where the EoT table on a dial allows these values to be read (or perhaps interpolated) to a second, these form by far the best means of identifying the source of the data. If this is not possible, the number of days around the October/November maxima when the EoT is equal or greater than l6m 0s is a very useful guide. After 1758 (conveniently close to the date of the calendar change), the July maximum exceeded 6 minutes and hence the day-count above 6m becomes useful. The method of day- counts can also be used on the two minima although it is less sensitive. When a dial gives the EoT in minutes only it is usually necessary to compare the table with possible sources individually.

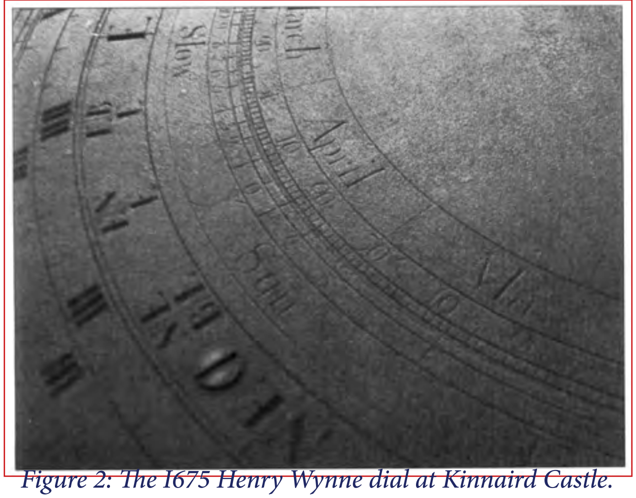


Figure 2: The 1675 Henry Wynne dial at Kinnaird Castle. Note the two engraving styles. Photo: The Earl of Southesk.

The large stone polyhedral dial at Glamis Castle[14], Angus, has a simple EoT table carved on its pedestal. The dial is recorded as being erected in its present position by the 3rd Earl of Kinghorn between 1671 and 1680[26] so it is a contender for the earliest appearance of the EoT on a dial. The table consists of one value, in minutes only, for every seventh day of the year. It is spread over six sides of the octagonal pedestal and is now covered with lichen and difficult to read. Analysis of the values shows that although they follow the correct form there are significant discrepancies between the values of the maxima and minima, and the dates of the zero values, when compared to any of the tables which would have been available at this early date. There also appears to be a step of several minutes in the values between the 5th and 12th of March, i.e. at the time of the Julian equinox. Although the table might have been independently calculated by an unknown astronomer, it is also possible that it was engraved after the dial was installed using one of the approximate tables published during the 18th century. It has also been suggested[26] that the table may have been obtained experimentally, using sidereal time as a reference. This is certainly possible but any claim for this to be the earliest table on a dial must be viewed as unproven with some suspicion.

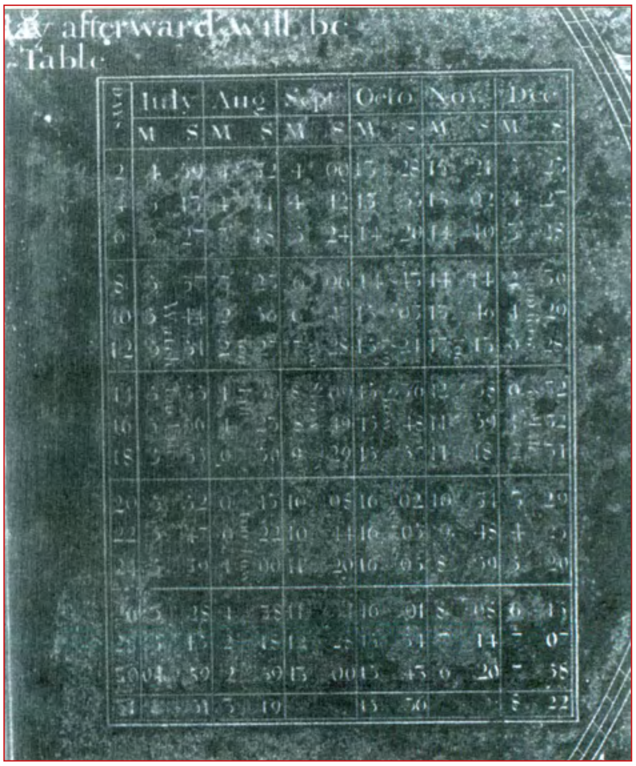


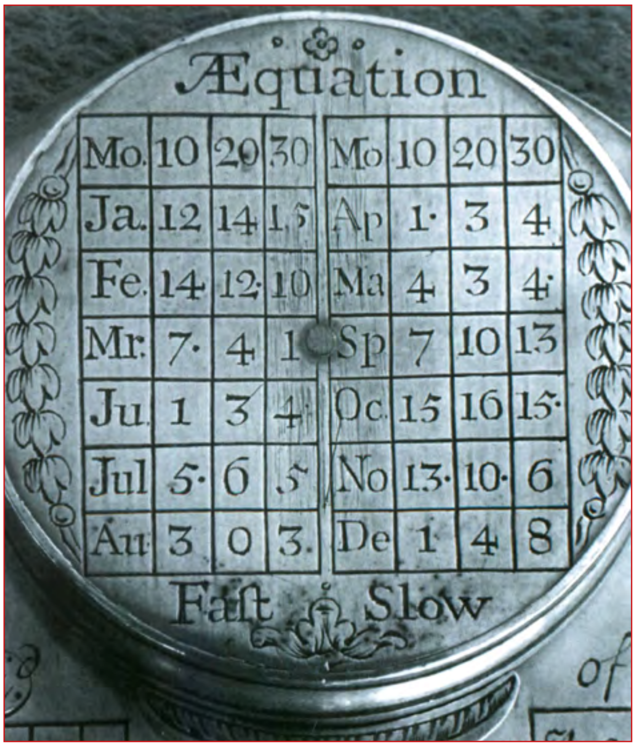
Figure 3: Half of the EoT table as engraved on a dial by Thomas Tompion. Photo: reprinted by courtesy of Sothebys.

The earliest dated dial with EoT data is the 1675 Henry Wynne dial at Kinnaird Castle, Brechin[13], shown in Fig 2. However, both the stylistic details and the actual values point to the EoT information being added about a century after Wynne made the original engraving. The data is in the form of a ring with the values unusually labelled "Sun fast(slow)", showing that clock time is now taking precedence over solar time. The months run anticlockwise and the engraving is oriented to be read from the outside whereas the rest of the dial, and Wynne's other dials, have the engraving oriented inwards. The style of the numerals (particularly the use of a flat-topped "8" and a very rounded "2") and the use of "J" rather than "I" for the initial letter of Jan, Jun and Jul, are not characteristic of Wynne. All of the minute marks for the EoT align exactly to a day marker, indicating that the information has been taken from a simplified table rather than by a proper interpolation of one in minutes and seconds. Finally, the values show that the table is using the Gregorian calendar (which Catholic Scotland adopted in 1599, well before the 1751 Act in England) with 12 days in October/November having values of over 16 minutes and 5 days in July having values over 6 minutes. Reference to Table 2 shows these values to point towards John Ferguson's 1785 table as the source although more detailed analysis shows an almost perfect match to a simplified table due to "Mr Smeaton" and included in Ferguson's *Astronomy*.

Another early table is on the Staunton Harold double horizontal dial by Wynne[15], confidently dated to 1685. Here, the table is uniquely in the form of a long strip along the top edge of the gnomon. It has a scalar presentation of the EoT, divided and numbered in whole minutes, set against a months scale divided down into individual days. Although the scale is non-linear by the very nature of the EoT, it is possible to interpolate values down to, perhaps, a tenth of a minute or 6s. In addition, the number of excess seconds is actually engraved at the maxima/minima giving, for example, 16m 5s on 23 October. The need for adopting a sign convention for the EoT is neatly side-stepped by labelling the values, for example, "Watch goes to Fast" (sic). Thus all the features of the circular "Æquation of Natural Days" scales found on a large number of high-quality dials of the 18th century[16] are already present. Although it is possible that Wynne added the EoT table after the dial had been finished, this seems unlikely and so this is probably the earliest extant example of EoT data on a dial. The values that Wynne has used seem to be the same as on the Tompion 1683 printed table.

The most obvious way of presenting the EoT data, if not the most efficient or stylish, is to provide a table with a column for each month and 28/30/31 rows for the days. This was the layout adopted by the unknown maker of the "Bacon" double horizontal dial[12]. In that case, the values are only given to the nearest minute but because every day has a value it is possible to match this table with reasonable certainty to Tompion's 1683 table. A more clear-cut example is the Henry Wynne dial at Drumlanrig Castle[25] where the full table for each day of the year can definitely be assigned to the Tompion 1683 table. A dial by Tompion himself was sold at Sothebys[24] in 2002 (Fig 3) and it featured this tabular format, echoing the versions that he printed for his clocks. To save space, the values (in

Figure 4: Part of the "Æquation of Natural Days" scale on the Thomas Wright dial at Lacovk Abbey (NT), Wiltshire. Note: the dates increasing right to left and the maximum of 30 Jan in excess of 14 minutes.


Æquation

Mo.	10	20	30	Mo	10	20	30
Ja.	12	14	15	Ap	1·	3	4
Fe.	14	12	10	Ma	4	3	4·
Mr.	7·	4	1	Sp	7	10	13
Ju.	1	3	4	Oc	15	16	15·
Jul	5·	6	5	No	13·	10·	6
Au	3	0	3.	De	1	4	8

Fast Slow

Figure 4 (b): "Æquation of Natural D ays"

return to "Table of Contents"

minutes and seconds) are given for every alternate day. Although the sale catalogue dates the dial as "circa 1705" it is easy to see that the values in this table are identical to the one he printed in 1683, rather than his improved 1690 version. Either the Sothebys' dating is a little out or Tompion has not bothered to convert his 1690 table back to the Julian calendar for this English dial.

Another example of the tabular format giving values in minutes and seconds is that at Cowham's Hospital, Stirling[18], dated 1727 and signed by Andr. Dickie. Andrew Dickie was a clockmaker in Stirling (w. 1723-39) and may also be the same man who was an official to the Board of Longitude in 1761-5. This dial was tragically re-engraved in 1910 to "freshen up" the appearance with the result that not all the values can be believed. Nevertheless, the values at the four maxima/minima are clear enough for the table to be identified as the Smart (Flamsteed) one.

In order to fit an EoT table on a portable dial, the data must be condensed significantly. An inventive method by John Rowley (Master of Mechanics to George I) is shown in Figure 4 where the table, on the back of the compass box of a Butterfield dial, has the months divided into two sets for "(watch) fast" and "(watch) slow". For August 30 (Julian), this results in the value having the wrong sign so Rowley has indicated this by use of a dot after the numeral. Some other numerals are also followed by dots, these stand in for half-minutes. The actual numbers of seconds for these values are between 25 and 38, rather than the 15 and 45 for a strict mathematical rounding. The extra resolution provided by this data allows the source to be identified as the Smart (Flamsteed) table of 1702, supporting a date of c. 1705 for the dial. Rowley made a similar dial, now at the NMM, Greenwich, for the Earl of Orrery[20]. Subtle differences to which EoT values have dots show that for this dial Rowley was working from the 1683 Tompion table indicating, for the first time, that this dial probably pre-dates that of Fig. 4.

One of the best extant examples of the Watch Faster/Slower or "Æquation of Natural Days" ring scales is that on the Thomas Wright dial at Lacock Abbey, Wiltshire (Fig 4). Here, the large size of the dial has allowed the scale to be divided to half-minutes so that, with care, it can be interpolated down to better than five seconds. This, together with the fact that it shows around 7 days in October with an EoT of 16m or more, allows the table to be identified as the early 1683 Tompion one. The dial is undated but Wright's working dates[19] of 1718-1747 show that he was using rather out-of-date data. Another, much later, Wright dial originally made for Haigh Hall, Wigan is now in a private collection. This, surprisingly, has an EoT scale in the Gregorian calendar which bears the added inscription "According to the New Style 1756 by Jno. Latham, Wigan". John Latham (w. 1730-1757) was a clockmaker in Wigan. The values of the EoT are difficult to read but, from the approximate 13 day separation of the two 16m marks in October/November, the values can be tentatively be attributed to Leadbetter's early Gregorian table.

Figure 5: A slate dial made by Patrick Fox of Churchtown in Ireland, 1829. The letters in the corners of the dial spell out "EQUATION".

Figure 6: Part of the EoT ring on a late 19th century dial by Troughton and Simms. The months are written anticlockwise so that the days increase left to right.

return to "Table of Contents"

Many dials, both garden and portable, made in the first half of the 18th century by London mathematical instrument makers used the 1710 Smart (Flamsteed) EoT table. For example, it seems that all the dials by the members of the Grocers' Guild used it. Some examples of garden horizontal dials by Benjamin Scott, Thomas Heath, Joseph Jackson and George Adams are illustrated in Ref 16. Other dials using this table include indoor inclining dials by John Sisson at the National Maritime Museum, illustrated in Ref 20, and one by Richard Glynne (another of Henry Wynne's apprentices) which was at the recent sale of items from The Time Museum[21]. Although the many examples of "Equation of Natural Days" rings are superficially similar, there are a number of detailed differences which give clues to their dates and makers and are worth noting when recording a dial. In addition to the key values of the EoT, the questions of which answers provide clues include the following:

- is the ring in a single arc or are there gaps at the north and south points?
- is the ring labeled "Æquation of Natural Days" or some similar title (later rings are usually unlabeled)?
- do the months lie on the inner rings with the EoT minutes on the outer ones, or vice versa?
- do the months run clockwise or anticlockwise? Note that if they are clockwise and oriented to be read from the outside, the days increase rather awkwardly from right to left.
- is the EoT labeled "Watch faster/slower" or "Clock faster/slower" or some similar expression?

The use of EoT data to date dials can sometimes produce anomalies. For example, a garden horizontal dial by Richard Glynne at the National Maritime Museum is said[20] to be dated 1753 but the reported values of its EoT maxima/minima are those of Tompion's long since replaced 1683 table. It is perhaps significant that Glynne is recorded[19] as having retired in 1730.

Although top-quality London dials continued to carry a full /Equation of Natural Days ring into the 19th century, provincial dials often only had a simplified table, if they showed the EoT at all. Fig 5 shows a beautifully engraved 1829 slate dial from Patrick Fox of Churchtown in Ireland. Fox is not otherwise known as a dial-maker. The EoT is shown as a large circular table round the outside of the dial with one segment per month and giving the dates for each integer number of minutes. It is worth noting that this method may not produce precisely the same table from the underlying data as giving the EoT to the nearest minute on pre-selected dates. This depends on the method adopted to round or truncate the data. Fox has clearly had difficulty in December where the rapidly-changing EoT has produced too many days for the available space so that he has had to erase one value and only show the even numbers of minutes. The data that Fox has used matches exactly that of the simplified table published by John Bonnycastle in his *Introduction to Astronomy* of 1796 although it is possible that there are other identical sources.

A very late example of a quality dial made by a mathematical instrument maker is one in Devon signed *Troughton & Simms*, shown in Fig 6. Edward Troughton's history can be traced back through several master instrument makers in the Grocers' Company to Benjamin Scott[16]. The engraving style on the dial shows that this influence has persisted although the design details and ornamentation have been simplified. Note in passing that the infill of the broad strokes of the main Roman numerals has been achieved by multiple short cuts from the edges, rather than the long parallel cuts that would have been used in the century. The EoT scale on this dial clearly shows six days above 6m in July as well as the thirteen days above 16m in October/November. This would date the table, if not the dial itself, to the beginning of the 1826-1922 working period ascribe[19] to Troughton & Simms.

Figure 7. The EoT graph on the 1921 dial in the Fellow's garden of Trinity College, Cambridge; possibly the earliest to use this format

return to "Table of Contents"

CLOSING REMARKS

Other presentation methods for the EoT are sometimes found on dials. For example, the analemma was engraved on a universal equatorial dial[20] by Johann Michael Vogler as early as the first quarter of the 18th century. A full study of analemmas on dials will be reserved for a future study. The use of an x-y (Cartesian) graph to show the EoT as a function of date seems to be a surprisingly modern feature. One of the earliest examples, shown in Fig. 7 and dating from 1921, was made by the Cambridge Instrument Company. It is in the Fellows' Garden of Trinity College, Cambridge The sign convention used in this graph is the same as that adopted in this paper (i.e., positive for the January maximum). A few other examples from the middle of the 20th century can be found in the BSS Sundial Register. In some more recent dials the correction graph also includes the local longitude correction but, strictly, this is not a true Equation of Time curve. Although the graphical format is easy to engrave and gives a fast appreciation of the correction, it does not give as good a resolution as a complete table or the scalar "Watch faster/slower" ring.

This paper has, I hope, shown that substantial historical information can be extracted from the EoT data on old dials. For modem designers of new dials, I suggest that the best data for the EoT is not that to be found printed in one of the standard mid-20th century texts (Waugh, Mayall & Mayall, Rohr etc.) or a modern computer-generated table for the year of manufacture. Instead, an averaged table for the projected life of the dial is proposed[7].

The author would be pleased to hear from readers who have other early examples of EoT tables from astronomy or navigation books, or pasted into longcase clocks.

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return to "Table of Contents"

ACKNOWLEDGMENTS

This paper would not have been possible without the substantial assistance of many people. It is a pleasure to acknowledge the diallists who have very kindly provided me with calculations, data, photographs and other information. These include Allan Chapman, Michael Cowhand, Christopher Daniel, Gordon Hendry, Martin Jenkins, The Earl of Southesk, Anthony Kitto, Michael Lowne, Kenneth MacKay, Eddie Mizzi, Peter Ransom, Anne Somerville, Margaret Stanier, David Young.

return to "Table of Contents"

More on the Equation of Time on Sundials

John Davis

This piece was first published in BSS Bulletin Volume 17 (ii), June 2005. Reproduced by kind permission.

In an earlier paper[1], the development of accurate Equation of Time (EoT) tables by astronomers in the 17th and 18th centuries, and the presentation of this data on sundials, was discussed. Further analysis has now been performed and other examples have come to light. The database of tables[2] currently contains over twenty printed versions and data from numerous dials.

Fig. 2. Part of the EoT table from the Tompion Wrest Park dial, showing the columns for each month of the year. Photo courtesy Sotheby,Äôs

Tompion and Flamsteed

The earlier paper described an EoT table printed, in Latin and using the Gregorian calendar, by Thomas Tompion in 1690. Although Tompion's earlier 1683 Julian table was widely used on dials by many makers, no examples of this later, improved version were known on dials. It is believed that both tables were actually calculated by the Astronomer Royal, John Flamsteed. More detailed examination has now shown that the underlying numerical values of the 1690 table are essentially the same as those of the (Julian) table calculated by Flamsteed in 1702 and reprinted from his papers by John Smart in 1710 (referred to as the 'Smart/Flamsteed' table in the earlier paper). However, both the Tompion 1690 and the Smart/Flamsteed tables have a discontinuity of around 7 seconds between the values on 31 December and l January, irrespective of the calendar system in use (see Fig. 1). As a result, the two tables show slightly different values from 22 to 31 December (Julian) but are otherwise identical. This allows an insight into the way that Flamsteed calculated the tables. He seems to have had a table for the daily differences in EoT (from one day to the next) which was the same for the 1690 and 1702 tables. For his 1702 table, he started from an EoT value of 8m 59s on 1 Jan (Julian), and then summed the differences each day of the year until he arrived at a value of 8m 29s on 31 Dec. The difference of 30s between these beginning and end values compares to the expected rate of change of 23 or 24 seconds per day at that time of the year: the discrepancy of approximately a quarter of the daily change is due to the slippage of the solar longitude with the leap year cycle. When calculating the 1690 Gregorian table for Tompion, Flamsteed has started with a value of 4m 42s on 22 Dec Julian (l Jan Gregorian) and then summed the same differences as before. Note that 1690 and 1702 were both LY+2 as 1700 was a leap year in the Julian calendar.

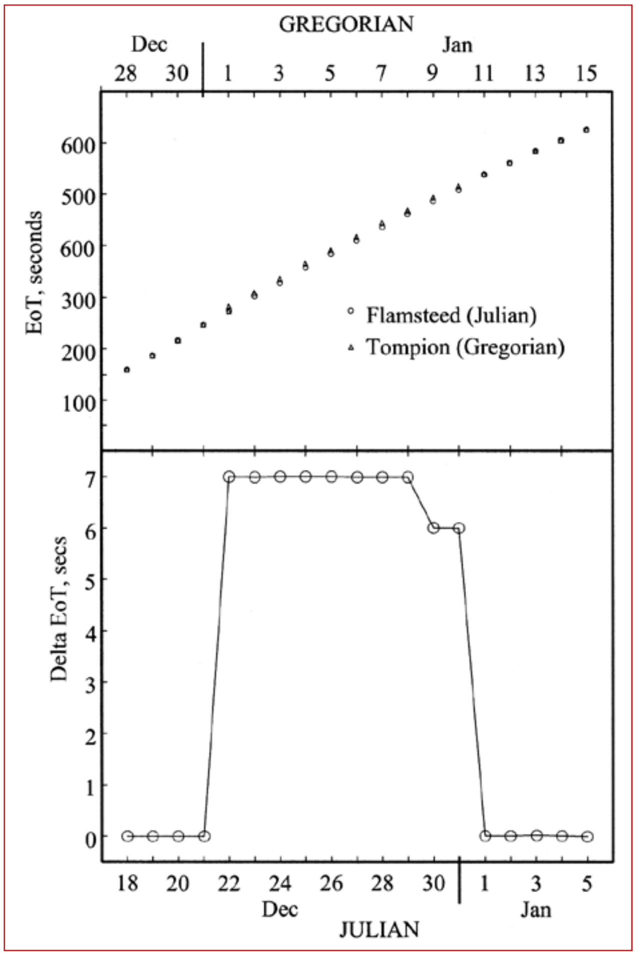


Fig. 1. Comparison of Tompion's 1690 Gregorian and Flamsteed's 1702 EoT values at the year end. Gregorian dates are 10 days different to Julian ones prior to 1700. Delta EoT is defined here as (Tompion's Flamsteed) values.

Fig. 3. Gabriel Stokes (1682-1768). Portrait by an unknown artist, courtesy of Trinity College Dublin.

In June 2004, Sotheby's sold a previously 'lost' sundial made by Tompion for Wrest Park, Beds.[3,4] This wonderful dial (illustrated in Fig. 2 and Refs. 3 and 4) includes a full table of the EoT in minutes and seconds for each day of the year. Although not all of the values are now easy to read (and the Sotheby's catalogue description unfortunately has reading errors in some of the key max/min values) enough of it has been tabulated to make it clear that the data is from the Smart/Flamsteed table. This is the only known example of Tompion using anything other than his 1683 table on a dial. Although this indicates that the dial is later than, for example, his "c.1705" dial (Fig. 3 in Ref. 1), it does not necessarily mean that the dial dates to after 1702 as Tompion may well have had access to Flamsteed's results before they were published. Being an English dial, Tompion has not used his 1690 Gregorian table which remains unknown on a sundial.

THOMAS TUTTELL

In 1698 the mathematical instrument maker Thomas Tuttell, one of Henry Wynne's apprentices, published a small book[5] on the analemmatic double dial which he had improved and was popularising. It included an EoT table with the title "A Table of Equation, shewing the Difference of a well Adjust'ned Pendulum and the Sun, every Day of the Year". The columns were listed with the unusual (unique?) labels "Pendulum Gains/Looses" rather than the more common "Watch Fast/Slow". The values of the max/mins in the table indicate that it has been calculated with the same set of orbital parameters as the Smart/Flamsteed table (the only difference is of l second to the October maximum). However, there are consistent differences in the daily values of up to 8 sec, equivalent to approximately a quarter of the daily change. This points to the table having been calculated, probably by Flamsteed, for the year 1697 which was LY+l.

Fig. 4. Analemmatic dial by Gabriel Stokes. Photo: M. Cowham

PORTABLE DIALS

The recent book 'A Dial in Your Poke' by Michael Cowham[6] shows several portable dials with EoT scales. The earliest and most important of these is a tabletop analemmatic and horizontal combination in silvered brass, in the format described by Thomas Tuttell. The dial has been shown previously in the Bulletin[7] and is by the relatively little-known maker Gabriel Stokes of Dublin[8-10] Stokes (1682-1768) was the son of a tailor but served his apprenticeship under Joseph Moland, a surveyor and mathematical instrument maker: his portrait (Fig. 3) shows him with a surveyor's cross-staff. Stokes went on to be Deputy Surveyor General of Ireland and to found a dynasty of important British scientists[8]. The name of one of his descendents, Sir George Gabriel Stokes, will be known to *Bulletin* readers through his association with the Campbell- Stokes sunshine recorder.

The EoT scale on Stokes' analemmatic dial (Fig. 4) is remarkable for at least two reasons. Firstly, it is laid out around the elliptical scale and in such a way that the (Julian) date scale is approximately linear, that is, not with an equi-angular arrangement around the centre of the ellipse. Laying this out is a clever piece of geometry.

The actual EoT scale on the Stokes analemmatic dial is divided down to 15s increments of EoT with every 2m numbered. This is an impressive piece of calculation as well as engraving. The max/min values are explicitly engraved as: +14m 51s, -4m 13s, +5m 53s and -16m 6s. These values are not an exact match to any of the published tables (see Table 2 in Ref. 1) although they are entirely consistent with values to be expected for around 1700. The closest match to known tabulations is the 1686 table of Stokes' Dublin compatriot William Molyneux. Molyneux had also been Surveyor General of Ireland a generation earlier and had corresponded with Flamsteed about the confusing number of EoT tables. Thus it is tempting to think that there had been some form of inheritance of the EoT data, possibly through the Dublin Philosophical Society or Trinity College Dublin, where both Molyneux's son Samuel and Stokes's two sons studied. Samuel Molyneux had revived the Dublin Philosophical Society in 1707 before moving to London and working with the astronomer Hadley. Stokes is knowns to have made detailed repairs and modifications to the 'Great Quadrant' at the Trinity College Observatory in 1715 and he clearly had contact with the Professor of Mathematics there. Thus it is quite possible that there is a yet-to-be-discovered early calculation of the EoT by the 'Dublin school' of astronomers waiting to be found.

Fig. 5. A mechanical equinoctial dial by Thomas Wright. Photo: M. Cowham

The mechanical equinoctial dial by Thomas Wright which Cowham[6] shows (Fig. 5) is similar to one by the same maker originally at the Time Museum and sold by Sotherby's in 2003.[12] Both dials have an EoT scale arranged to be read from the periphery of the dial and using data identified as the Smart/Flamsteed table. However, the scale runs clockwise on the Sotheby's dial but anticlockwise on the dial illustrated by Cowham. Wright[13] was one of the last major makers to adopt the more logical anticlockwise format so, although both dials are undated, this indicates that the Cowham dial is slightly later than the Sotheby's one. This supposition is supported by the signatures: on the Sotherby's dial Wright signs himself as "lnstrmt maker to his MAJESTY" whereas on the Cowham dial he is "Instructt maker to ye KING". Another guide to the date of the Cowham dial is the magnetic variation for London of 13.5°W indicated on its compass, a value which is compatible with around 1730.

Fig. 6. A magnetic compass dial by Fraser. Photo: M. Cowham

Cowham also shows a small magnetic compass dial signed "Fraser, London" which has an EoT scale running around the outside (Fig. 6). The dial is probably by William Fraser who worked from New Bond Street[14] 1780-1805, so naturally the date scale is for the Gregorian calendar. Although the engraving is very fine, with the date shown to every 2 days and the EoT to 1m increments, the actual data is a muddle, especially in July where the maximum value of +6m 46s cannot possibly be right for any date of that millennium. The value of -l6m (0)s in December is too small for an 1800 date and the -4m 13s for May is more suited to a date in the 1730s. It appears that there may have been a rather poor attempt at converting an old Julian table, rather than using one from a recent Nautical Almanac.

A similar magnetic compass dial, made for S. America, is also shown by Cowham[6]. It has an EoT table printed on paper pasted into the lid (Fig. 7). The format in this instance is to give the dates when the EoT is a whole number of minutes, resulting in more entries for those months where the EoT is changing rapidly. The general format is that used by the engineer

return to "Table of Contents"

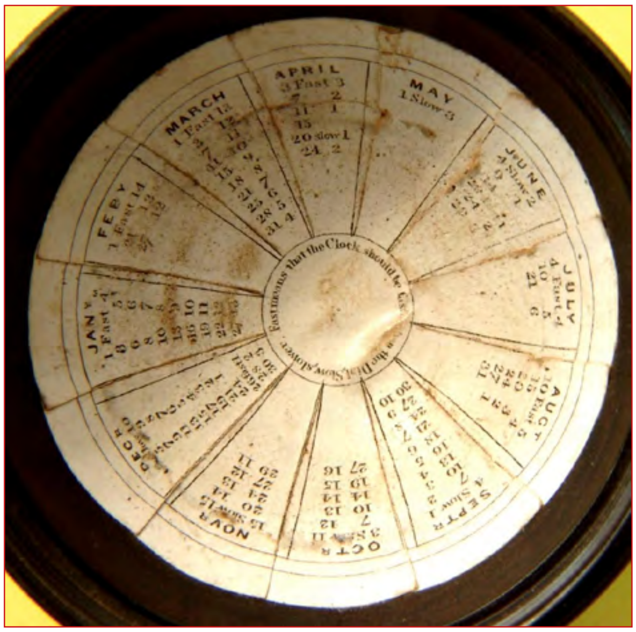
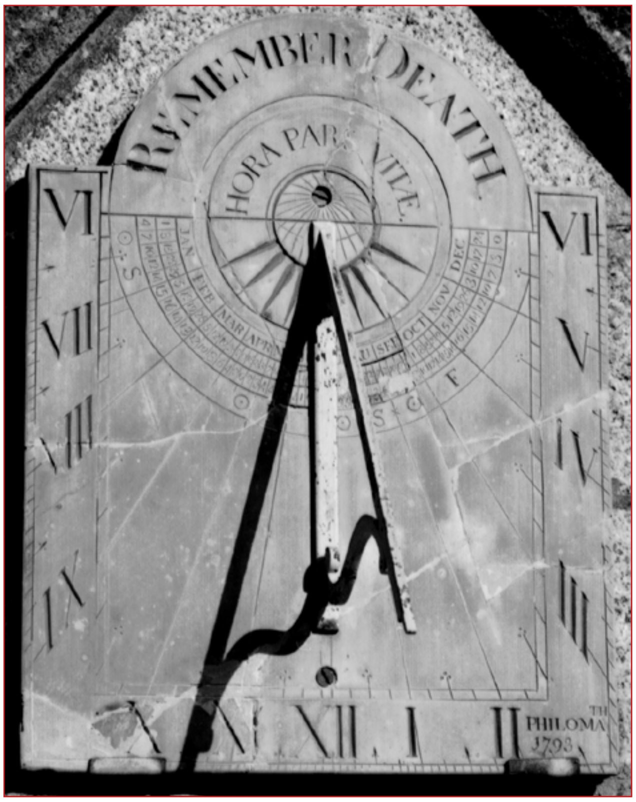


Fig. 7. An EoT table pasted into the lid of a magnetic compass dial. Photo: M. Cowham

John Smeaton in his printed tables (e.g. in Ferguson's 'Astronomy'[33]) in the last quarter of the 18th century. The table on this dial shows a number of mistakes, such as values repeated at the end of February and the beginning of March, and missing values in May. There are other subtle differences which point to this being a poor copy of a table similar to Smeaton's. Strangely, the table associated with the 1892 vertical stone dial at Sandringham, Norfolk, has the same data with the same errors! This latter dial is by the respected firm of F. Barker & Son, 12 Clerkenwell Rd., London. They were a major manufacturer both of dials and of EoT plates[10] so there is clearly more research to be done in this area.

Vertical Dials

Relatively few vertical dials have EoT scales. This is probably partly because they were generally made by local artisans not familiar with the latest scientific discoveries. A second reason is that the users of the dials would not be using them to set their clocks but simply to get a general indication of the local time. Because vertical dials are normally viewed at a substantial distance, the letter-cutting has to be bold to be legible and so EoT data has to be presented in a condensed format. This is usually done by giving the values, in whole minutes only, either on selected dates or on the occasions when the value changes. For the former method, there is a wide variety of ways to choose the dates: it may be certain days of each month, such as the 7th, 14th, 21st and 28th, or it may be for every fifth (for example) day of the year, giving different day numbers in each month. With this wide variety of methods, it is difficult to make comparisons between the underlying data sources.



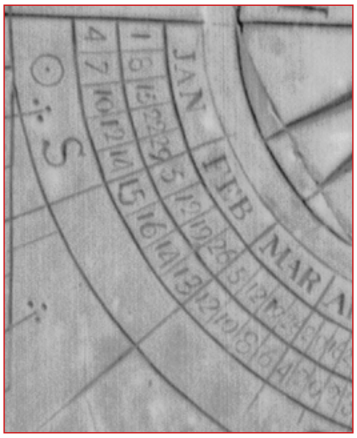


Fig. 8a (above): Camborne church dial, Cornwall.

Fig. 8b (left): closeup of the EoT scale.

Photos: L. Burge.

One quite comprehensive table is that on the vertical south dial at Camborne church as shown in Len Burge's book Cornwall Church Sundials 15 (Fig. 8). The maker's name on this 1793 dial cannot now be read but was drawn as " mes or" by Jeannie Crowley in 1957.[16] The maker describes himself as a 'philomath' - a lover of learning - which perhaps explains the presence of the EoT table. The values, for every seventh day of the year, are fairly standard but there is a major error in February where values reaching 16 minutes are clearly shown. This mistake could have been a simple misreading of the source data but it may be significant that another Cornish dial has a similar error. This latter case is a brass horizontal dial of 1792 from Helston[15] and it also shows every seventh day from 1 January. Could it be that these makers expected the February maximum to mirror the November minimum?

return to "Table of Contents"

Another Cornish vertical dial[15] with an EoT table is at St. Blazey and is dated 1839 (Fig. 9). Here, values are given for four days of each month. The drawback with this scheme is seen by the step of four minutes between consecutive entries in early December, when the EoT is changing at its fastest. The actual values are unremarkable and are accurate for that era.

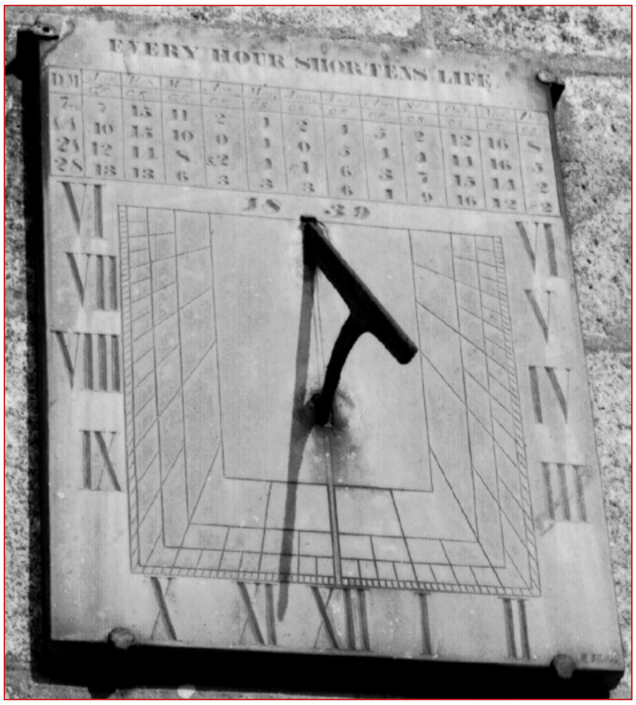


Fig. 9. St. Blazey church dial, Cornwall. Photo: L. Burge

Horizontal Dials

In 1819 a good quality horizontal dial made by W & S Jones of Holborn was installed in the tower of the church at Benenden, Kent, for the purpose of regulating the new clock.[17] The dial has an EoT ring which specifies the values of the maxima/minima as +l4m 37s, -3m 57s, +6m 7s and -16m l6s. From the data of Fig. 1 of Ref. 1, it can be deduced that these values come from a table calculated for the year l825±-15years. Thus it seems that the dial- makers, working to the orders of the clockmakers Thwaites and Reed, have used the most up-to-date table available to them, probably from the Nautical Almanac. What is perhaps surprising, though completely normal for the period, is that no attempt has been made to anticipate the variation in the EoT over the likely lifetime of the dial. Using the tables to set the clock today can result in an error of up to about ±30 seconds, depending on the time of year.

A well-made c.l825 bronze horizontal dial by A. Adie of Edinburgh[19] has a traditional 'Watch Faster/Slower' EoT scale running in an anticlockwise arc, as had become standard by that time: see Fig. 10. Close examination shows that each minute mark on the EoT scale corresponds exactly to a day on the date scale. The scale also has, for example, only one mark for 16mins in Oct/Nov where a pair of marks, one either side of the minimum, is required. This indicates that the source of the data was an as-yet unidentified simplified table, in minutes only, in the form published by Smeaton. This shows how the importance of sundials as scientific instruments had declined since the 18th century, where the integer-minute values of EoT were carefully interpolated to fractional days from daily tables giving the values in minutes and seconds.

Fig. 10. A. Adie dial. Photo © The Trustees of The National Museums of Scotland.

The very fine dial by Troughton & Simms at St. Michael's Mount, Cornwall[20], has a ring which is actually engraved with the legend "Equation of Time" rather than the older "Æquation of Natural Days". The working dates for this company are given[14] as 1826-1922 and the EoT values on the dial, in contrast to those on the Adie one, are properly interpolated to fractional days. The combination of 11 days in July with an EoT above 6mins and 13 days in Oct/Nov where it is above l6mins, dates the data, if not the dial itself, towards the beginning of this period. As makers of astronomical instruments, Troughton & Simms clearly took the EoT seriously.

Slate Horizontal Dials

A recently renovated 1825 Irish slate dial by Samuel Eason has an unusual EoT scale (Fig. 11). The completely circular inner ring gives the names of the constellations and is evenly subdivided into one degree

return to "Table of Contents"


Fig. 11. An 1825 slate dial by Samuel Eason:
(top) prior to its recent restoration;
(bottom)part of EoT scale in degrees of solar longitude.
Courtesy of David Harber Sundials.
Photos: Harriet James

increments of solar longitude, numbered (0), 10, 20, (30). The next ring has the names of the months but these are not subdivided at all. They occupy slightly variable angular spaces to correspond to the zodiac ring. The EoT values are then shown, numbered in minutes. The intervals between the minutes are divided into five but these are *not* 12 second increments as there are the same number of divisions even when the consecutive minutes numbers are the same (e.g. at the maximums and minimums). Instead, it appears that Eason has simply divided the gaps evenly, giving a false impression of the resolution of his scale. The source of his data is unknown: although ‘s first printed EoT Flamsteed table in 1672 was calculated for degrees of solar longitude subsequent tables all give the values as a function of date. Eason is definitely using a Gregorian calendar so he has not simply used the old Flamsteed table. There are other anomalies in the scale which indicate that Eason did not fully understand the properties of the EoT.

Another Irish slate dial, of similar date but by Robert Connell, was sold[21] in 2002 and is shown in Fig. 12. It has a wealth of engraved detail including an EoT table in a four-days-per-month format. In this ease the choice of days is the 1st , 8th, 16th and 24th of each month, suggesting either a different source to that used fer the St. Blazey dial or an extraction from a full table.


Fig. 12. An 1815 slate dial by Robert Connell.
Photo: M. Cowham

The most prolific maker of slate horizontal dials was Richard Melville.[19,22] He seems to have first introduced EoT tables into his multiple-gnomon dials in the 1840s. An example is shown in Fig. 13. He generally gives values starting on 10 January and then for every tenth day. The layout depends on the number of subsidiary dials on the plate and is usually either in two or four arc segments. Not surprisingly, the same data is used on many dials but an exception is the value for 20 May which is variously given as 3 mins or 4 mins (the correct value is 4 mins). Two dials which show 3 mins for 20 May are the 1843 ‘Dunmore’[22] dial and an 1848 one in a private collection, suggesting that he later corrected this particular error. Melville also has


Fig. 13. Part of the EoT ring on a multiple-gnomon slate dial by Richard Melville. This Sussex dial, from relatively late in Melville’s career, has the correct value of 4 mins for May 20. Photo: Harriet James

serious trouble with indicating the sign of the EoT in cases where it changes during the month. In addition to the EoT, later Melville dials also sometimes specify the time offset from Greenwich: a useful feature for finding the original design location.

Fig. 14. The EoT graph on the west side of the Abbey Gardens, Bury St Edmunds, pillar dial. Photo: M. Cowham

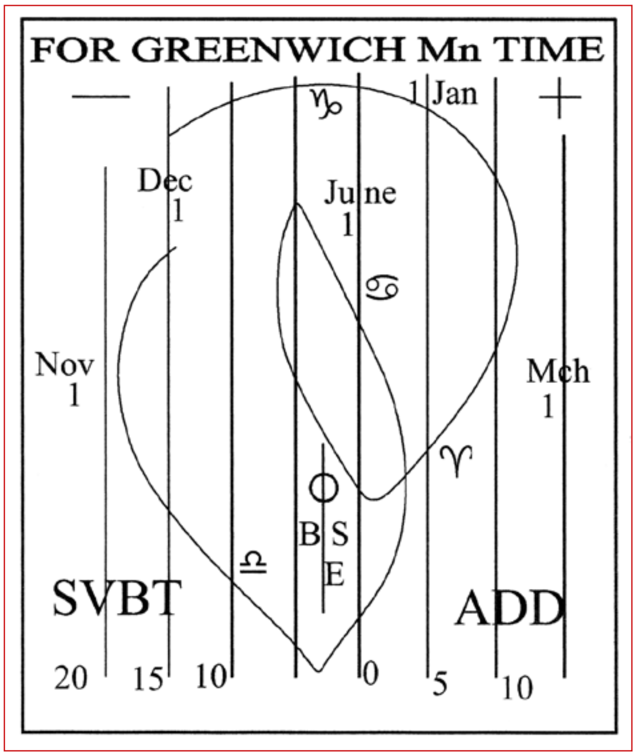


Fig. 15. An interpretation of the EoT graph on the Bury St Edmunds pillar dial.

GRAPHICAL PRESENTATIONS

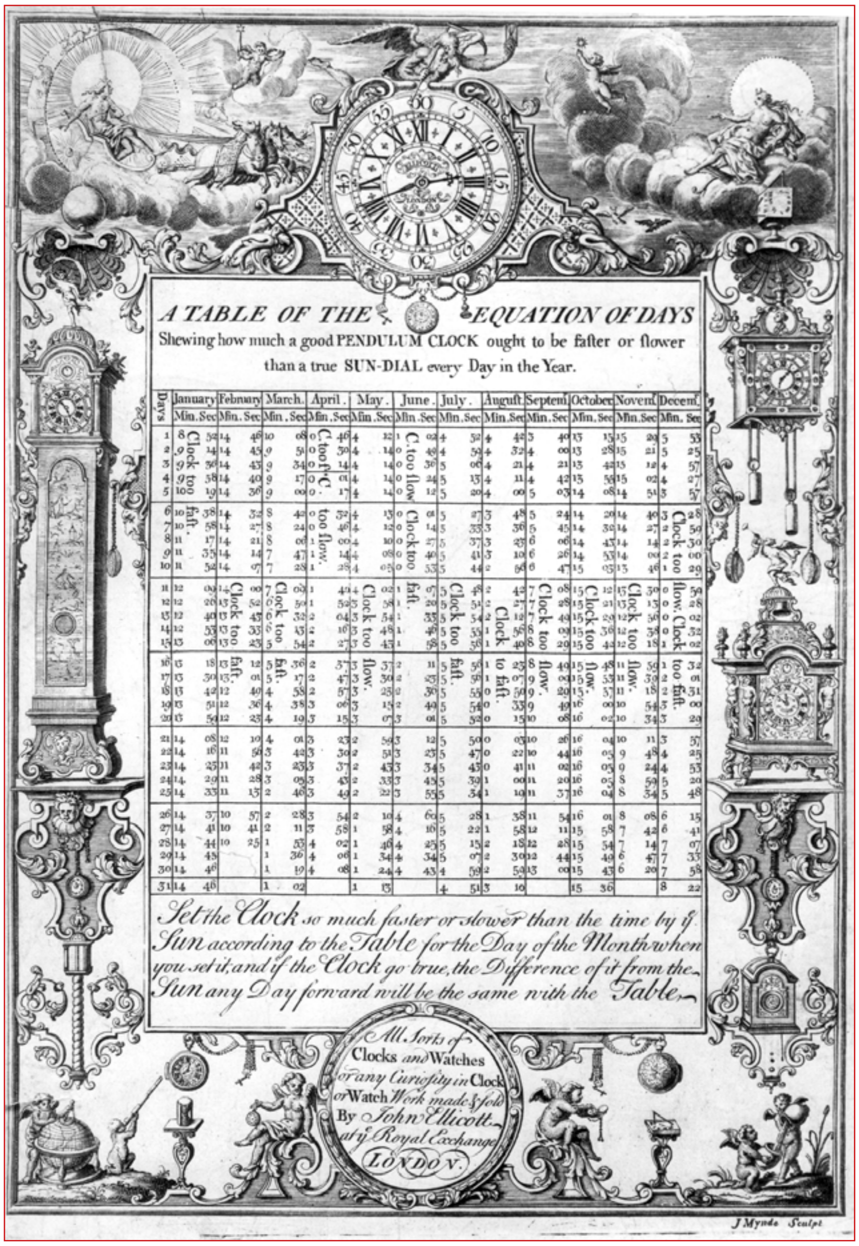
The 1870 pillar dial[23] in the Abbey Gardens, Bury St Edmunds (Figs. 14 and 15) features a single, direct south, dialface with inscriptions of the other vertical faces of the cuboid. That on the west face is a most unusual representation of the EoT, appearing at a first glance to be a 'folded analemma'. Although not a proper x-y Cartesian graph of the EoT, this representation must rank as one of the earliest graphical forms other than true analemmas. The horizontal axis is the time difference but the vertical axis seems to be arbitrary, with the curve being discontinuous at 1 December. The local longitude of 0° 43' E gives a time offset of 2m 52s fast compared to Greenwich solar time. This offset is actually shown on the engraving so the curve really shows the total correction of EoT + longitude.

A presentation in a more standard Cartesian form was shown on the 1936 cast concrete dial by Robert McClintock (see Fig. 5 of Ref 22). In that case, however, it can be seen that the data used only had values for the beginning and middle of each month and simple straight-line interpolations between these values have been used.

CLOCKMAKERS

Some clockmakers followed the lead set by Thomas Tompion and printed EoT tables to be pasted into their longcase clocks. Usually this was only done by the best London makers but one example of a country maker with an EoT table is John Calver (c.1695-1750) of Woodbridge, Suffork.[24] Working prior to 1751, his table showed the value, to minutes and seconds, for just six days each month. The values follow those of the 1702 Smart/Flamsteed table very closely although a significant percentage of the values show a 1 second difference, perhaps the difference between rounding and truncating values from a table calculated to tenths of a second. Calver printed a lengthy introduction to his table, transcribed as an addendum to this paper, which gives a good flavour of the wordiness of the time but which is, nevertheless, remarkably accurate. It also gives the good advice of trying to compare the clock with a sundial near to noon to minimize errors due to refraction or to errors in the layout of the dial. The clockmaker John Ellicott jnr., working at the Royal Exchange, London, around 1740, had a trade card which included a full EoT table (Fig. 16). The data came from Tompion's 1683 table and so was very out of date especially in contrast to Calver's use of more recent data. Its continued use indicates the esteem in which Tompion was held. Ellicott probably inherited the printing plate ("J Mynde Sculpt") from his clockmaker father.

In contrast to Ellicott, William Graham "clock and watchmaker at the Dial in Lombard-Street" printed[25] an EoT table which followed the Smart/Flamsteed



A TABLE OF THE EQUATION OF DAYS
Shewing how much a good PENDULUM CLOCK ought to be faſter or ſlower than a true SUN-DIAL every Day in the Year.

Set the Clock so much faster or slower than the time by ye Sun according to the Table for the Day of the Month when you set it; and if the Clock go true, the Difference of it from the Sun any Day forward will be the same with the Table.

All Sorts of
Clocks and Watches
or any Curioſity in Clock
or Watch Work made & ſold
By John Ellicott
at ye Royal Exchange
LONDON.

J Mynde Sculpt

Fig. 16. John Ellicott junior,Äôs trade card. Note the cherub checking his watch by the sundial in the lower border, enlarged above. (Heal Collection, courtesy of the Trustees of the British Museum.)

data very closely: other than two obvious typographical errors there were just four days where the data differed by 1s. The table, dated 1725, is laid out in exactly the same format as Tompion used for his 1683 table including very similar titles and general wording.

By 1836, the chronometer makers Webster and Hunter of Cornhill, London, were able to print an "Equation Table (for the information of our scientific friends)" which gave the EoT to a resolution of 0.1s for every day of the (leap) year.[24] They neglected to specify, though, at what time of the day the values were for: since the value might be changing at a rate of well over a second an hour, this would have been important if the full accuracy of the table was to be employed. It mattered, for example, whether the value was for (solar) noon at Greenwich or for 12 o'clock.

As an aside, there was an interesting case[26] at the Old Bailey on 7 December 1743 when Thomas Jones was sentenced to transportation for the theft of numerous articles from his employer, the clockmaker Francis de la Balle. Amongst the items found hidden in Jones's room was a copper plate for printing an Equation Table, valued at 8s. What is perhaps surprising is that the plate was held by the clockmaker, rather than a printer. It would be good to find a copy of the printed table.

20th Century Tables

The four most commonly read sundial books (Mayall & Mayall[27] Waugh[28],Cousins[29] Rohr[30]) all give daily EOT tables. Mayall & Mayall and Rohr both give the values in decimal minutes so that the resolution is only 6 seconds which is sufficient for most sundial work but makes accurate comparisons difficult. The values in the 1994 (3rd) edition of Mayall & Mayall are "compiled from the American Ephemeris" and look as though they have not been updated from the 1938 first edition. Rohr specifies that his values come from the 1963 Ephémérides Nautiques and are for true noon. Waugh states that his are "averaged values" but he includes a value for 29 February which results in a 10 second discontinuity in ordinary years. Cousins gives his source as "JGP" and has an optional value for 29 February: the values are appropriate for the 1969 publication date. Of the four books, three use the sign convention of EoT = mean time - solar time, used in the BSS Glossary[31] whilst only Cousins adopts the opposite convention now favoured by professional astronomers[32]. The use of any of these 20th century sources in the first half of the 21st century can typically give errors of up to 10 seconds.

Printer's Errors

A surprisingly large number of printers' errors have been found while examining various historical EoT tables for this study. These can usually be detected as a deviation from the smooth change of EoT from day to day, often caused by a single digit being wrong, such as when the number of minutes does not increment

return to "Table of Contents"

as the seconds in- crease above 59. The misreadings between '3' and '5', and between '6', '9' and '0' are the other most common errors. These errors often persist even in tables which go through many editions, such as Ferguson's 'Astronomy Explained'. The frustration of Charles Babbage, who strived so hard to remove human involvement in the printing of mathematical tables, is easily understood. Modern readers are warned to take care when using old tables. On the other hand, the errors could prove useful in the future when trying to establish if an author or dialmaker copied an early table.

CONCLUSIONS

Continued study of the history of the Equation of Time has shown that a surprising amount of information can be extracted from tables which appear superficially similar. As the records of tables in both printed form and engraved on dials increases, so the value of existing records will grow.

ACKNOWLEDGMENTS

It is a pleasure to thank the many people who have supplied the data, photographs or other information which have made this study possible. They include: Mike Cowham, Andrew James, Jeff Darken (AHS), Catherine Southon (Sotheby's), John Foad, Alan Midleton, John Milburn, Michael Sandford, Michael Harley, Harriet James, Marcella Senior (Trinity College Dublin), Allison Morrison-Low (National Museums of Scotland), David Harber, Len Burge, Peter Ransom, Alison Wright (British Museum), Fred Sawyer, Patrick Powers, Ian Butson.

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return to "Table of Contents"

ADDENDUM

Transcription[24] of the introduction to an EoT table on the door of a longcase clock by John Colver of Woodbridge, c.1720.

> A SHORT Account of the Equation of Time, and Directions for adjusting Pendulum Clocks, &c.
>
> True or absolute Time, called also Duration, constantly flows equally, and in the same Manner, but its Quantity cannot be measur'd but by Motion, for if all Things were as rest, we could by no Means know the Flux, or Quantity of Time, and the Duration of all things would go on without Perception. The Diurnal and Annual Revolutions of the Sun, as having been from the beginning of Nature, suppos'd constant, regular, and universally observable by all Mankind, are generally made use of for the Measure of Duration, But although they seem design'd by the Author of Nature, for Signs, and for Seasons, for Days and for Years, yet the nice Enquiries of Astronomers have found Irregularities in the Sun's apparent Motion, which are by them clearly accounted for, and proceed from a double Cause, (to wit) the Obliquity of the Ecliptick, that is the Circle wherein the Sun moves lying obliquely, or making an Angle of 23 Degrees 29 Minutes with the Equator, or the Circle wherein equal Time is Measur'd, equal Arches of the one cannot answer to equal Arches of the other, And from the Eccentricity of the Earth's Orbit, that is, the Earth moving in an Ellipsis (or Oval), and describing equal Arches in equal Times, those Parts cannot agree with the equal Parts of a Circle. Hence proceeds the Difference between a well regulated clock and a Sun-Dial, the one being suppose'd the true or mean Time, and the other the relative or apparent Time, which Differences are called the Equation of Time, and are calculated in the following Table to every Five Days, which will be found of good Use for the better regulating of Clocks and Watches. The first and last Column shew the Days of the Month, then against the Day you want, and under each respective Month, your have how many Minutes and Seconds your Clock or Watch ought to be faster or slower than the Sun-Dial. In observing the Time by your Dial, it is best to do it near Noon, by reason of the Refractions, or some error of the Dial, and if by Observation you find your Clock have got or lost more than is express'd in the Table, you must alter the Pendulum by the Screw at the Bottom, raising the Bob to make it go faster, and letting it down when you would have it go slower.
>
> Here followeth The TABLE of Equation of Natural Days

return to "Table of Contents"

Early Days & Ptolemy's Invention of the Equation of Time

adapted from the author's paper in NASS The Compendium 16(4), December 2009

Note on Terminology

There is no direct translation of Equation of Time in Greek. The nearest related concept is νυχθημερον = nychthemeron = the duration of day and night e.g. from one solar noon to the next. In Arabic, EoT is taᶜdīl al-ayyām bi layālayhā. In medieval Latin, Æquatio Dierum = Equation of Days.

The concept of mean days/years and the mean Sun was something well understood and extensively used in ancient and mediaeval astronomy. However the ancient mean Sun is one that moves along the Ecliptic and thus is only indirectly associated with the modern mean Sun, which keeps our time and tracks the Equator.

Introduction

The Greeks knew all about the Equation of Time - though not under that name and not for the purpose of telling the time as we understand it. Their knowledge related to astronomical, and in particular, lunar prediction, rather than time telling. There has been extensive study of the EoT from the astronomical perspective from Greek to Renaissance times: see References & Related Reading.

In 1669, Christiaan Huygens (1629 - 1695) published the first 'modern' table of the Equation of Time in his "Instructions Concerning the Use of Pendulum-Watches for finding the Longitude at Sea". Thereafter the Equation of Time became the concern of navigators, time tellers and thus sundial makers. It was around this time that the term Æquatio Dierum translated to Equation of Days or Equation of Time.

Brief Astronomical History

The Babylonians were aware that the Sun's motion was not uniform. Between Virgo and Pisces, they reckoned that the Sun moved at 30° per lunar month, while in the other half of the year, it moved at 28;7.30° (= degrees/mins/secs in their sexagesimal system). The non-uniformity was certainly known to the Greek Callippus as early as 330 BC.

Babylonian astronomers used to measure events. Heavenly events were directly connected to omens – either good or bad. Omens were an important part of the ruler's tool kit of government. So good predictions were vital. Thus, detailed daily records of astronomical phenomena were made as early as 1500 BC. They were primarily concerned with matching observed data to simple arithmetical rules so that predictions could be made. They noted, for example, that eclipses occured in the so-called Saros cycle – every 18.03 years.

Greek astronomy began as a means to tell the seasons rather than the time:

> "When the Pleiades, daughter of Atlas, are rising,
> begin the harvest, the ploughing when they set"
> *Hesiod: Works and Days.*

With the rise of reasoned thought, their approach to astronomy became concerned with why things happen and with *'saving the phenomena'*. In this context, the word 'phenomenon' is used for an observed fact that must be described in terms of a given philosophy.

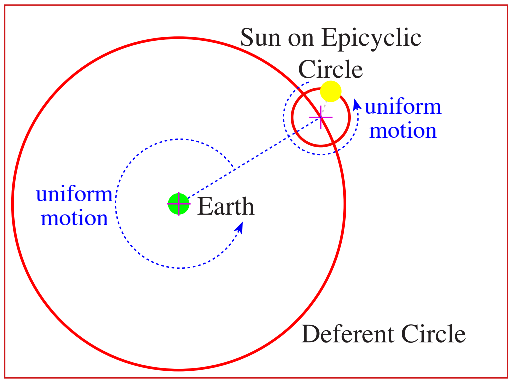
The philosophy used was that derived by Aristotle from Plato's a priori assumption that the Heavens were a perfect sphere and everything therein must be moving in circles at a uniform rate. This philosophy – coined as the "most regressive step in the history of science" - remained extant for almost two millennia, until Kepler's theories were finally accepted. Such was Aristole's influence that it has been said that

> "Science, up to the Renaissance
> consists in a series of footnotes to Aristotle"
> *Koestler: Sleepwalkers.* *'Footnote on Aristolean Cosmology' on page 78..*

'Saving the phenomena' implied producing some kind of working model that could describe the movements of the 'Wandering Stars' – the Planets and the Moon. The Fixed Stars were easy – they just obeyed Aristotle's demands. The Planets, however, usually move in the direction of the Sun, but sometimes they go backwards - their so-call retrograde movement. This was hard to match to a philosophy of uniformity and perfection.

The first really viable general models, that could explain the non-uniform motions of the Sun and the Planets, were developed by Apollonius of Perga 262 - 190 BC – famous amongst mathematicians as the geometer of conic sections. He developed two models:

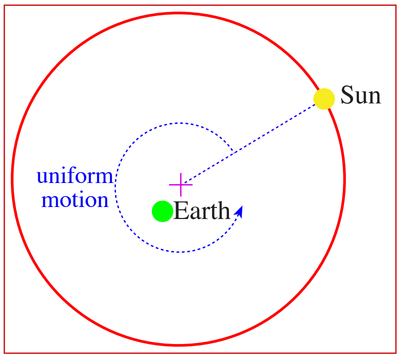
- Epicyclic theory, which works well for sun, moon and planets, in which the heavenly body rotates uniformly around a small 'epicycle' circle, which is centred on the circumference of a large 'deferent' circle. The deferent is centred on the Earth. The epicycle centre moves uniformly around the deferent. This fulfills the philosophical requirement.



Epicyclic Theory

return to "Table of Contents"

- Eccentric theory, which works well for the Sun, in which the heavenly body moves uniformly around a circle, but that the Earth is not the centre of the circle. This simpler model only somewhat fulfills the philosophical requirement.

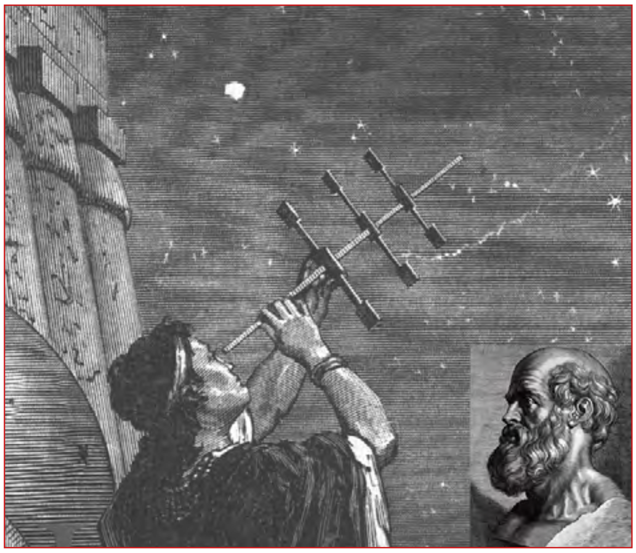


Eccentric Theory

The two models, as shown, are mathematically the same. However, the Epicyclic model is more 'versatile' inasmuch as it is easy to describe the retrograde motion of the inferior planets and more complicated motions can be built by making an epicyclic circle to be the deferent of a yet smaller epicycle.

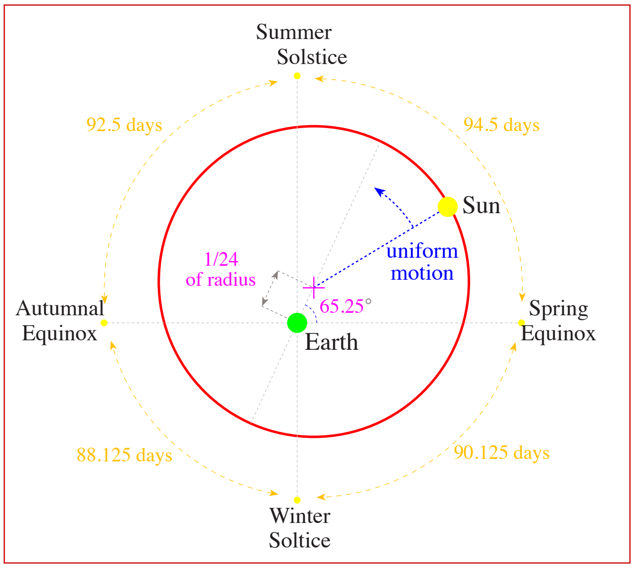
HIPPARCHOS AND THE SUN'S MOTION

After Alexander's conquest of Mesopotamia, the extensive astronomical records of the Babylonian astronomers became available to the Greeks, and the next leap forward was made by the great Hipparchos of Rhodes, 190 -120 BC, who invented trigonometry and who discovered Precession of the Equinoxes. Hipparchos is considered by many as the greatest ancient astronomer. He was not only a theoretician but one who backed his theories by measurement. He also invented the astrolabe and the equatorial ring to find the time of the equinoxes, but had nothing more sophisticated than a vertical gnomon with which the find the solstices, which is difficult since the Sun's declination changes very slowly around those times.



Hipparchos measuring the night sky

Using many measurements of his own and by collating data that stretched back to Babylonian times, he set down the seasonal lengths as follows: Winter Solstice > Vernal Equinox > Summer Solstice > Autumn Equinox > Winter Solstice as 90 $^{1}/_{8}$days > 94 ½ days > 92 ½ days > 88 $^{1}/_{8}$ days > = adding up to 365 ¼ days in total.



Hipparcus Solar Theory

Using these figures, he constructed an Eccentric model that adequately described the Sun's motion. Next, it appears that Geminus of Rhodes 110-40 BC became aware that the time between one solar noon and the next - the nythchemeron - was not constant and that he considered this to be the result of the angle between the Equatorial & Ecliptic planes, which is partially correct.

PTOLEMY AND UNEQUAL DAYS

It was, however, the great Claudius Ptolemy of Alexandria, 85-165 AD, who first provided a comprehensive description of the phenomenon of 'unequal nythchemeron'.

The crowned Ptolemy being guided by the muse Astronomy, from Margarita Philosophica by Gregor Reisch, 1508

return to "Table of Contents"

He provided the means to calculate the difference between nythchemeron and the length of a mean day – which is exactly what we understand by the daily change in the Equation of Time. Furthermore his description clearly delineated between the two elements that we now call the Eccentricity & the Obliquity components of the Equation of Time Here are Ptolemy's own words on the subject…

> "On the Inequality of Solar Days
>
> …. it seems appropriate to add a brief discussion of the subject of the inequality of the solar day.
>
> A grasp of this topic is a necessary prerequisite, since the mean motions which we tabulate for each body are all arranged on the simple system of equal increments, as if all solar days were of equal length. However, it can be seen that this is not so……..for two reasons:
>
> firstly, because of the sun's apparent anomaly:
>
> secondly, because equal sections of the ecliptic do not cross ….. the meridian in equal times.
>
> Neither of these effects causes a perceptible difference between the mean and the anomalistic return for a single solar day, but the accumulated difference over a number of solar days is quite noticeable……"
>
> *adapted from Almagest III 9, translated by G.J. Toomer*

The *'apparent anomaly'* is the difference between the sun's actual movement along the ecliptic compared with its mean movement: this is the Eccentricity component of the EoT.

The *'equal sections of the ecliptic do no cross either the horizon or the meridian in equal times'* describes the Obliquity component of the EoT.

With only a sundial and a water clock to tell the time, why was this important to Ptolemy, again, quoting his own words…

> "… to neglect a difference (in the nythchemeron) of this order would, perhaps, produce no perceptible error in the computation of phenomena associated with the sun and other planets; but in the case of the moon, since its speed is so great, the resulting error could no longer be overlooked,…
>
> Both of these (effects) produce a maximum additive or subtractive effect, which is composed of…..
>
> …. about 3 2/3° due to the effect of the Solar Anomaly...
>
> …. about 4 2/3° due to the variation in the time of Meridian crossing…
>
> *adapted from Almagest III 9, translated by G.J. Toomer*

The ability to foretell solar, planetary and *in particular* lunar events was of great interest to both astronomers and their political paymasters. It gave them the edge that knowledge provides over the common people whose understanding reached only as far as the realms of Astrology. With observations spanning many centuries and events timed by either a gnomon or by the rising and setting of the fixed stars, they achieved more precision than one might expect.

HOW GOOD WAS THE THEORY?

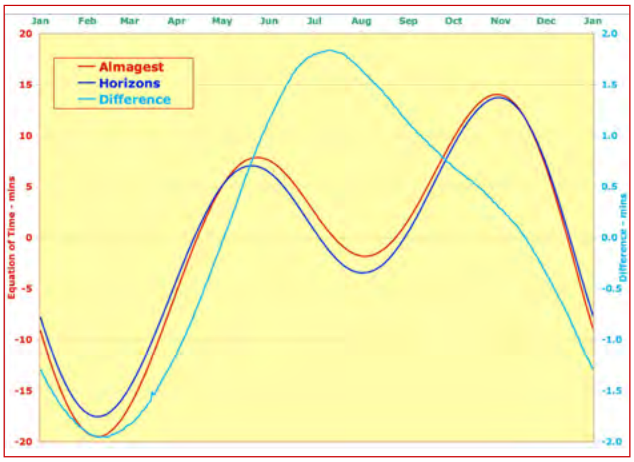
His measurements were made at best with a gnomon, alidade, quadrant and equinoctial ring, But, having mastered the theory, how good were Ptolemy's calculations. Bearing in mind that he used the sexagesimal numbering system and used 'chords' rather than our familiar sines & cosines, his calculations are somewhat opaque for the lay person to follow. However, R.H. van Gent of Utrecht University has handily provided a web-based calculator that embodies all of Ptolemy's calculations.

http://www.phys.uu.nl/~vgent/astro/almagestephemeris.htm

The figure below shows the difference between the EoT derived there from and that calculated using the Jet Propulsion Lab's Horizons program for the year 150 AD. The similarity in shape is remarkable, and the error has a standard deviation of 1 ¼ mins.

Note. Ptolemy started his whole cosmology at Noon in Alexandria on Day 1 of the Month Throth of the Era Nabonassar = 26th February 746 BC. From this date he considered that there was a continuous reliable record of astronomical events covering both the Babylonian and Greek eras. At that time, his Equation of Time was zero. In 150 AD, 900 years later, his calculated values had all shifted so that they spanned between -1 and 28 minutes.

To make the comparison between modern and ancient calculations, Ptolemy's figures have been linearly shifted to make his annual average EoT the same as the annual average EoT calculated by the Horizons program.



Ptolemy's Equation -v- NASA JPL Horizons.
- not much improvement over the years -

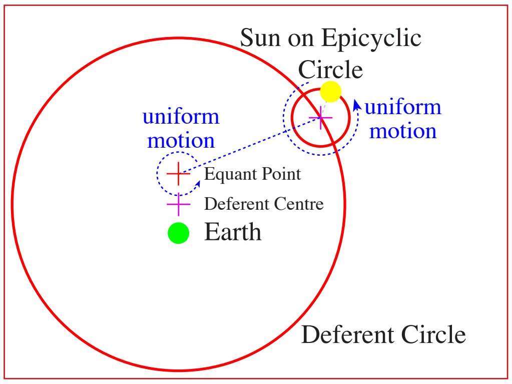
One may be amazed at how good the Greek theory was. But, on the other hand, the degree of accuracy obtained reflects our tiny orbital eccentricity and the fact that Ptolemy's method to obtain the Obliquity component was no better or worse than that found using modern trigonometry.

The Almagest

Ptolemy's greatest legacy was his massive work, The Almagest.

Μαθηματικὴ Σύνταξις = Mathematical Syntacsis =>
المجسطي = Al-Majisṭī = The Greatest = The Almagest

In this, he developed a complete and predictably 'accurate' theory of planetary and lunar movement. In order to 'save the phenomenon' for moon and planets - i.e. to fit theory to observation - Ptolemy found he needed to move the Earth away from the centre of the deferent and to move the centre of epicyclic uniform motion an equal amount in the opposite direction to an imaginary Equant Point (Punctum Aequans). In so doing, he deviated from Plato's philosophical assumption that everything moved uniformly in circles.



The Equant Point

We can now recognise that the Equant Point to be the fore-runner of Kepler's ellipse focal point.

The Almagest was the standard textbook on astronomy for some 1400 years until the end of the Renaissance – a period of time only superseded in the technical world by Euclid on geometry and Galen on medicine.

Ptolemy's - Geographia - printed 1482 - British Library.

Ptolemy was a man of towering intellect and influence. Throughout mediaeval times, Claudius Ptolemy was confused as one of the Ptolemies who ruled Egypt after the conquest of Alexander. Hence the crown. But also, the title 'King Ptolemy' is generally viewed as a mark of respect for Ptolemy's elevated standing in science.

Ptolemy, crowned, with the globe - representing his work on Geography - with an earlier Greek astronomer Hipparchos, with the celestial globe.

From Raphael's 1510 fresco 'The School of Athens'.

Other than the Almajest, his works are recorded in a number of books – one on Optics (lost), Tetrabiblos (which is on Astrology), Geographica (on map making and with maps), Handy Tables (the original Astronomical Almanac – which tabulates the Equation of Time and much else).

Ptolemy - along with Euclid (Geometry), Galen (Medicine), Aristotle (Natural Philosophy & Cosmology) - defined what we now call science until the Enlightenment in the late 16th C and early 17th Century. (That canon has since been increased by Newton, Darwin and Einstein.)

Ptolemy's Legacy

Although, there was much work on the equation of time by Islamic and Mediaeval astronomers - for example: al-Battānī (C9), al-Khwārizmī (C9) Kūshyār (C10), al-Kāshī (C14), there was nothing more substantial than improvements to the computation of solar longitude and the value of obliquity. Their focus was on navigational methods and the finding the Quibla : (the direction to Mecca).

return to "Table of Contents"

There were many reasons for the longevity of the Ptolemy's legacy – the dark ages, the burning of the Alexandrian library, the great schism, scholastic dogmatism towards Aristotle's precepts and the neatness with which Aristotle's cosmology fitted the requirements of both Christian and Muslim faiths.

However, there were three technical reasons why all of Ptolemy's works survived so long.

Firstly, there was nothing better.

Secondly, Ptolemy's theory for the planets (if not for the Sun) was technically very demanding to understand.

When Alphonso X The Wise of Castile (1221 - 1284 AD), who was a great patron of astronomers, was being taught the Ptolemaic system, he is quoted as saying...

"If the Lord Almighty had consulted me before embarking on creation thus, I should have recommended something simpler"

Thirdly, the imaginary Equant Point point deeply offended both Islamic and mediaeval western astronomers. It flew directly in the face of the Aristotelean philosophical requirements.

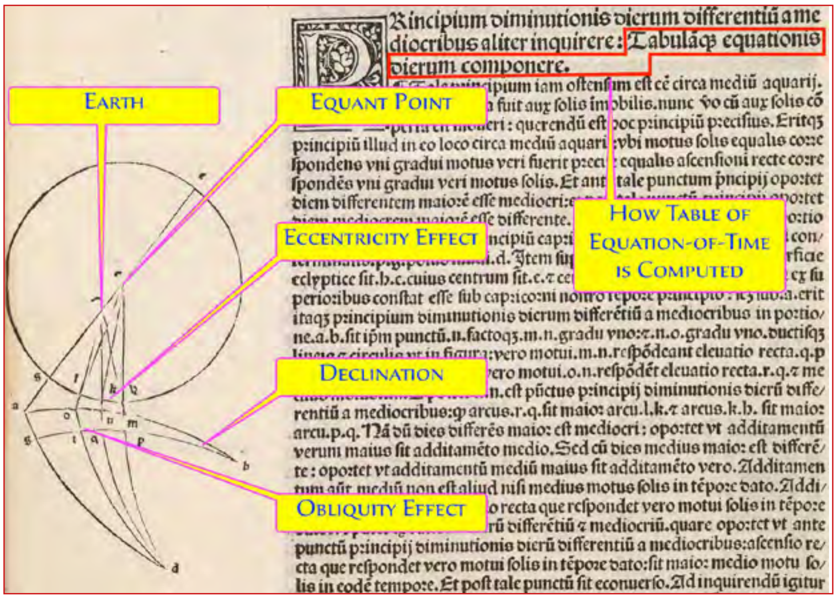
As a result, the astronomers - notably, the Persian al-Tūsī (C13), the Syrians al-'Urdi (C13) and Ibn al-Shatir (C14) and Nicolaus Copernicus (C16) - spent an *inordinate* amount of intellectual energy trying to eradicate it by introducing equants on top of equants.

The Almagest was translated from its original Greek into Syriac and then into a number of Arabic versions as part of the great Translation Movement in the 8th C. The earliest known version of the Almagest is in the Vatican library. It is in C9 Greek and presumably from the libraries of Constantinople. There is little doubt, however, that the main influence of the Almagest in Europe came via the Arabic translations.

From Arabic, it was translated to Latin in Toledo in 1175. The work entered the mainstream of western astronomy with the commentary by Peurbach and Regiomontanus in 1475, and its printing in Venice in 1515.

For the first time, this translation opened up the wonders of Greek astronomy to a wider audience.

Page from Ptolemy's Handy Tables showing the EoT.
C9 Greek from the Vatican Library



Principium diminutionis dierum differentiũ a mediocribus aliter inquirere: Tabulãqꝫ equationis dierum componere.

1496 - Epytoma Ioannis de Monte Regio (Regimontanus)
Almagestum Ptolomei.
Regimontanus produced a synthesis of the Almagest, which, for the first time, made the Almagest accessible to everyone.

return to "Table of Contents"

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8. Benno van Dalen: 'On Ptolemy's Table for the Equation of Time', *Centaurus*, 37 97-153 (1994). A most detailed and scholarly analysis on the subject of the EoT.
9. E.S. Kennedy: 'Two Medieval Approaches to the Equation of Time', *Centaurus*, 31 1-8, (1998). Discusses the computational approach of Kūshyār and Al-Kāshī
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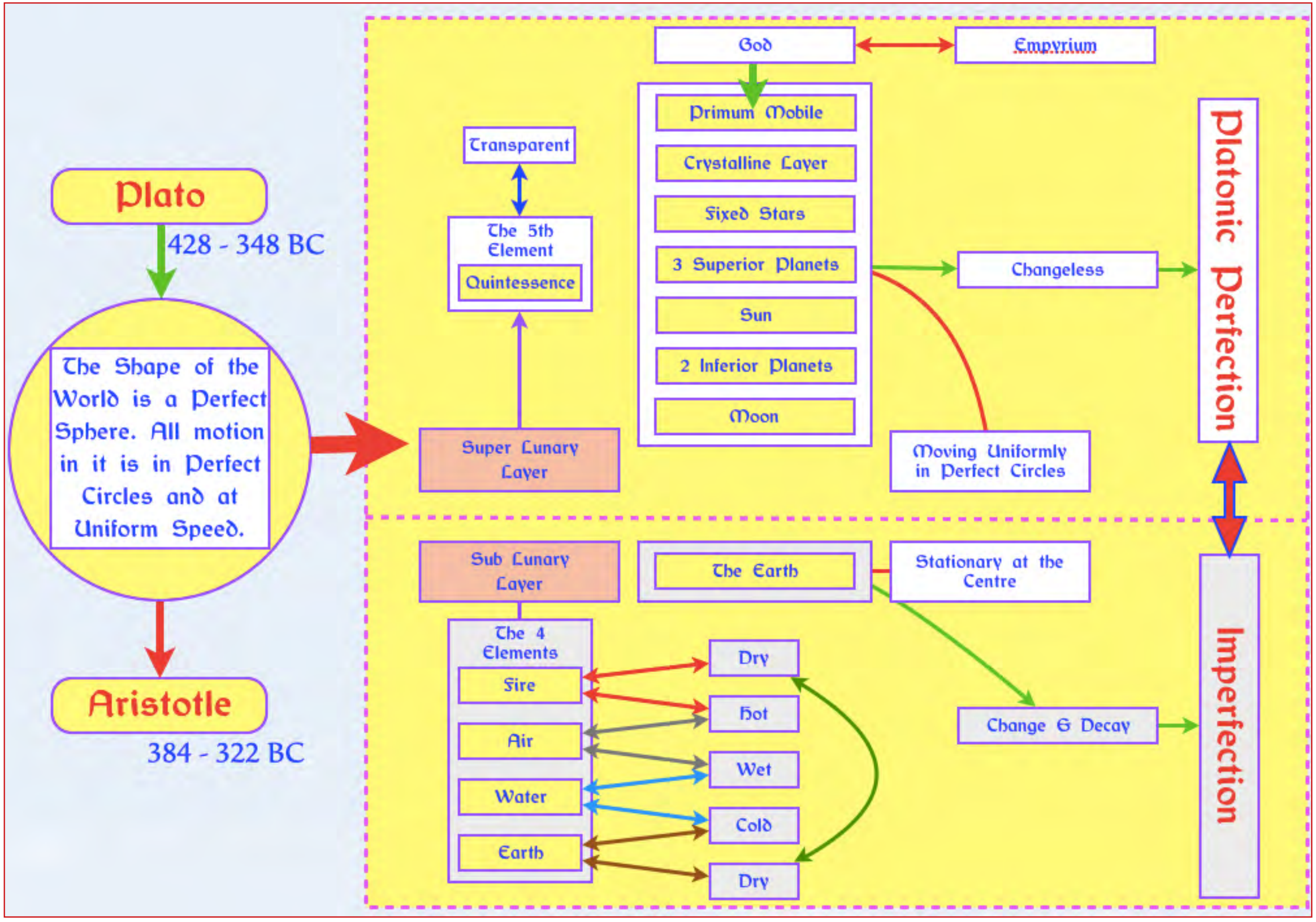
11. Jet Propulsion Laboratory: http://ssd.jpl.nasa.gov/horizons.cgi, Horizon's Ephemeris calculator, which uses the DE405/406 routines. The DE405 routines therein are directly called for calculations published in the annual Astronomical Almanac.
12. Kevin Lee(?), University of Nebraska-Lincoln: http://astro.unl.edu/naap/ssm/animations/ptolemaic.swf. A good animation to show the Equant Point.
13. Dennis Duke: http://people.sc.fsu.edu/~dduke/arabmars.html. A good animation of Islamic attempts to eradicate the Equant Point.

Footnote on Aristolean Cosmology

The elegance and completeness of Aristolean Cosmology, together with how well it fitted with the precepts of both Christian and Muslim faiths, was largely responsible for the lack of significant progress in natural philosophy (i.e. science) until 14th/15th C. This in turn allowed Ptolemy's astronomy, albeit deviant in the introduction the Equant Point, to be substantially unchallenged. In the diagram below, the top two items, God & Empyrium (= where God is) were Christaian inventions.There were two major additions of this cosmology : the introduction of additional layers, being Purgatory and Hell, codified by Dante in 14th C.



return to "Table of Contents"

The Equation of Time in Astronomy & Navigation

There are many uses to which the Equation of Time has been used.

EARLY ASTRONOMICAL TABLES

Since the publication of Ptolemy's Almagest, written around 150 CE, See 'Early Days & Ptolemy's Invention of the Equation of Time' on page 73., astronomers have been tabulating and using the Equation of Time - primarily for predicting lunar movement. There was little advance on Ptolemy's work, since there few who could fully understand his complex work. And, those who did understand the work were philosophically offended by the his introduction of the imaginary Equant Point that he used to simulate what we would now call the focus of an ellipse.

In the 8th C, the Zij al-Sindhind tables were compiled in Baghdad based on the work of Indian and Persian astronomers. ca. 1090, the Toledan tables were compiled in Spain by a group of Muslim and Jewish astronomers, but these contained many errors.

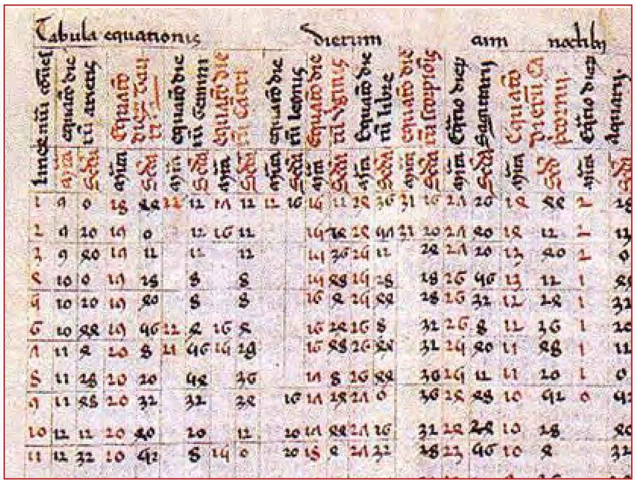
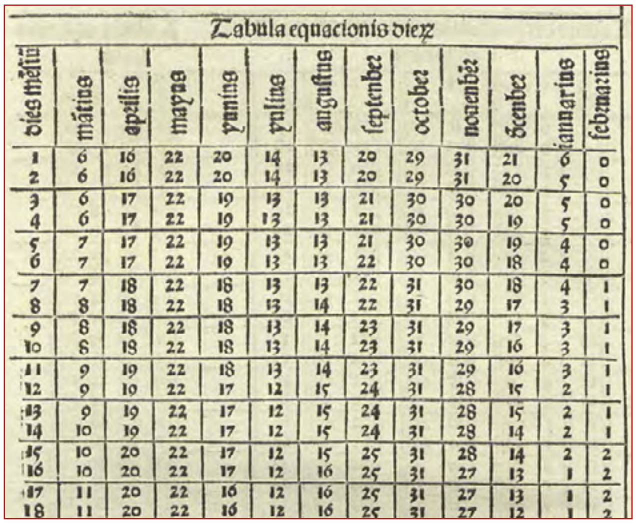


Fig 1 -Part of Tabula Equationis Dierum from the Alfonsine Tables - 1492. Note the EoT in the last column.



Tabula equacionis diez

dies mẽſiũ	mãtius	apꝛilis	mayus	yunius	yulius	auguſtus	ſeptenbꝛ	octobꝛ	nouẽbꝛ	decẽbꝛ	ianuarius	febꝛuarius
1	6	16	22	20	14	13	20	29	31	21	6	0
2	6	16	22	20	14	13	20	29	31	20	5	0
3	6	17	22	19	13	13	21	30	30	20	5	0
4	6	17	22	19	13	13	21	30	30	19	5	0
5	7	17	22	19	13	13	21	30	30	19	4	0
6	7	17	22	19	13	13	22	30	30	18	4	0
7	7	18	22	18	13	13	22	31	30	18	4	1
8	8	18	22	18	13	14	22	31	29	17	3	1
9	8	18	22	18	13	14	23	31	29	17	3	1
10	8	18	22	18	13	14	23	31	29	16	3	1
11	9	19	22	18	13	14	23	31	29	16	3	1
12	9	10	22	17	12	15	24	31	28	15	2	1
13	9	19	22	17	12	15	24	31	28	15	2	1
14	10	19	22	17	12	15	24	31	28	14	2	1
15	10	20	22	17	12	15	25	31	28	14	2	2
16	10	20	22	17	12	16	25	31	27	13	1	2
17	11	20	22	16	12	16	25	31	27	13	1	2
18	11	20	22	16	12	16	25	31	27	12	1	2

Fig 2 - Tabula Equacionis Diez from Almanach Perpetuum - 1496 by Abraham Zacut.
Note how the year begins in March - the 1st Point of Aries

Tabula æquationis dierum cum nocti

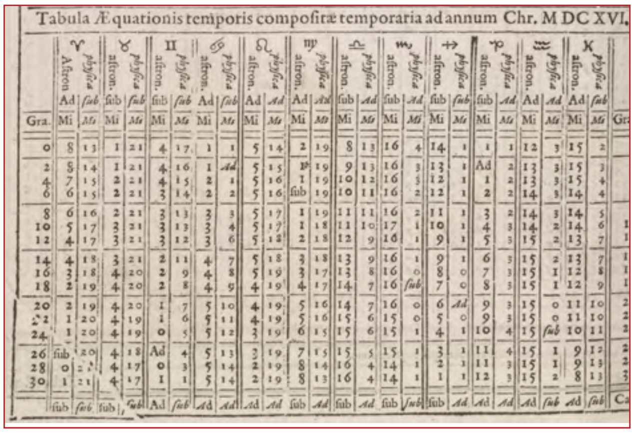
	♈ 0		♉ 1		♊ 2		♋ 3		♌ 4		♍ 5		♎ 6		♏ 7		♐ 8		♑ 9
	m.	se.	m	se.	m.	se	m	se.	m	se.	m	se.	m	se.	m	se.	m	se.	m
0	7	48	17	8	20	48	16	20	11	44	15	24	25	8	32	24	29	40	16
1	8	8	17	24	20	44	16	8	11	44	15	36	25	24	32	28	29	20	15
2	8	28	17	40	20	36	15	56	11	44	15	52	25	44	32	32	29	4	15
3	8	48	17	52	20	32	15	44	11	40	16	8	26	4	32	36	28	48	14
4	9	8	18	4	20	28	15	28	11	40	16	24	26	24	32	40	28	28	14
5	9	28	18	16	20	24	15	16	11	40	16	40	26	44	32	44	28	4	13
6	9	48	18	28	20	16	15	4	11	44	16	52	27	4	32	48	27	40	13
7	10	8	18	40	20	12	14	52	11	44	17	8	27	24	32	52	27	20	12
8	10	28	19	0	20	8	14	36	11	48	17	28	27	40	32	52	26	56	12
9	10	48	19	16	20	0	14	20	11	25	17	48	27	56	32	48	26	32	11
10	11	8	19	28	19	52	14	8	11	56	18	4	28	16	32	44	26	4	11
11	11	28	19	36	19	44	13	56	12	0	18	24	28	32	32	40	25	40	11
12	11	48	19	44	19	36	13	48	12	4	18	44	28	48	32	36	25	16	10
13	12	8	19	56	19	28	13	36	12	8	19	4	29	4	32	34	24	48	10
14	12	28	20	4	19	20	13	28	12	12	19	24	29	20	32	32	24	20	9
15	12	48	20	12	19	16	13	16	12	20	19	44	29	36	32	28	23	52	9
16	13	8	20	16	19	8	13	4	12	28	20	4	29	52	32	24	23	24	8
17	13	32	20	20	19	0	12	56	12	36	20	24	30	8	32	16	23	0	8
18	13	48	20	28	18	48	12	48	12	44	20	44	30	20	32	8	22	32	7
19	14	4	20	36	18	36	12	44	12	52	21	8	30	36	32	0	22	4	7

Fig 3 - The Alfonsine Tables
from Tabulae resolutae de supputandis siderum motibus clarissimi mathematici - 1542 by Ioannes Virdung
Note how the Zodiac signs are used in place of months.

Alfonso the Wise, King of Castile, sponsored his Tables, in order to correct the Todelan tables and update the astronomy. These were used for many years.

With the publication of Copernicus's famous 'De Revolutionibus', Erasmus Reinhold set out to re-calculate afresh, from Copernicus's basic parameters, a new set of tables. These were the Prussian Tables (1551), dedicated to Albert, Duke of Prussia.

The next significant step was the Rudophine Tables, These were started by Tycho Brahe but completed by Joannes Kepler in 1627, and named in honour of their patron Rudolf II, Holy Roman Emperor.



Tabula Æquationis temporis compoſitæ temporaria ad annum Chr. M DC XVI.

Fig 4 - Tabula Aequationis Temporis from The Rudolphine Tables - 1627 by Joannes Kepler.
Note again the Zodiac signs are used in place of months, and the first entry is the 1st Point of Aries

return to "Table of Contents"

MARINE NAVIGATION

> …nothing is so much wanted and desired at sea,
> as the discovery of the longitude,
> for the safety and quickness of voyages,
> the preservation of ships, and the lives of men…
> *(British) Longitude Act, 1714*

It it is easy to find the latitude of a ship at sea, using an altitude of the noon sun and a declination table provided by the astronomers. But there was no reliable means to find longitude without a reliable clock. If noon could be measured at sea, this could be compared the time shown on an clock, which was set at a port of known longitude. If the measured time was at 1 p.m. clock time, the ship was 15° east of the port longitude. However, noon measured from the sun, must be corrected for the Equation of Time, so that mean time can be compared to mean time.

Fig 5 - Christiaan Huygens 1629 - 1696

THE GENIUS OF CHRISTIAAN HUYGENS

Christiaan Huygens was the greatest western scientist straddling the time between Galileo and Newton. Although a Dutchman, most of his working life was spent in Paris, where he was largely responsible for the creation of the French Academy of Science. He was the first overseas member of the Royal Society. He was a true polymath and an international communicator with interests in geometry, optics, astronomy, mechanics, musical theory - and clocks...

The idea that longitude at sea could be found with a good clock was first proposed by the Dutch astronomer Gemma Frisius in 1530. 100 years later, a blind Galileo invented the pendulum clock - never built, but a picture of which was drawn by his son.

Fig 6 - Bruce-Oosterwijck Longitude Pendulum clock 1656.

In 1656, Huygens made the first pendulum clock. He solved the problem of non-isochronicity of large swing pendulums – needed for the verge and foliate escapement then used – by using 'cheeks' or 'chops'. See also 'Footnote 1 - Huygens' Pendulums' on page 90.. 6 years later, he collaborated with Alexander Bruce, the Scottish Earl of Kincardine and a scientist, living in exile is Holland.

Four clocks were designed by them, two of which were made by the leading clockmaker, Severyn Oosterwijck, in the Hague. See Fig 6.

Ships were used to test these and other clocks. On one, the captain forgot to wind the clock and faked the results. Another voyage seemed to have been successful – but subsequent studies suggests that the results were erroneous, not least because Huygens was unaware of the latitude effect on gravity, discovered a few years later by the French.

Huygens turned his attention to spring driven watches. However, he made little progress, baffled by the temperature effect on springs.

In 1665, he issued 'Short Instruction Regarding the Use of Clocks for Finding the Longitudes of East and West'.

In this, he provided a complete and detailed set of instructions of how to use clocks and obtain longitude at sea – including a first accurate table of the Equation of Time, which he called the Æquation, see Fig 9. The days were in the Julian calendar, 11 days different from the present Gregorian. And the values are all positive - making the EoT for 31st Jan/ 1st Feb zero rather than their usual minimum negative values

The instructions must be the first cogent and complete scientific description of a process in history of science, including exactly what records to keep.

These are given in *a highly summarised* form overleaf. The images are from the Royal Society's translation, which starts...

return to "Table of Contents"

> Whereas 'tis generally esteemed that there is no Practice for the Finding of the Longitude at sea comparable to that of those Watches, which instead of a Ballance-wheele are regulated by a Pendulum, as now they are brought to great perfection, and made to measure time very equally.
>
> *Royal Society : Philosophical Transactions (1665-1678), 1669, Vol. 4 (1669)*
>
> *https://www.jstor.org/stable/100996*

Instructions Concerning the Use of Pendulum-Watches for Finding the Longitude at Sea

1) One needs to have at least two watches on-board, in case one stopped or in need of cleaning.
2) the person in charge of the watches should go to a watchmaker for training in winding and setting the hands.
3) the watches should be situated on board in a place safe from disturbance, which is as clean and dry as possible
4) before going on-board, the watches should be set by a watchmaker, who has a timepiece that is known to be correct. If this is not possible, follow item 5)
5) ***How to set the watch and check by how much it is gaining or losing***

   The reason why the sun does not tell mean time is given, see Fig 8, and a tabulation of 365 days (based on Julian time) of the Æquation is provided - see Fig 9 overleaf - and its fundamental use is explained.

   When solar time is first used to set the watch, the Æquation for that day must be ***subtracted*** from the observed solar time to get Local Mean Time (LMT) at noon.

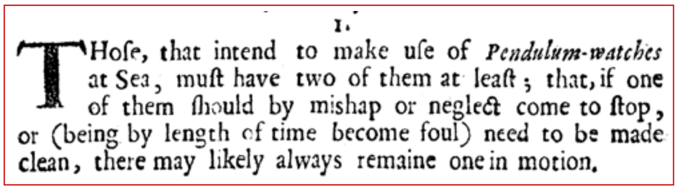
   Subsequently, on another day, the appropriate Æquation value must be ***added*** to solar time to get LMT.

- If the watch was not initially set by a watchmaker to mean time, then...

  When on land, draw a N/S line on the ground (it is assumed one knows how to do this and accuracy is not essential), hang two plumb bobs over the line and measure (through a darkened glass or one smoked with a candle) the moment when the centre of the sun is lined up to the plumb lines. Then set the watch to 12 hours less the value of the Æquation for that day. This is LMT at port.

- When in the harbour, (it not being possible at sea), with the clock running, measure the time between sunrise and sunset. Then calculate Sunrise time + half the time difference between Sunrise and Sunset. That is the clock time when the sun is due south. Add the appropriate Æquation to that time, which gives noon LMT at port.

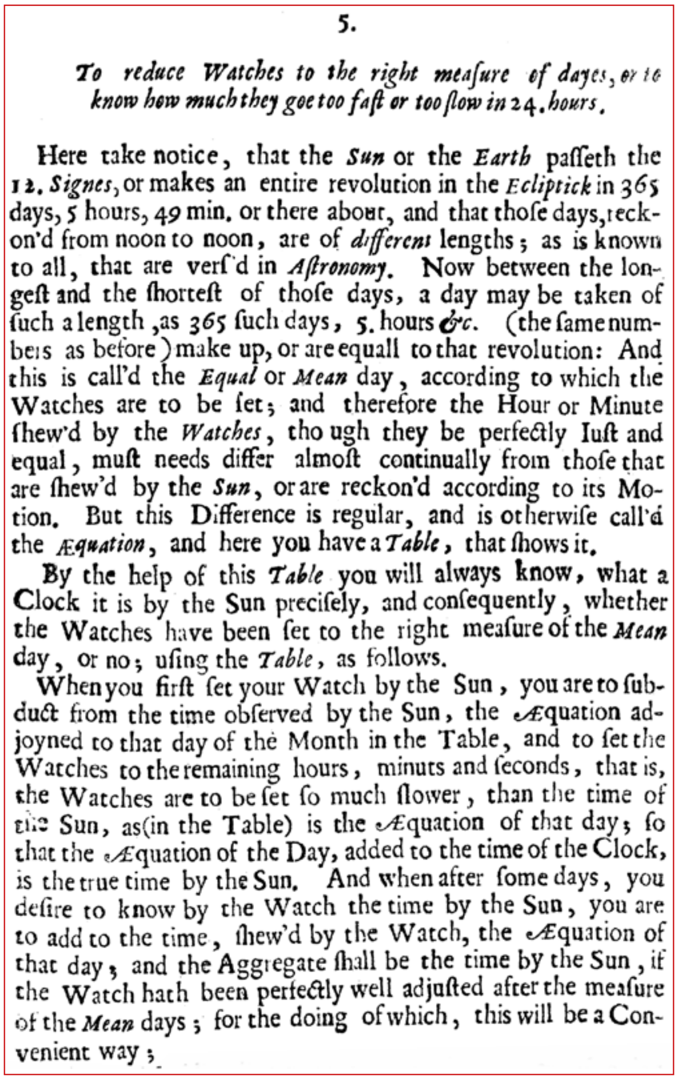
  After as many days as possible, repeat the exercise. If clock time of mean noon is the same, the clock is well adjusted. If not, either adjust the pendulum or just note the daily gain/loss.



1.

THoſe, that intend to make uſe of *Pendulum-watches* at Sea, muſt have two of them at leaſt; that, if one of them ſhould by mishap or neglect come to ſtop, or (being by length of time become foul) need to be made clean, there may likely always remaine one in motion.

Fig 7 - Step 1



5.

To reduce Watches to the right meaſure of dayes, or to know how much they goe too faſt or too ſlow in 24. hours.

Here take notice, that the *Sun* or the *Earth* paſſeth the 12. *Signes*, or makes an entire revolution in the *Eclíptick* in 365 days, 5 hours, 49 min. or there about, and that thoſe days, reckon'd from noon to noon, are of *different* lengths; as is known to all, that are verſ'd in *Aſtronomy*. Now between the longeſt and the ſhorteſt of thoſe days, a day may be taken of ſuch a length, as 365 ſuch days, 5. hours *&c.* (the ſame numbers as before) make up, or are equall to that revolution: And this is call'd the *Equal* or *Mean* day, according to which the Watches are to be ſet; and therefore the Hour or Minute ſhew'd by the *Watches*, though they be perfectly Iuſt and equal, muſt needs differ almoſt continually from thoſe that are ſhew'd by the *Sun*, or are reckon'd according to its Motion. But this Difference is regular, and is otherwiſe call'd the *Æquation*, and here you have a *Table*, that ſhows it.

By the help of this *Table* you will always know, what a Clock it is by the Sun preciſely, and conſequently, whether the Watches have been ſet to the right meaſure of the *Mean* day, or no; uſing the *Table*, as follows.

When you firſt ſet your Watch by the Sun, you are to ſubduct from the time obſerved by the Sun, the Æquation adjoyned to that day of the Month in the Table, and to ſet the Watches to the remaining hours, minuts and ſeconds, that is, the Watches are to be ſet ſo much ſlower, than the time of the Sun, as (in the Table) is the Æquation of that day; ſo that the Æquation of the Day, added to the time of the Clock, is the true time by the Sun. And when after ſome days, you deſire to know by the Watch the time by the Sun, you are to add to the time, ſhew'd by the Watch, the Æquation of that day; and the Aggregate ſhall be the time by the Sun, if the Watch hath been perfectly well adjuſted after the meaſure of the *Mean* days; for the doing of which, this will be a Convenient way;

Fig 8 - Step 5

- Alternatively, use the equal altitude of the sun before/after noon, instead of sunrise/set. The effect of refraction being negligible.

6) How to find longitude at sea

Before setting sail, for each watch, set the time to solar time reduced by the Æquation for that day (i.e. port LMT). Record the date, so any watch gaining/losing can be corrected

When at sea, measured the solar time on each watch, correct for daily gain/loss, add the Æquation for that day. This gives sea LMT

The difference (LMT at sea – LMT at port) x 15°/hr gives the longitude difference from the port. If the difference is positive, the value is East of the port.

return to "Table of Contents"

7) *To find the time of day at sea*

An error of 1 minute in measuring solar time is equivalent 1/4° of longitude (or 15 miles at the equator). Thus it is important to measure solar time as accurately as possible and since that altitude of the sun varies very little around noon, measuring time around noon is not advised. It is better to observe the sun's altitude when is as close as possible to east or west, when the altitude is varying more quickly with time.

Knowing the height of the pole star, the sun's declination and a measured altitude, solar time can be calculated - but in a cumbersome and error prone way. *It is better to use the following.*

8) ***How by Observing the Rising and Setting of the Sun, and the Time by the Watches, the Longitude at Sea may be found.***

Take the watch time at sunrise and sunset, take half the interval between sunrise/set, add that to sunrise time. Add the Æquation for that day. This gives noon LMT.

Alternatively, measure sunset and the following sunrise and find midnight LMT

9) *If you are far north or south...*

you can also take the time a star is rises and sets. Half way between these times, the star is due south (or north). Then knowing the Right Ascension of both the star and the sun, one can calculate the time when the star was at zenith and thus the longitude at that midnight.

10) It can happen that the watches, that have been running consistently, start to differ. In this case, if no other reason is obvious, use the fastest watch, since the wire on which the pendulum hangs can only get longer by stretching : it can never get shorter.

11) When you get in sight of land, check how this differs from any map that may or may not be accurate. This can then help improve the accuracy of the map. It can also show how far has been sailed east or west.

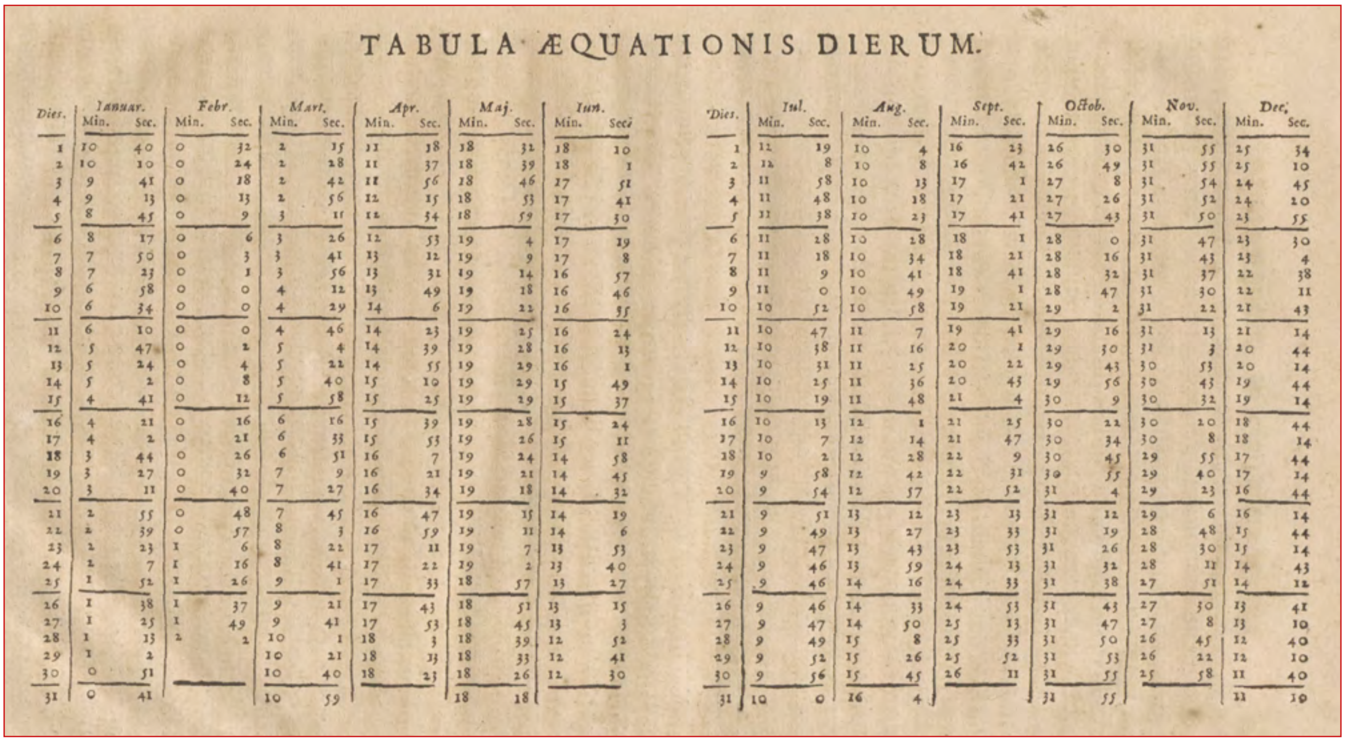
If by mischance or carelessness, all the watches have stopped, you can restart them again and use that place of known longitude as the origin

12) *If all the watches stop, get then going as soon as possible. Thereby, one can at least know the relative longitudinal movement you make from the place where the watches are set*

13) gives the exact format for recording all the watch readings.

Of interest in this methodology is its simplicity. While clocks had to tick accurately, they did not require to be set to the exact mean time as we now know it. And, since his table was zero-ed on the day of minimum EoT, the conversion from solar to LMT was always an addition, and vice versa, a subtraction

Many other methods of finding longitude were proposed involving, inter-alia, the moon, the moons of Jupiter and other astronomical events. All of these were difficult to measure at sea during the night and were complicated to compute.



TABULA ÆQUATIONIS DIERUM.

Dies.	Januar.		Febr.		Mart.		Apr.		Maj.		Iun.	
	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
1	10	40	0	32	2	15	11	18	18	32	18	10
2	10	10	0	24	2	28	11	37	18	39	18	1
3	9	41	0	18	2	42	11	56	18	46	17	51
4	9	13	0	13	2	56	12	15	18	53	17	41
5	8	45	0	9	3	11	12	34	18	59	17	30
6	8	17	0	6	3	26	12	53	19	4	17	19
7	7	50	0	3	3	41	13	12	19	9	17	8
8	7	23	0	1	3	56	13	31	19	14	16	57
9	6	58	0	0	4	12	13	49	19	18	16	46
10	6	34	0	0	4	29	14	6	19	22	16	35
11	6	10	0	0	4	46	14	23	19	25	16	24
12	5	47	0	2	5	4	14	39	19	28	16	13
13	5	24	0	4	5	22	14	55	19	29	16	1
14	5	2	0	8	5	40	15	10	19	29	15	49
15	4	41	0	12	5	58	15	25	19	29	15	37
16	4	21	0	16	6	16	15	39	19	28	15	24
17	4	2	0	21	6	33	15	53	19	26	15	11
18	3	44	0	26	6	51	16	7	19	24	14	58
19	3	27	0	32	7	9	16	21	19	21	14	45
20	3	11	0	40	7	27	16	34	19	18	14	32
21	2	55	0	48	7	45	16	47	19	15	14	19
22	2	39	0	57	8	3	16	59	19	11	14	6
23	2	23	1	6	8	22	17	11	19	7	13	53
24	2	7	1	16	8	41	17	22	19	2	13	40
25	1	52	1	26	9	1	17	33	18	57	13	27
26	1	38	1	37	9	21	17	43	18	51	13	15
27	1	25	1	49	9	41	17	53	18	45	13	3
28	1	13	2	2	10	1	18	3	18	39	12	52
29	1	2			10	21	18	13	18	33	12	41
30	0	51			10	40	18	23	18	26	12	30
31	0	41			10	59			18	18		

Dies.	Iul.		Aug.		Sept.		Octob.		Nov.		Dec.	
	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
1	12	19	10	4	16	23	26	30	31	55	25	34
2	12	8	10	8	16	42	26	49	31	55	25	10
3	11	58	10	13	17	1	27	8	31	54	24	45
4	11	48	10	18	17	21	27	26	31	52	24	10
5	11	38	10	23	17	41	27	43	31	50	23	55
6	11	28	10	28	18	1	28	0	31	47	23	30
7	11	18	10	34	18	21	28	16	31	43	23	4
8	11	9	10	41	18	41	28	32	31	37	22	38
9	11	0	10	49	19	1	28	47	31	30	22	11
10	10	52	10	58	19	21	29	2	31	22	21	43
11	10	47	11	7	19	41	29	16	31	13	21	14
12	10	38	11	16	20	1	29	30	31	3	20	44
13	10	31	11	25	20	22	29	43	30	53	20	14
14	10	25	11	36	20	43	29	56	30	43	19	44
15	10	19	11	48	21	4	30	9	30	32	19	14
16	10	13	12	1	21	25	30	22	30	20	18	44
17	10	7	12	14	21	47	30	34	30	8	18	14
18	10	2	12	28	22	9	30	45	29	55	17	44
19	9	58	12	42	22	31	30	55	29	40	17	14
20	9	54	12	57	22	52	31	4	29	23	16	44
21	9	51	13	12	23	13	31	12	29	6	16	14
22	9	49	13	27	23	33	31	19	28	48	15	44
23	9	47	13	43	23	53	31	26	28	30	15	14
24	9	46	13	59	24	13	31	32	28	11	14	43
25	9	46	14	16	24	33	31	38	27	51	14	12
26	9	46	14	33	24	53	31	43	27	30	13	41
27	9	47	14	50	25	13	31	47	27	8	13	10
28	9	49	15	8	25	33	31	50	26	45	12	40
29	9	52	15	26	25	52	31	53	26	22	12	10
30	9	56	15	45	26	11	31	55	25	58	11	40
31	10	0	16	4			31	55			11	10

Fig 9 - Horologium Oscillatorium - 1673 by Christiaan Huygens.
Note that the dates are in the Julian calendar and the zero value is when the Equation is minimum at end of Gregorian January

return to "Table of Contents"

HUYGENS' LEGACY

Three technological requirements were required before Huygens' legacy was finally realised.

1) the replacement of the mariner's astrolabe with the sextant, see Figs 10 and 11 The sextant's mirror doubles the apparent vertical sensitivity and its mirrors allow the image of the sun to be 'dropped' to the horizon. Noon sightings are thus much more accurate. A modern sextant is quoted as providing longitudinal accuracy of less than 100 metres at the equator (though it is doubtful whether a modern mariner could achieve such results).
2) the early works of John Harrison, Pierre le Roy, Thomas Earnshaw and John Arnold - which finally made chronometers manufacturable. It was not until the early 19th century that virtually all ships carried one (or preferable three) chronometers.
3) the availability of national astronomical institutes, firstly to provide astronomical almanacs and secondly to provide calibration services for marine chronometers.

It is said that, in the British navy, if a captain owned a chronometer of his own (i.e. he was wealthy), the admiralty would supply another two. But, if he was poor, he would be given just one!

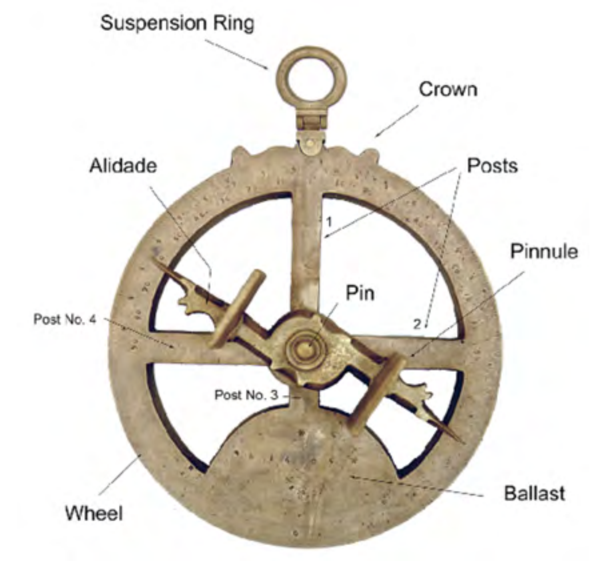


Fig 10- Mariner's Astrolabe - ca 1650

Fig 11 - The Marine Sextant. This works by optically bringing the sun's image of the down towards the horizon. The navigator tracks the sun until it 'bumps' the horizon - that is the moment of solar noon

Fig 12 - French Naval Chronometer - 1815 by 'Horloger de la Marine' Abraham-Louis Breguet

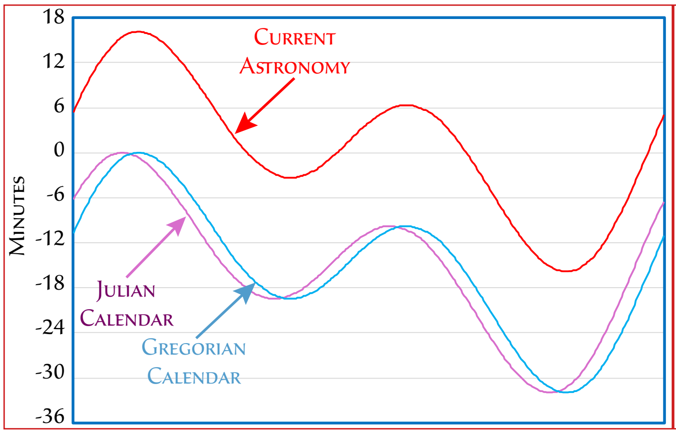


Fig 13 - Adjusting Huygens' Table

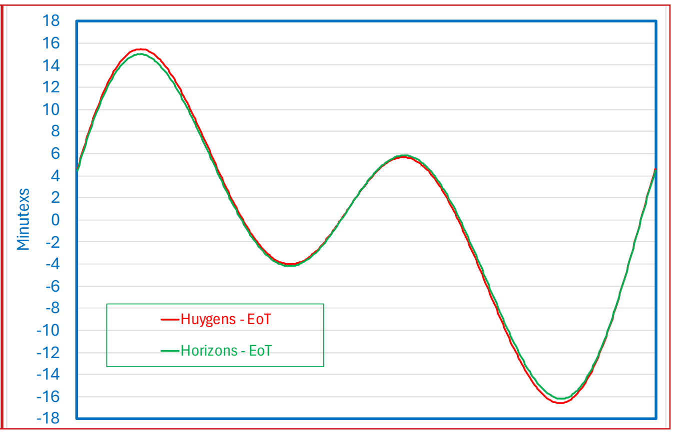


Fig 14 - Comparing Huygens with Horizons

Note that...

i) Huygens' table relates to the Julian Calendar, 10 days different from the Gregorian Calendar (the Calvinistic Dutch province of Holland made the change in 1682)

ii) zero (at end of January) equates to the day when the EoT is minimum by modern reckoning.

Adjusting for these differences (see Fig 14) and comparing with results from Horizons for 1665 shows good agreement, but some differences.

Huygens' astronomy was very good. Fourier analysis to 1st to 4th harmonics of both Huygens and Horizons reveals that the only significant difference is in the 1st Harmonic - see Figs 15 - 18.

For the pedant only.

If one assumes that the 1st harmonic represents the eccentricity while ignores the tiny 2nd eccentricity harmonic. See *'Further Analysis' on page 22.*. Back calculating the eccentricity from the harmonic magnitude indicates the value used by Huygens was 0.017 and not the (possibly) correct value indicated by Horizons of 0.165. Small variations in eccentricity have a marked change in the size of the eccentricity effect.

It is easy to surmise that – in the absence of calculators – Huygens use the numerically simple value.

See also 'Footnote 2 - The mean of Huygens' EoT values' on page 90..

1st Harmonic Magnitude					
Huygens - 1665			Horizons - 1665		
Eccentricity Effect	>>>	Eccentricity	Eccentricity Effect	>>>	Eccentricity
7.80436 mins	>>>	0.0170	7.37865 mins	>>>	0.0165

Fig 15 - Fourier Results

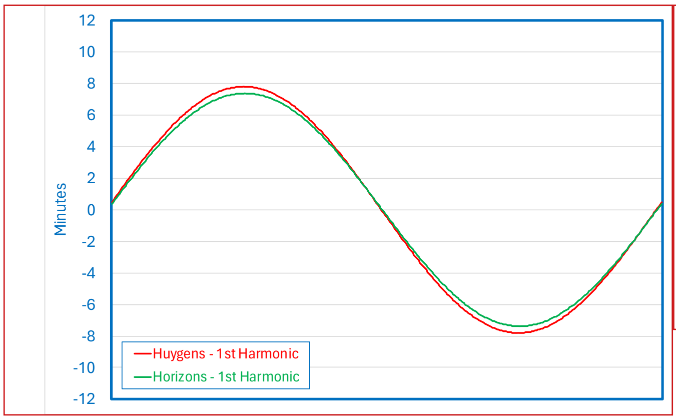


Fig 16 - 1st Harmonic - slight variance

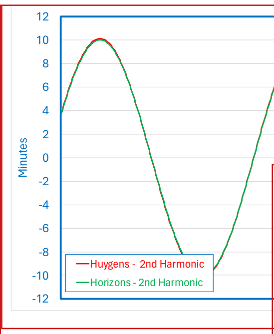


Fig 17 -2nd Harmonic almost identical

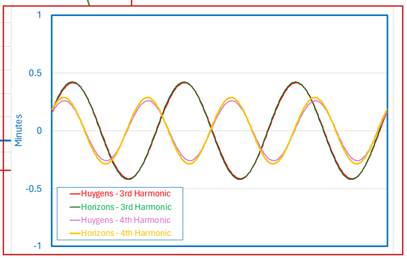
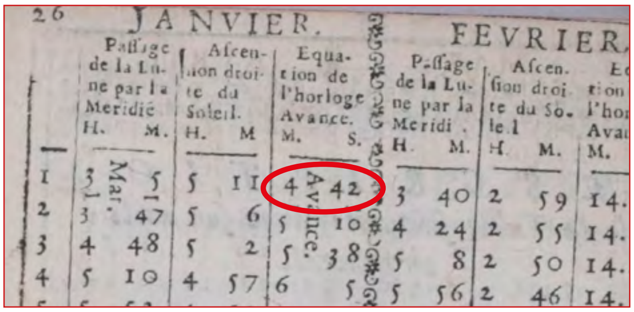


Fig 18 -3rd & 4thHarmonic almost identical

return to "Table of Contents"

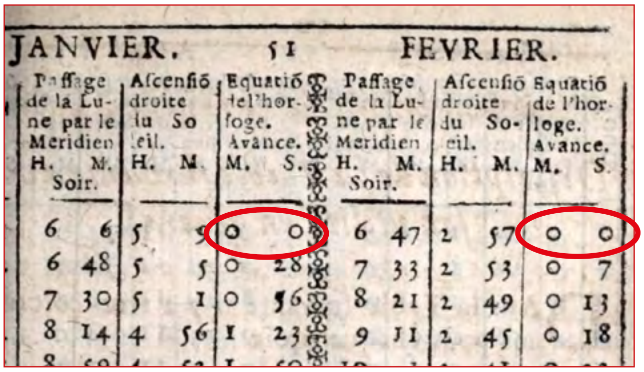
LATER ASTRONOMICAL TABLES: THE FRENCH VIEW



26 JANVIER. FEVRIER.

	Paſſage de la Lune par la Meridié H. M.	Aſcenſion droite du Soleil. H. M	Equation de l'horloge Avance. M. S.	Paſſage de la Lune par la Meridi . H. M.	Aſcenſion droite du Soleil H. M.	E tion l'hor Ava M.
1	3 Mar. 5	5 11	4 Avance. 42	3 40	2 59	14.
2	3 47	5 6	5 10	4 24	2 55	14.
3	4 48	5 2	5 38	5 8	2 50	14.
4	5 10	4 57	6 5	5 56	2 46	14.

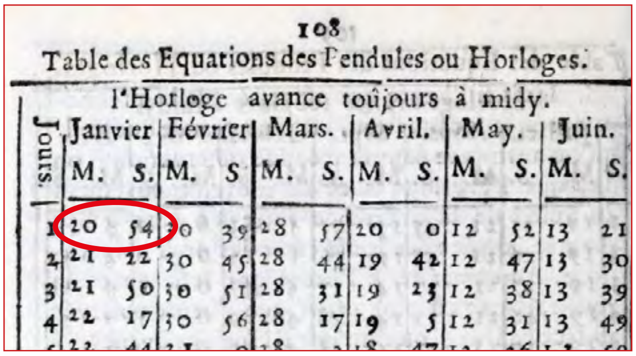
*Fig 19 - **1679** Connoissance de Temps*
- note the terminology of the third column is "Equation of the Clock"



JANVIER. 51 FEVRIER.

Paſſage de la Lune par le Meridien H. M. Soir.	Aſcenſiõ droite du Soleil. H. M.	Equatiõ de l'horloge. Avance. M. S.	Paſſage de la Lune par le Meridien H. M. Soir.	Aſcenſiõ droite du Soleil. H. M.	Equatiõ de l'horloge. Avance. M. S.
6 6	5 9	0 0	6 47	2 57	0 0
6 48	5 5	0 28	7 33	2 53	0 7
7 30	5 1	0 56	8 21	2 49	0 13
8 14	4 56	1 23	9 11	2 45	0 18

*Fig 20 - **1686** Connoissance de Temps*
- note the 1st of each month is now zero.
This assumes one sets one's clock correctly to 12:00 solar the on 1st of each month, the tabulated amount is the correction to be applied to one's clock to read God's (i.e. Solar) time.



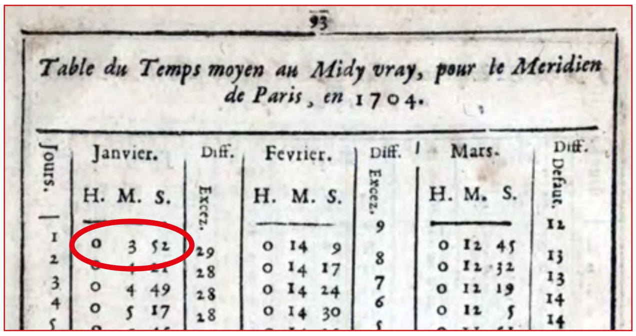
108

Table des Equations des Pendules ou Horloges.

l'Horloge avance toûjours à midy.

Jours.	Janvier M. S.	Février M. S	Mars. M. S.	Avril. M. S.	May. M. S.	Juin. M. S.
1	20 54	30 39	28 57	20 0	12 52	13 21
2	21 22	30 45	28 44	19 42	12 47	13 30
3	21 50	30 51	28 31	19 23	12 38	13 39
4	22 17	30 56	28 17	19 5	12 31	13 49

*Fig 21 - **1702** Connoissance de Temps*
- note the EoT value is now based with zero at 1st November (its max value by our reckoning). This assumes that one sets the clock accurately then and corrects daily by the tabulated amount.



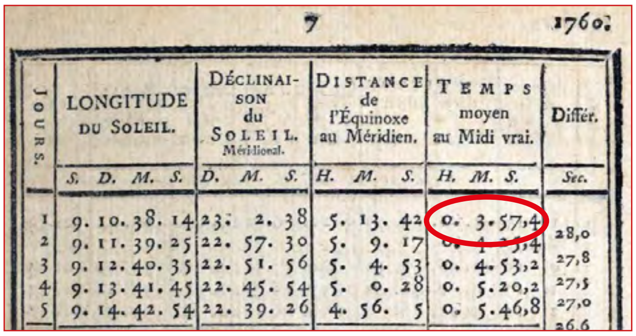
93

Table du Temps moyen au Midy vray, pour le Meridien de Paris, en 1704.

Jours.	Janvier. H. M. S.	Diff. Excez.	Fevrier. H. M. S.	Diff. Excez.	Mars. H. M. S.	Diff. Defaut.
				9		12
1	0 3 52	29	0 14 9	8	0 12 45	13
2	0 4 21	28	0 14 17	7	0 12 32	13
3	0 4 49	28	0 14 24	6	0 12 19	14
4	0 5 17	28	0 14 30		0 12 5	14

*Fig 22 - **1704** Connoissance de Temps*
In addition to the format shown above, a new table is added in the format we now recognize.

Ref Denis Savoie - 'Le équation du temps au fil des éphémérides' - Cadrans Info no35 - 2017
All the above images are from https://gallica.bnf.fr/

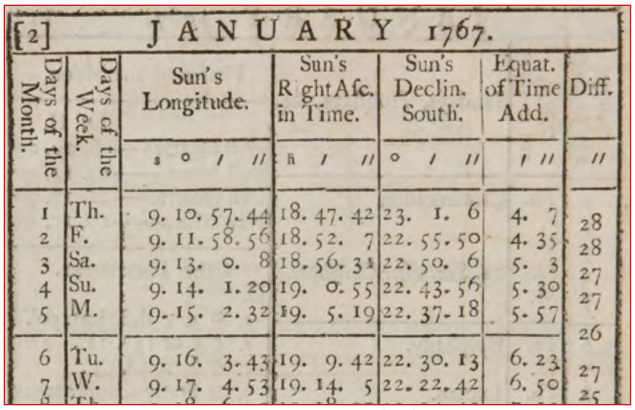


7 1760.

Jours.	LONGITUDE DU SOLEIL. S. D. M. S.	DÉCLINAISON du SOLEIL. Méridional. D. M. S.	DISTANCE de l'Équinoxe au Méridien. H. M. S.	TEMPS moyen au Midi vrai. H. M. S.	Differ. Sec.
1	9. 10. 38. 14	23. 2. 38	5. 13. 42	0. 3. 57,4	
2	9. 11. 39. 25	22. 57. 30	5. 9. 17	0. 4. 25,4	28,0
3	9. 12. 40. 35	22. 51. 56	5. 4. 53	0. 4. 53,2	27,8
4	9. 13. 41. 45	22. 45. 54	5. 0. 28	0. 5. 20,2	27,5
5	9. 14. 42. 54	22. 39. 26	4. 56. 5	0. 5. 46,8	27,0
					26,6

*Fig 23 - **1760** Connoissance de Temps*
- note the terminology of "Equation of the Clock" is lost

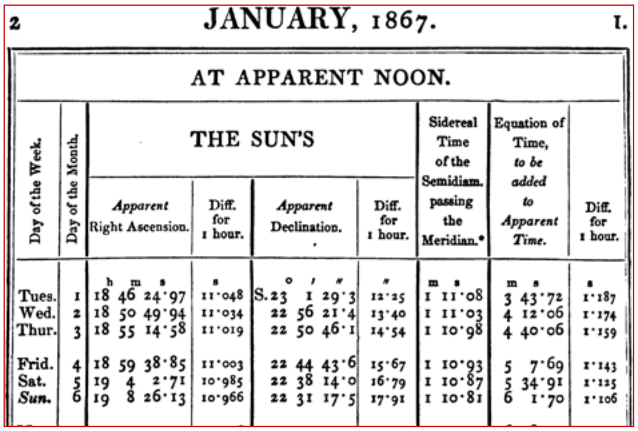
LATER ASTRONOMICAL TABLES: THE ANGLO-SAXON VIEW



[2] JANUARY 1767.

Days of the Month.	Days of the Week.	Sun's Longitude. s ° ' "	Sun's Right Aſc. in Time. h ' "	Sun's Declin. South. ° ' "	Equat. of Time Add. ' "	Diff. "
1	Th.	9. 10. 57. 44	18. 47. 42	23. 1. 6	4. 7	28
2	F.	9. 11. 58. 56	18. 52. 7	22. 55. 50	4. 35	28
3	Sa.	9. 13. 0. 8	18. 56. 31	22. 50. 6	5. 3	27
4	Su.	9. 14. 1. 20	19. 0. 55	22. 43. 56	5. 30	27
5	M.	9. 15. 2. 32	19. 5. 19	22. 37. 18	5. 57	26
6	Tu.	9. 16. 3. 43	19. 9. 42	22. 30. 13	6. 23	27
7	W.	9. 17. 4. 53	19. 14. 5	22. 22. 42	6. 50	25

*Fig 24 - **1767** Nautical Almanac & Astronomical Ephemeris*

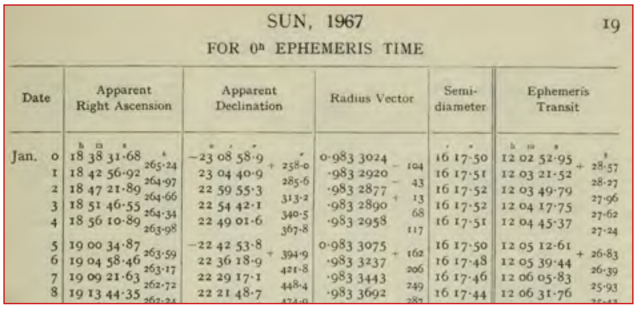


2 JANUARY, 1867. I.

AT APPARENT NOON.

Day of the Week.	Day of the Month.	THE SUN'S Apparent Right Ascension.	THE SUN'S Diff. for 1 hour.	THE SUN'S Apparent Declination.	THE SUN'S Diff. for 1 hour.	Sidereal Time of the Semidiam. passing the Meridian.*	Equation of Time, to be added to Apparent Time.	Diff. for 1 hour.
		h m s	s	° ' "	"	m s	m s	s
Tues.	1	18 46 24·97	11·048	S.23 1 29·3	12·25	1 11·08	3 43·72	1·187
Wed.	2	18 50 49·94	11·034	22 56 21·4	13·40	1 11·03	4 12·06	1·174
Thur.	3	18 55 14·58	11·019	22 50 46·1	14·54	1 10·98	4 40·06	1·159
Frid.	4	18 59 38·85	11·003	22 44 43·6	15·67	1 10·93	5 7·69	1·143
Sat.	5	19 4 2·71	10·985	22 38 14·0	16·79	1 10·87	5 34·91	1·125
Sun.	6	19 8 26·13	10·966	22 31 17·5	17·91	1 10·81	6 1·70	1·106

*Fig 25 - **1867** Nautical Almanac & Astronomical Ephemeris. 100 years later. Note that **Apparent** Times are given which include astronomical Nutation & Aberration.*



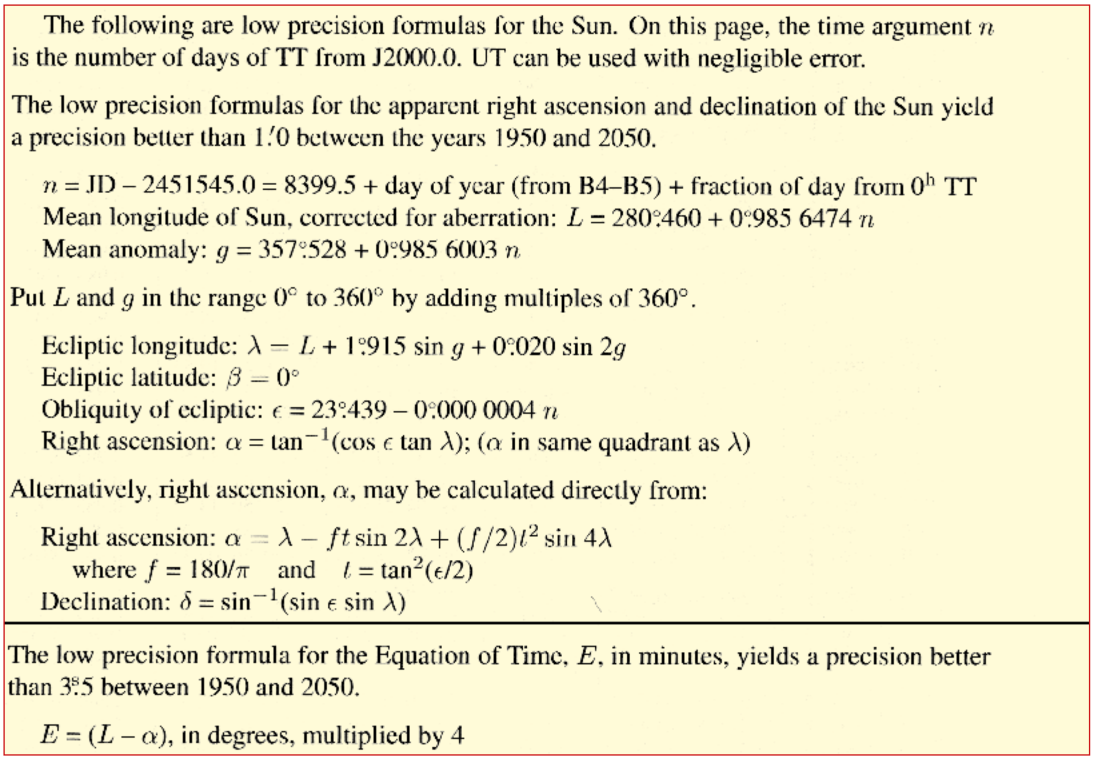
SUN, 1967 19

FOR 0^h EPHEMERIS TIME

Date		Apparent Right Ascension		Apparent Declination		Radius Vector		Semi-diameter	Ephemeris Transit	
		h m s	s	° ' "	"			' "	h m s	s
Jan.	0	18 38 31·68	265·24	−23 08 58·9	+258·0	0·983 3024	−104	16 17·50	12 02 52·95	+28·57
	1	18 42 56·92	264·97	23 04 40·9	285·6	·983 2920	−43	16 17·51	12 03 21·52	28·27
	2	18 47 21·89	264·66	22 59 55·3	313·2	·983 2877	+13	16 17·52	12 03 49·79	27·96
	3	18 51 46·55	264·34	22 54 42·1	340·5	·983 2890	68	16 17·52	12 04 17·75	27·62
	4	18 56 10·89	263·98	22 49 01·6	367·8	·983 2958	117	16 17·51	12 04 45·37	27·24
	5	19 00 34·87	263·59	−22 42 53·8	+394·9	0·983 3075	+162	16 17·50	12 05 12·61	+26·83
	6	19 04 58·46	263·17	22 36 18·9	421·8	·983 3237	206	16 17·48	12 05 39·44	26·39
	7	19 09 21·63	262·72	22 29 17·1	448·4	·983 3443	249	16 17·46	12 06 05·83	25·93
	8	19 13 44·35		22 21 48·7		·983 3692		16 17·44	12 06 31·76	

*Fig 26 - **1967** Nautical Almanac & Astronomical Ephemeris. 100 years later. Now LORAN radio signals are available, so navigators no longer need the sextant (& thus EoT). To find the EoT, it is now necessary to subtract 12 hrs from the Ephemeris Transit time (& multiply by 60) to find the EoT*

return to "Table of Contents"



The following are low precision formulas for the Sun. On this page, the time argument n is the number of days of TT from J2000.0. UT can be used with negligible error.

The low precision formulas for the apparent right ascension and declination of the Sun yield a precision better than $1\overset{\prime}{.}0$ between the years 1950 and 2050.

n = JD – 2451545.0 = 8399.5 + day of year (from B4–B5) + fraction of day from 0^h TT
Mean longitude of Sun, corrected for aberration: $L = 280\overset{\circ}{.}460 + 0\overset{\circ}{.}985\,6474\, n$
Mean anomaly: $g = 357\overset{\circ}{.}528 + 0\overset{\circ}{.}985\,6003\, n$

Put L and g in the range 0° to 360° by adding multiples of 360°.

Ecliptic longitude: $\lambda = L + 1\overset{\circ}{.}915 \sin g + 0\overset{\circ}{.}020 \sin 2g$
Ecliptic latitude: $\beta = 0°$
Obliquity of ecliptic: $\epsilon = 23\overset{\circ}{.}439 - 0\overset{\circ}{.}000\,0004\, n$
Right ascension: $\alpha = \tan^{-1}(\cos\epsilon \tan\lambda)$; ($\alpha$ in same quadrant as λ)

Alternatively, right ascension, α, may be calculated directly from:

Right ascension: $\alpha = \lambda - ft \sin 2\lambda + (f/2)t^2 \sin 4\lambda$
where $f = 180/\pi$ and $t = \tan^2(\epsilon/2)$
Declination: $\delta = \sin^{-1}(\sin\epsilon \sin\lambda)$

The low precision formula for the Equation of Time, E, in minutes, yields a precision better than $3\overset{s}{.}5$ between 1950 and 2050.

$E = (L - \alpha)$, in degrees, multiplied by 4

Fig 27 - ***2023*** *Almanac Method of finding the Equation of Time, without using the Ephemeris Transit*

The Astronomical Almanac for the Year 2023 - Page C5

SOLAR COMPASS BACKGROUND

When driving steel vehicles or when land surveying in areas where iron ore is found, magnetic compasses are unreliable. In these cases, a solar compass can be used. These require...

- the date to provide the EoT- either from tables or mechanically
- longitude, time zone and standard time to provide local mean time
- either the latitude or solar azimuth

BURTS SOLAR COMPASS - 1835

William Burt (1792 - 1858) was an inventor, postmaster, circuit court judge, state legislator and deputy U.S. surveyor - for which he won acclaim for his accurate work on public land surveys. Burt's compass, (see Figs 39 & 30) was typically attached to a surveyor's transit Such was its success that it was specified by the US government for all border surveys. Compared to other surveying methods, it was cheap to use. Survey costs for the boundary line between Iowa and Minnesota were $120 per mile with old-fashioned instruments, while with Burt's compass they were only $15 per mile.

Burt exhibited his compasses at the Great Exhibition in 1851 where it received praise from both the astronomer John Herschel and the Prince of Wales,

The compass continued to be used until the advent of GPS.

Fig 28 -Hon Wm. A. Burt

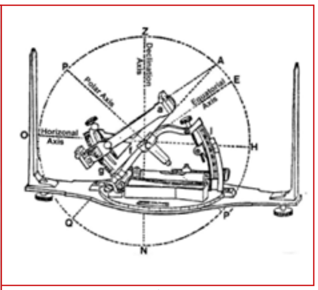


Fig 29 - Burt's US Patent - 1835

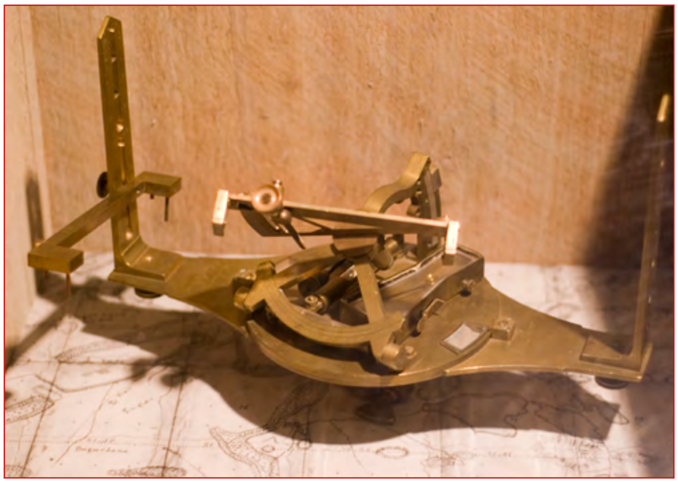


Fig 30 - Commercial version of Burt's Compass

return to "Table of Contents"

The Astro Compass

These were used extensively by aircraft in World War II, where either the Sun or stars could be used. The EoT was not directly used, since the L.H.A. (local hour angle) scale would be set using Astronomical Almanacs. However, the author has used this compass, knowing the local mean time and the EoT to set the L.H.A. scale and thus find true North.

Fig 31 - 1943 Astro Compass

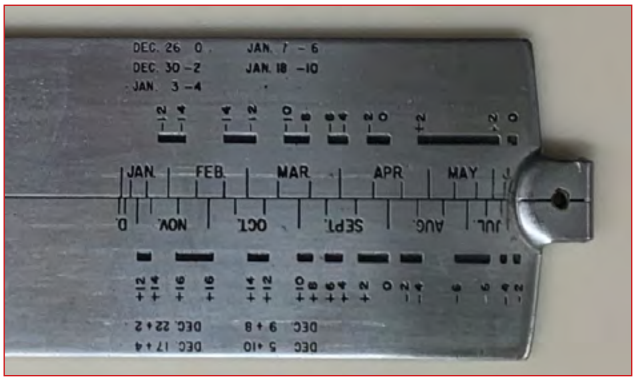


Fig 33 - Abrams Date/EoT Plate

The Abrams & Cole Compasses - WW II

The compasses were designed to be used in the Sahara desert during World War II where steel vehicles precluded the use of magnetic compasses and where there were no topographical maps or local topography to provide an indication of direction. They required a watch adjusted from local standard time to local mean time according to longitude.

The American and the South African Cole model had a fixed 360°-marked base that was aligned with the vehicle's direction of travel. In the centre of this was a vertical gnomon and a central alidade, see Figs 32 & 35 overleaf. A moveable 'dial' plate could slide up and down a date/EoT scale, see Figs 33, 34. & 36

To use, the 'dial' plate was moved up and down match date/EoT scale, the central alidade was turn to the intersection of time and latitude on the dial plate. The date/EoT scale and the 'dial' plate would be turned until the gnomon shadow coincided with alidade. The scale would then be pointing true N/S.

These instruments are universal mean time analemmatic sundials. See 'Moving Style Equiangular Dials' on page 130.

Fig 32 - US Abrams Sun Compass - 1943

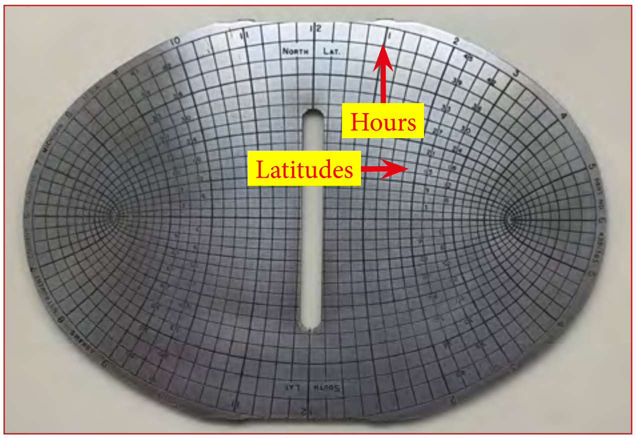


Fig 34 - Abrams Dial Plate

return to "Table of Contents"

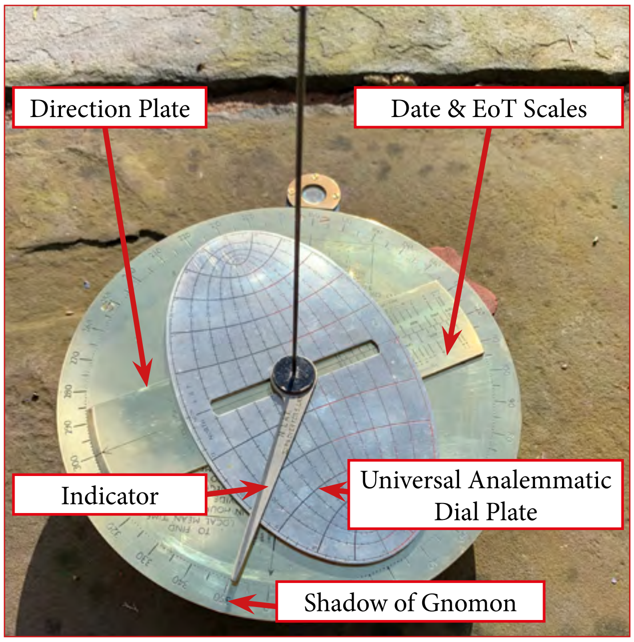


Fig 35 - South African Cole Sun Compass

1. At Base, Set Watch to LMT in area of operations
2. In Field, read EoT from Direction Plate
3. Set Direction Plate to required Azimuth
4. Find Solar Time from Watch & EoT
5 Set Indicator to intersection of Latitude Lines & Solar Time on Dial Plate
6. Align vehicle so gnomon shadow lies on Indicator & Drive!

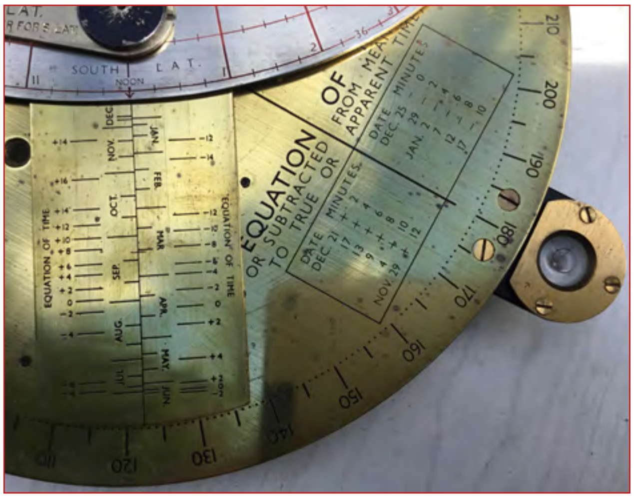


Fig 36 - Date and EoT Scale on Cole Sun Compass: note the EoT scale along the edge of the alidade and also the EoT table covering December and January when the value is changing fastest

The Bagnold Sun Compass

Major Ralph Bagnold was an engineer, academic, desert explorer, expert in desert transport and specialist in dry sand-dune dynamics : (he has a dune field on Mars named after him.) He formed the Long Range Desert Group (LRDG), which scouted German forces in the Libyan desert in World War II. The SAS called LRDG the 'Libyan Desert Taxi Service', who delivered and recovered them from their enemy targets.

He invented the Bagnold Sun Compass. See Fig 38.

The compass required cards of EoT, solar azimuth for different latitudes and seasons, see Fig 39 overleaf.

The compass index and gnomon were alighted to the front of the vehicle. The navigator used the appropriate card to set his watch to LAT, by correcting for longitude and EoT. The course setting disk was rotated to desired bearing and gnomon shadow provided acted as a compass. Over a half-hour period, the thumb drive had to be rotated ¼ turn every 2 minutes to correct the sun's movement.

Bagnold's compass, although it required a data card, had the advantage that it showed the true bearing of the course followed at any moment, whereas the Cole and Abrams had to have a time consuming readjustment if the vehicle had to change course to avoid obstacles as happened all the time in rough country or sand dunes.

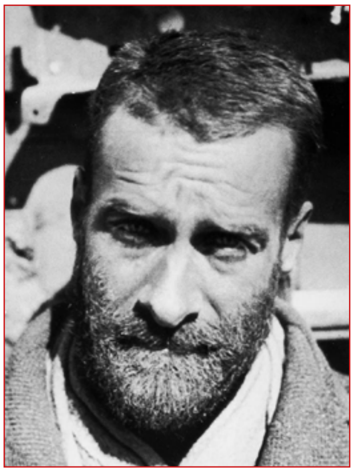


Fig 37 - Dr Ralph Bagnold FRS, OBE, KL

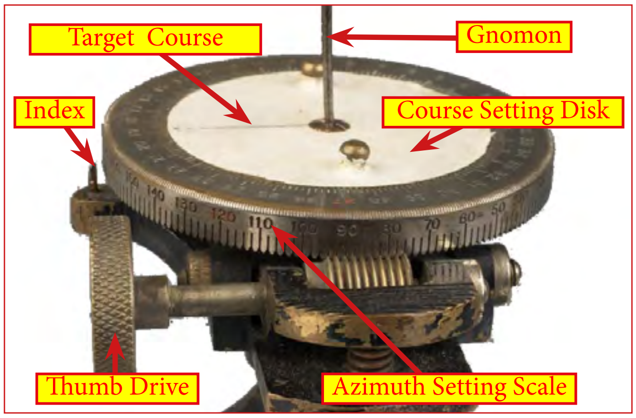
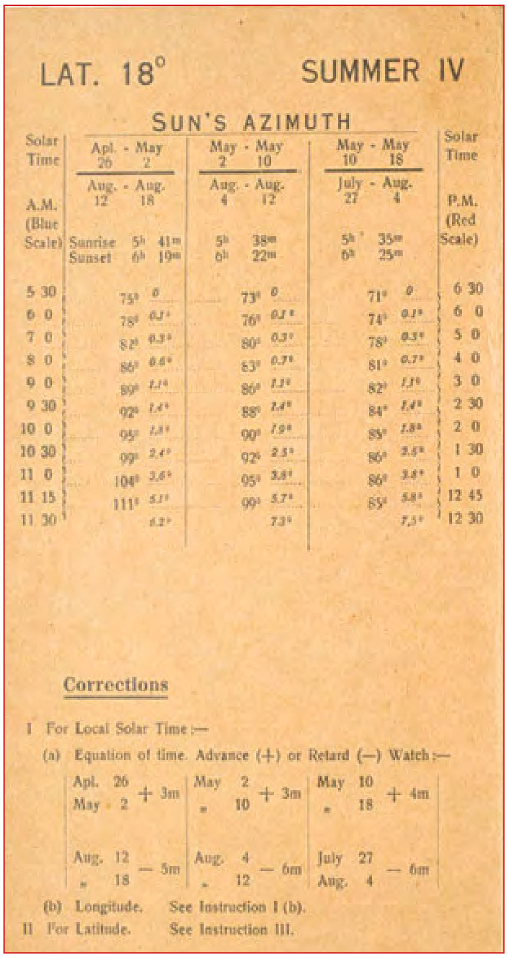


Fig 38 - Bagnold Sun Compass



LAT. 18° SUMMER IV

SUN'S AZIMUTH

Solar Time	Apl. - May 26 2	May - May 2 10	May - May 10 18	Solar Time
	Aug. - Aug. 12 18	Aug. - Aug. 4 12	July - Aug. 27 4	
A.M. (Blue Scale)	Sunrise 5h 41m Sunset 6h 19m	5h 38m 6h 22m	5h 35m 6h 25m	P.M. (Red Scale)
5 30	75° 0	73° 0	71° 0	6 30
6 0	78° 0.1°	76° 0.1°	74° 0.1°	6 0
7 0	82° 0.3°	80° 0.3°	78° 0.3°	5 0
8 0	86° 0.6°	83° 0.7°	81° 0.7°	4 0
9 0	89° 1.1°	86° 1.1°	82° 1.1°	3 0
9 30	92° 1.4°	88° 1.4°	84° 1.4°	2 30
10 0	95° 1.8°	90° 1.9°	85° 1.8°	2 0
10 30	99° 2.4°	92° 2.5°	86° 2.5°	1 30
11 0	104° 2.6°	95° 3.8°	86° 3.8°	1 0
11 15	111° 5.1°	99° 5.7°	85° 5.8°	12 45
11 30	6.2°	7.3°	7.5°	12 30

Corrections

I For Local Solar Time :—

(a) Equation of time. Advance (+) or Retard (—) Watch :—

Apl. 26 May 2	+ 3m	May 2 " 10	+ 3m	May 10 " 18	+ 4m
Aug. 12 " 18	— 5m	Aug. 4 " 12	— 6m	July 27 Aug. 4	— 6m

(b) Longitude. See Instruction I (b).

II For Latitude. See Instruction III.

Fig 39 - Setting Card for Bagnold Compass

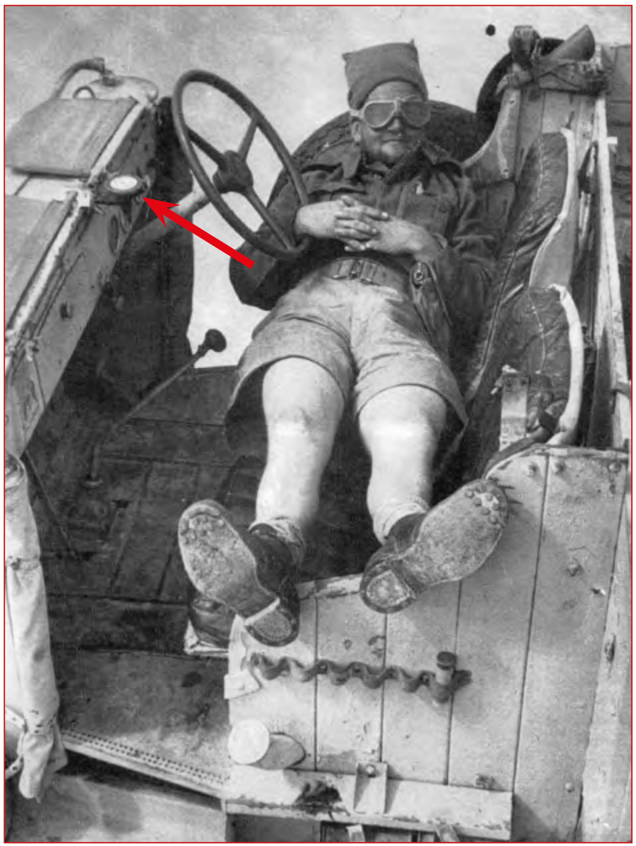


Fig 40 - LRDG trooper McCullogh

resting in his Chevrolet truck

- note the Bagnold compass

photo M. Carr, via I. Chard, via Kuno Gross

'The Bagnold Sun Compass - History & Utilization' by Kuno Gross

Fig 41 - The SAS in Libya - note the Bagnold compass

The Howard Sun Compass -1980

Until the arrival of GPS, the Howard Sun Compass was issued to NATO forces.

It was hand-held with a spirit level and made of anodised aluminium, perspex and plastic. The white plastic plastic plate was premarked with EoT corrected hour lines suitable for a small specific location range and a short specific date range.

Fig 42 - Howard Sun Compass, pre-marked for mid-February and a Saudi Arabian location in the 1991 Gulf War

Footnote 1 - Huygens' Pendulums

Huygens noted that pendulums are not isochronous (keeping even time) if the swing of the pendulum is large. This exercised his mind since, before the advent of the anchor escapement, the verge escapement required a large swing. His significant invention was that if the top of the pendulum was flexible, constrained by 'cheeks' and if the cheeks have the curve of an cycloid (the locus of a point on a circle rolling around another shape), the pendulum will be isochronous.

The fact that Huygens was able to find a geometrical solution to a problem set by Blaise Pascal is extraordinary, since, without the benefits of calculus, the problem cannot be solved algebarically.

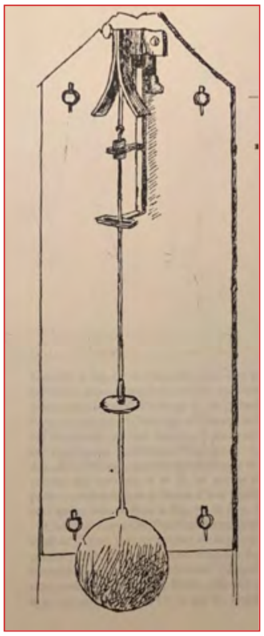


Fig 43 - Huygens' original drawing of his clock showing the Cheeks

Fig 44 - Pendulum clock by Salomon Coster - 1659. It is thought that Salomon was unable to accurately shape the cheeks but was just copying Huygens' general idea.

Rijksmuseum, Leiden

Footnote 2 - The mean of Huygens' EoT values

The values of Huygens' EoT were such that zero equated to the minimum value in modern terminology, i.e.

0^{mins} Huygens = $-16^{\text{mins}}\ 13^{\text{secs}}$ the 1665 minimum EoT (in modern terminology)

To rectify his values in this way and taking the average value over the whole year does not yield zero as expected, but rather $-15^{\text{mins}}\ 42^{\text{secs}}$. This is an unexplained dilemma, but does not negate the Fourier results

Footnote 3 - Does this Zij contain the Equation?

The author hopes that this beautiful Zij contains the Equation....

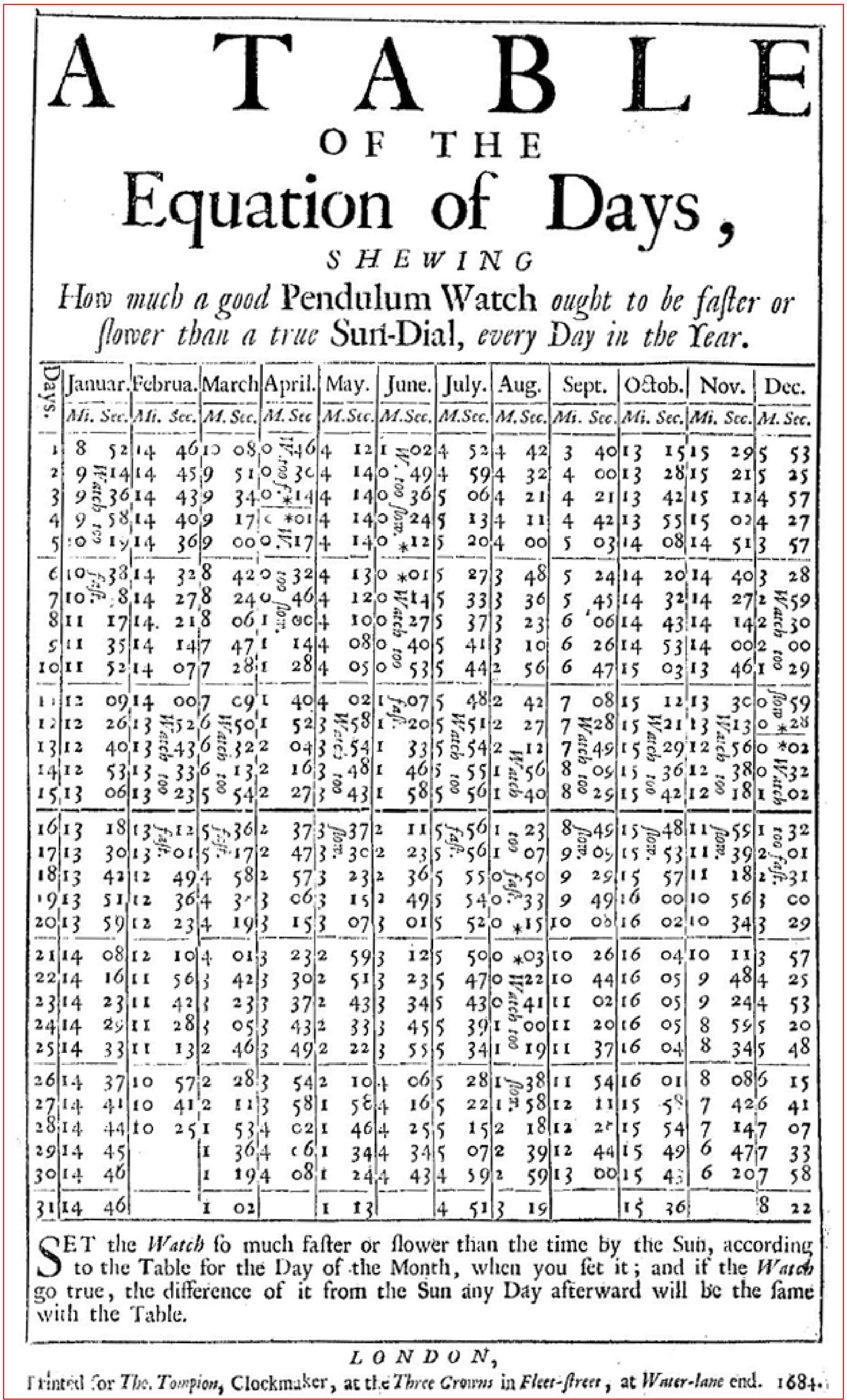
Sanjufini Zij by Samarkandi astronomer Khwaja Ghazi al-Sanjufini. Compiled -1363

Displaying the Equation of Time

The Equation of Time may be displayed on or near a Sundial or clock (or anywhere else) in a number of ways...

For historical information, see the two chapters written by Dr John Davis. See ‘The Equation of Time as Shown on Sundials’ on page 53. and ‘More on the Equation of Time on Sundials’ on page 63..

... As a Full Table



A TABLE

OF THE

Equation of Days,

SHEWING

How much a good Pendulum Watch *ought to be faſter or ſlower than a true* Sun-Dial, *every Day in the Year.*

Days.	Januar.		Februa.		March		April.		May.		June.		July.		Aug.		Sept.		Octob.		Nov.		Dec.	
	Mi.	*Sec.*	*Mi.*	*Sec.*	*M.*	*Sec.*	*M.*	*Sec*	*M.*	*Sec.*	*M.*	*Sec.*	*M.*	*Sec.*	*M.*	*Sec.*	*Mi.*	*Sec.*	*Mi.*	*Sec.*	*Mi.*	*Sec.*	*M.*	*Sec.*
1	8	52	14	46	10	08	0	46	4	12	1	02	4	52	4	42	3	40	13	15	15	29	5	53
2	9	14	14	45	9	51	0	30	4	14	0	49	4	59	4	32	4	00	13	28	15	21	5	25
3	9	36	14	43	9	34	0	*14	4	14	0	36	5	06	4	21	4	21	13	42	15	12	4	57
4	9	58	14	40	9	17	0	*01	4	14	0	24	5	13	4	11	4	42	13	55	15	02	4	27
5	10	19	14	36	9	00	0	17	4	14	0	*12	5	20	4	00	5	03	14	08	14	51	3	57
6	10	38	14	32	8	42	0	32	4	13	0	*01	5	27	3	48	5	24	14	20	14	40	3	28
7	10	58	14	27	8	24	0	46	4	12	0	14	5	33	3	36	5	45	14	32	14	27	2	59
8	11	17	14	21	8	06	1	00	4	10	0	27	5	37	3	23	6	06	14	43	14	14	2	30
5	11	35	14	14	7	47	1	14	4	08	0	40	5	41	3	10	6	26	14	53	14	00	2	00
10	11	52	14	07	7	28	1	28	4	05	0	53	5	44	2	56	6	47	15	03	13	46	1	29
11	12	09	14	00	7	09	1	40	4	02	1	07	5	48	2	42	7	08	15	12	13	30	0	59
12	12	26	13	52	6	50	1	52	3	58	1	20	5	51	2	27	7	28	15	21	13	13	0	*28
13	12	40	13	43	6	32	2	04	3	54	1	33	5	54	2	12	7	49	15	29	12	56	0	*02
14	12	53	13	33	6	13	2	16	3	48	1	46	5	55	1	56	8	05	15	36	12	38	0	32
15	13	06	13	23	5	54	2	27	3	43	1	58	5	56	1	40	8	25	15	42	12	18	1	02
16	13	18	13	12	5	36	2	37	3	37	2	11	5	56	1	23	8	49	15	48	11	55	1	32
17	13	30	13	01	5	17	2	47	3	30	2	23	5	56	1	07	9	05	15	53	11	39	2	01
18	13	42	12	49	4	58	2	57	3	23	2	36	5	55	0	50	9	29	15	57	11	18	2	31
19	13	51	12	36	4	3[illegible]	3	06	3	15	2	49	5	54	0	33	9	49	16	00	10	56	3	00
20	13	59	12	23	4	19	3	15	3	07	3	01	5	52	0	*15	10	05	16	02	10	34	3	29
21	14	08	12	10	4	01	3	23	2	59	3	12	5	50	0	*03	10	26	16	04	10	11	3	57
22	14	16	11	56	3	42	3	30	2	51	3	23	5	47	0	22	10	44	16	05	9	48	4	25
23	14	23	11	42	3	23	3	37	2	43	3	34	5	43	0	41	11	02	16	05	9	24	4	53
24	14	29	11	28	3	05	3	43	2	33	3	45	5	39	1	00	11	20	16	05	8	59	5	20
25	14	33	11	13	2	46	3	49	2	22	3	55	5	34	1	19	11	37	16	04	8	34	5	48
26	14	37	10	57	2	28	3	54	2	10	4	06	5	28	1	38	11	54	16	01	8	08	6	15
27	14	41	10	41	2	11	3	58	1	58	4	16	5	22	1	58	12	11	15	5[illegible]	7	42	6	41
28	14	44	10	25	1	53	4	02	1	46	4	25	5	15	2	18	12	2[illegible]	15	54	7	14	7	07
29	14	45			1	36	4	06	1	34	4	34	5	07	2	39	12	44	15	49	6	47	7	33
30	14	46			1	19	4	08	1	24	4	43	4	59	2	59	13	00	15	43	6	20	7	58
31	14	46			1	02			1	13			4	51	3	19			15	36			8	22

SET the *Watch* ſo much faſter or ſlower than the time by the Sun, according to the Table for the Day of the Month, when you ſet it; and if the *Watch* go true, the difference of it from the Sun any Day afterward will be the ſame with the Table.

LONDON,
Printed for *Tho. Tompion*, Clockmaker, at the *Three Crowns* in *Fleet-ſtreet*, at *Water-lane* end. 1684.

The clockmaker, Thomas Tompion's Equation table of 1683.
Some 12 years later, the first Astronomer Royal, John Flamsteed, wrote to Sir Isaac Newton "Tompion's a true table of equations, but made for a particular year perhaps, fits not the present" !

A Table

Shewing for every Day in the Year at Noon, how much the Equal or Mean Time is faster or slower than the Apparent or Sun's Time, Or how much a Clock should be faster or slower than a true Sun-Dial.

Days	JAN. Min. Sec. Add.	FEB. Min. Sec. Add.	MAR. Min. Sec. Add.	APRIL. Min. Sec. Add.	MAY. Min. Sec. Sub.	JUNE. Min. Sec. Sub.	JULY. Min. Sec. Add.	AUG. Min. Sec. Add.	SEPT. Min. Sec. Sub.	OCT. Min. Sec. Sub.	NOV. Min. Sec. Sub.	DEC. Min. Sec. Sub.	Days
1	4 14	14 8	12 41	3 56	3 10	2 37	3 11	5 46	0 19	10 27	16 14	10 29	1
2	4 42	14 15	12 29	3 38	3 17	2 28	3 23	5 43	0 38	10 45	16 14	10 6	2
3	5 10	14 21	12 16	3 20	3 24	2 19	3 34	5 39	0 57	11 4	16 14	9 41	3
4	5 37	14 27	12 2	3 1	3 30	1 9	3 45	5 34	1 16	11 22	16 12	9 17	4
5	6 4	14 32	11 48	2 44	3 36	1 58	3 56	5 28	1 36	11 39	16 10	8 52	5
6	6 30	14 35	11 34	2 26	3 41	1 48	4 6	5 22	1 55	11 56	16 7	8 26	6
7	6 56	14 38	11 19	2 8	3 45	1 37	4 16	5 16	2 15	12 13	16 4	7 59	7
8	7 22	14 41	11 4	1 51	3 49	1 26	4 25	5 8	2 35	12 30	15 59	7 33	8
9	7 47	14 42	10 49	1 34	3 52	1 14	4 34	5 1	2 56	12 46	15 54	7 6	9
10	8 11	14 43	10 33	1 17	3 55	1 2	4 43	4 52	3 16	13 1	15 47	6 38	10
11	8 35	14 43	10 17	1 1	3 57	0 50	4 51	4 43	3 37	13 16	15 40	6 10	11
12	8 58	14 42	10 0	0 44	3 59	0 38	4 59	4 34	3 57	13 31	15 32	5 42	12
13	9 20	14 40	9 43	0 28	4 0	0 25	5 7	4 24	4 18	13 45	15 24	5 13	13
14	9 42	14 38	9 26	0 13	4 0	0 13	5 14	4 13	4 39	13 58	15 14	4 44	14
15	10 3	14 35	9 9	Sub. 4	4 0	0 0	5 20	4 2	5 0	14 11	15 4	4 15	15
16	10 24	14 31	8 52	0 19	3 59	Ad. 12	5 26	3 50	5 21	14 24	14 52	3 46	16
17	10 44	14 26	8 34	0 33	3 58	0 16	5 32	3 38	5 42	14 36	14 40	3 17	17
18	11 3	14 21	8 16	0 47	3 56	0 29	5 36	3 25	6 3	14 47	14 28	2 47	18
19	11 21	14 15	7 58	1 1	3 54	0 42	5 41	3 12	6 24	14 58	14 14	2 17	19
20	11 39	14 8	7 40	1 14	3 51	0 55	5 45	2 58	6 45	15 8	13 59	1 47	20
21	11 56	14 1	7 21	1 27	3 47	1 8	5 48	2 44	7 6	15 18	13 44	1 17	21
22	12 12	13 53	7 3	1 40	3 43	1 21	5 51	2 30	7 27	15 26	13 28	0 47	22
23	12 27	13 45	6 44	1 52	3 39	1 33	5 53	2 14	7 48	15 34	13 11	0 17	23
24	12 42	13 36	6 26	2 3	3 34	1 46	5 55	1 59	8 9	15 42	13 54	Ad. 12	24
25	12 56	13 26	6 7	2 14	3 29	1 59	5 56	1 43	8 2	15 48	12 35	0 42	25
26	13 8	13 15	5 48	2 24	3 23	2 11	5 56	1 27	8 49	15 54	12 16	1 12	26
27	13 20	13 4	5 29	2 34	3 16	2 24	5 56	1 10	9 9	16 0	11 56	1 41	27
28	13 31	12 53	5 11	2 44	3 9	2 36	5 55	0 53	9 29	16 4	11 35	2 11	28
29	13 42		4 52	2 53	3 2	2 48	5 54	0 36	9 49	16 8	11 14	2 40	29
30	13 51		4 33	3 2	2 54	3 0	5 52	0 18	10 6	16 11	10 52	3 10	30
31	13 59		4 15		2 46		5 50	Sub. 0		16 13		3 38	31

The Figure of Mathematics is drawing a Curtain, diſcovering Time with Truth.

The Boy near the Clock is an Emblem of the Mechanic Powers.

LONDON:
Published by Robert Sayer,
53 Fleet Street

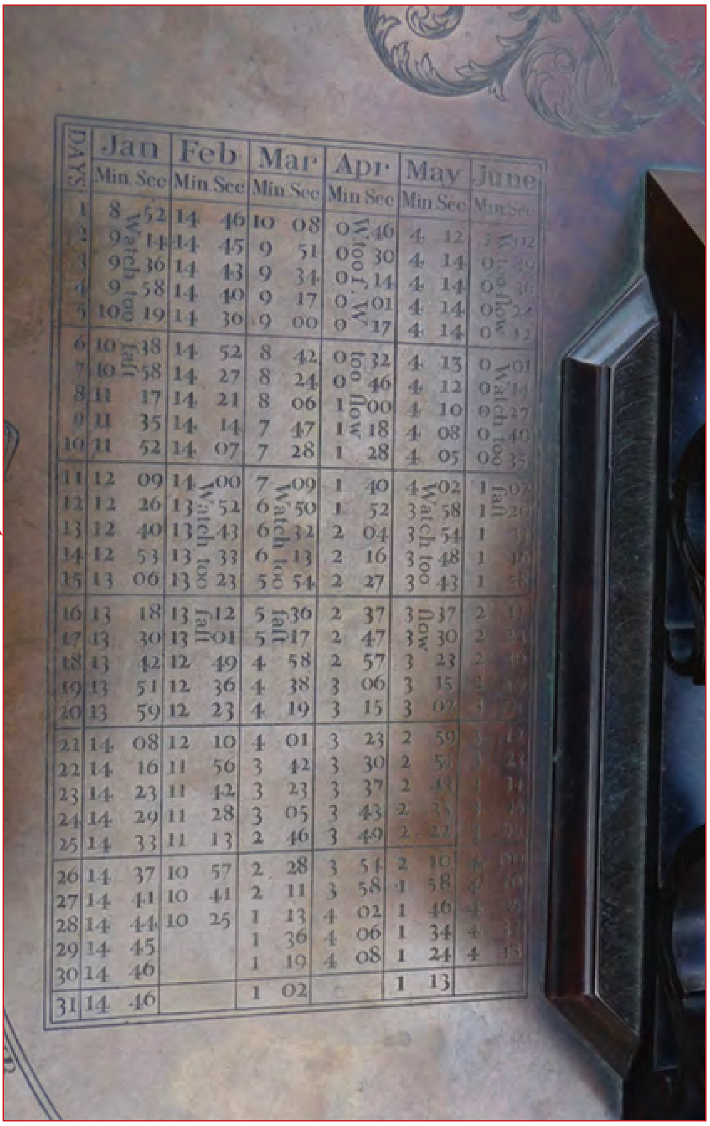
This Table is carefully computed to the Sun's true Excentricity, and to the present Place of his Apogee & will hold good 20 Years without any sensible Error; being adapted to the New Stile.

Price 1^s

Broadsheet by Samuel Wale - 1760

The text at the bottom of the image reads....
'The Figure of Mathematics is drawing a Curtain discovering Time with Truth.
The Boy near the Clock is an Emblem of Mechanic Powers.
This Table is carefully computed to the Sun's true Eccentricity and to the present Place of its Apogee & will hold good for 20 Years without any sensible Error; being adapted to the New Stile.   Price 1^s

return to "Table of Contents"

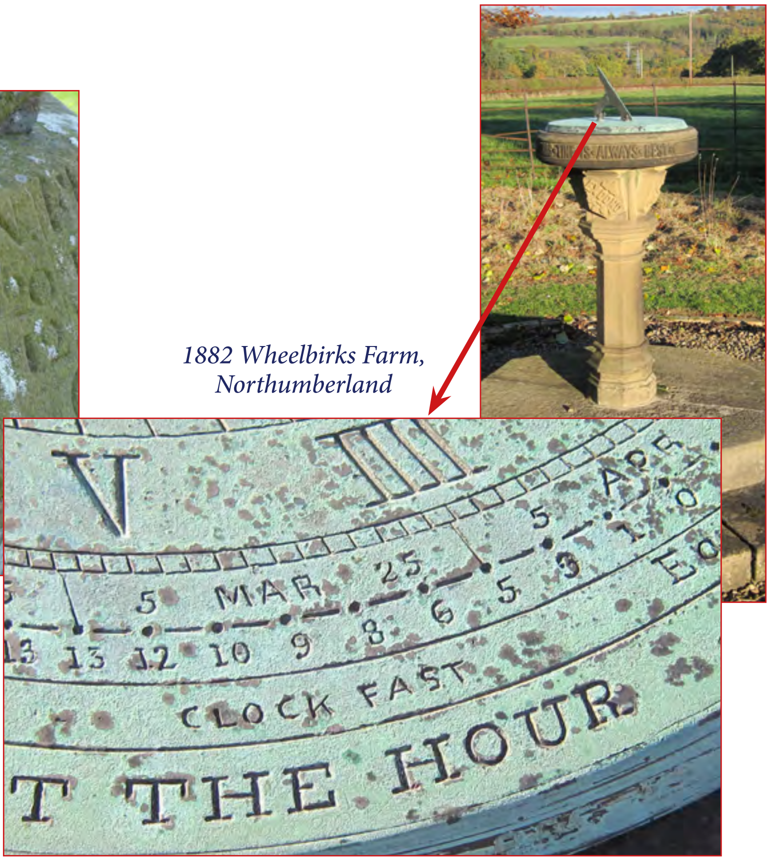


DAYS	Jan Min	Jan Sec	Feb Min	Feb Sec	Mar Min	Mar Sec	Apr Min	Apr Sec	May Min	May Sec	June Min	June Sec
1	8 Watch too fall	52	14	46	10	08	0 Watch too slow	46	4	12	[illegible] Watch too slow	02
2	9	14	14	45	9	51	0	30	4	14	0	[illegible]
3	9	36	14	43	9	34	0	14	4	14	0	[illegible]
4	9	58	14	40	9	17	0	01	4	14	0	[illegible]
5	10	19	14	30	9	00	0	17	4	14	0	12
6	10	38	14	52	8	42	0	32	4	13	0 Watch too	01
7	10	58	14	27	8	24	0	46	4	12	0	[illegible]
8	11	17	14	21	8	06	1	00	4	10	0	27
9	11	35	14	14	7	47	1	18	4	08	0	40
10	11	52	14	07	7	28	1	28	4	05	0	35
11	12	09	14 Watch too fall	00	7 Watch too fall	09	1	40	4 Watch too slow	02	1 fall	07
12	12	26	13	52	6	50	1	52	3	58	1	20
13	12	40	13	43	6	32	2	04	3	54	1	[illegible]
14	12	53	13	33	6	13	2	16	3	48	1	[illegible]
15	13	06	13	23	5	54	2	27	3	43	1	[illegible]
16	13	18	13	12	5	36	2	37	3	37	2	[illegible]
17	13	30	13	01	5	17	2	47	3	30	2	[illegible]
18	13	42	12	49	4	58	2	57	3	23	2	[illegible]
19	13	51	12	36	4	38	3	06	3	15	[illegible]	[illegible]
20	13	59	12	23	4	19	3	15	3	07	[illegible]	[illegible]
21	14	08	12	10	4	01	3	23	2	59	[illegible]	[illegible]
22	14	16	11	56	3	42	3	30	2	[illegible]	[illegible]	[illegible]
23	14	23	11	42	3	23	3	37	2	[illegible]	[illegible]	[illegible]
24	14	29	11	28	3	05	3	43	2	[illegible]	[illegible]	[illegible]
25	14	33	11	13	2	46	3	49	2	22	[illegible]	[illegible]
26	14	37	10	57	2	28	3	54	2	10	[illegible]	[illegible]
27	14	41	10	41	2	11	3	58	1	58	[illegible]	[illegible]
28	14	44	10	25	1	13	4	02	1	46	4	[illegible]
29	14	45			1	36	4	06	1	34	4	[illegible]
30	14	46			1	19	4	08	1	24	4	[illegible]
31	14	46			1	02			1	13		

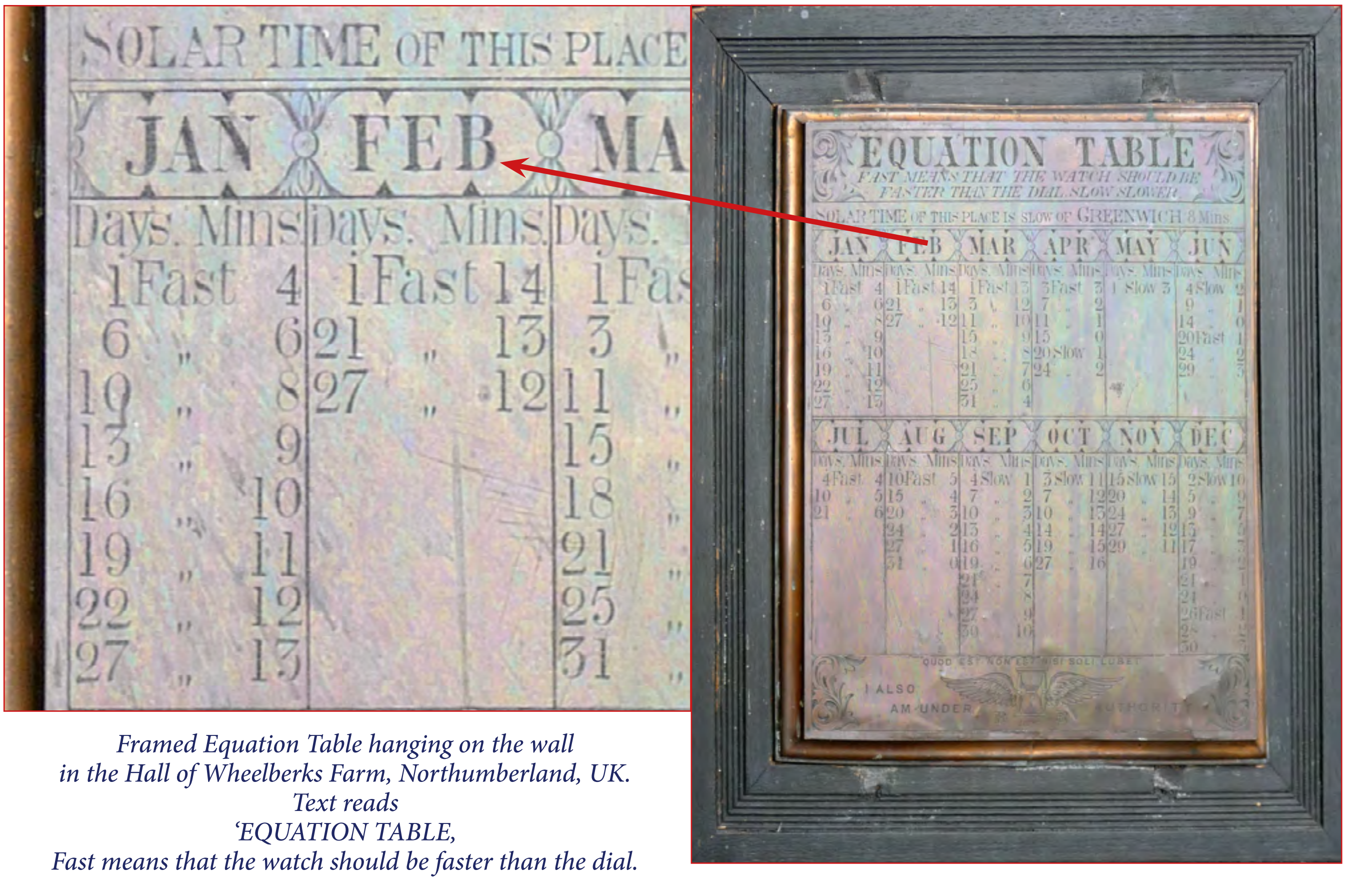
Full Year Table from Tompion's Dial at Hampton Court Palace. This is a replica dial - the original is in the museum.

... As a Partial Table

Gillamore, Yorkshire - EoT Table

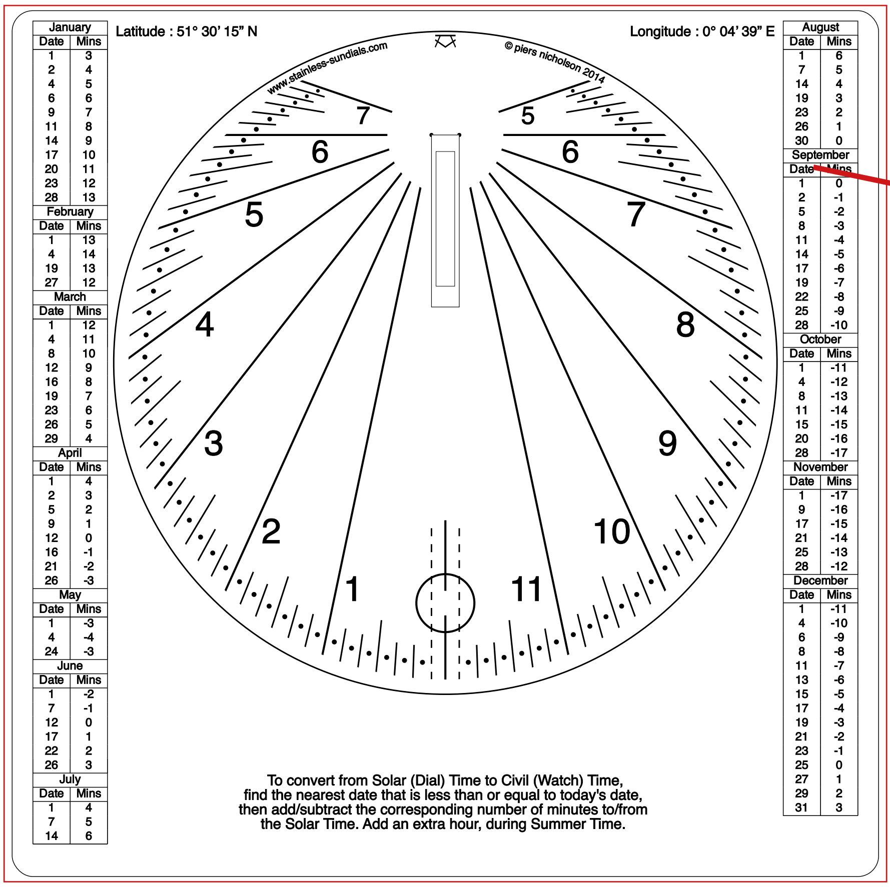


1882 Wheelbirks Farm, Northumberland



Days.	Mins	Days.	Mins	Days.
1 Fast	4	1 Fast	14	1 Fas
6 „	6	21 „	13	3
10 „	8	27 „	12	11
13 „	9			15
16 „	10			18
19 „	11			21
22 „	12			25
27 „	13			31

Framed Equation Table hanging on the wall
in the Hall of Wheelberks Farm, Northumberland, UK.
Text reads
‘EQUATION TABLE,
Fast means that the watch should be faster than the dial.
Slow Slower.
SOLAR TIME of this Place is slow of GREENWICH 8 mins’.



January	
Date	Mins
1	3
2	4
4	5
6	6
9	7
11	8
14	9
17	10
20	11
23	12
28	13

February	
Date	Mins
1	13
4	14
19	13
27	12

March	
Date	Mins
1	12
4	11
8	10
12	9
16	8
19	7
23	6
26	5
29	4

April	
Date	Mins
1	4
2	3
5	2
9	1
12	0
16	-1
21	-2
26	-3

May	
Date	Mins
1	-3
4	-4
24	-3

June	
Date	Mins
1	-2
7	-1
12	0
17	1
22	2
26	3

July	
Date	Mins
1	4
7	5
14	6

August	
Date	Mins
1	6
7	5
14	4
19	3
23	2
26	1
30	0

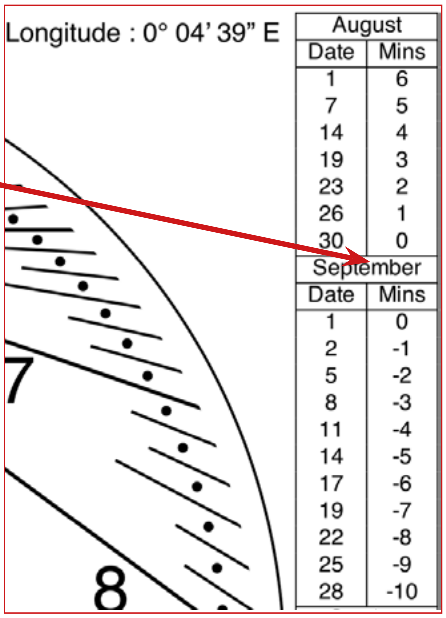
September	
Date	Mins
1	0
2	-1
5	-2
8	-3
11	-4
14	-5
17	-6
19	-7
22	-8
25	-9
28	-10

October	
Date	Mins
1	-11
4	-12
8	-13
11	-14
15	-15
20	-16
28	-17

November	
Date	Mins
1	-17
9	-16
17	-15
21	-14
25	-13
28	-12

December	
Date	Mins
1	-11
4	-10
6	-9
8	-8
11	-7
13	-6
15	-5
17	-4
19	-3
21	-2
23	-1
25	0
27	1
29	2
31	3

To convert from Solar (Dial) Time to Civil (Watch) Time,
find the nearest date that is less than or equal to today's date,
then add/subtract the corresponding number of minutes to/from
the Solar Time. Add an extra hour, during Summer Time.



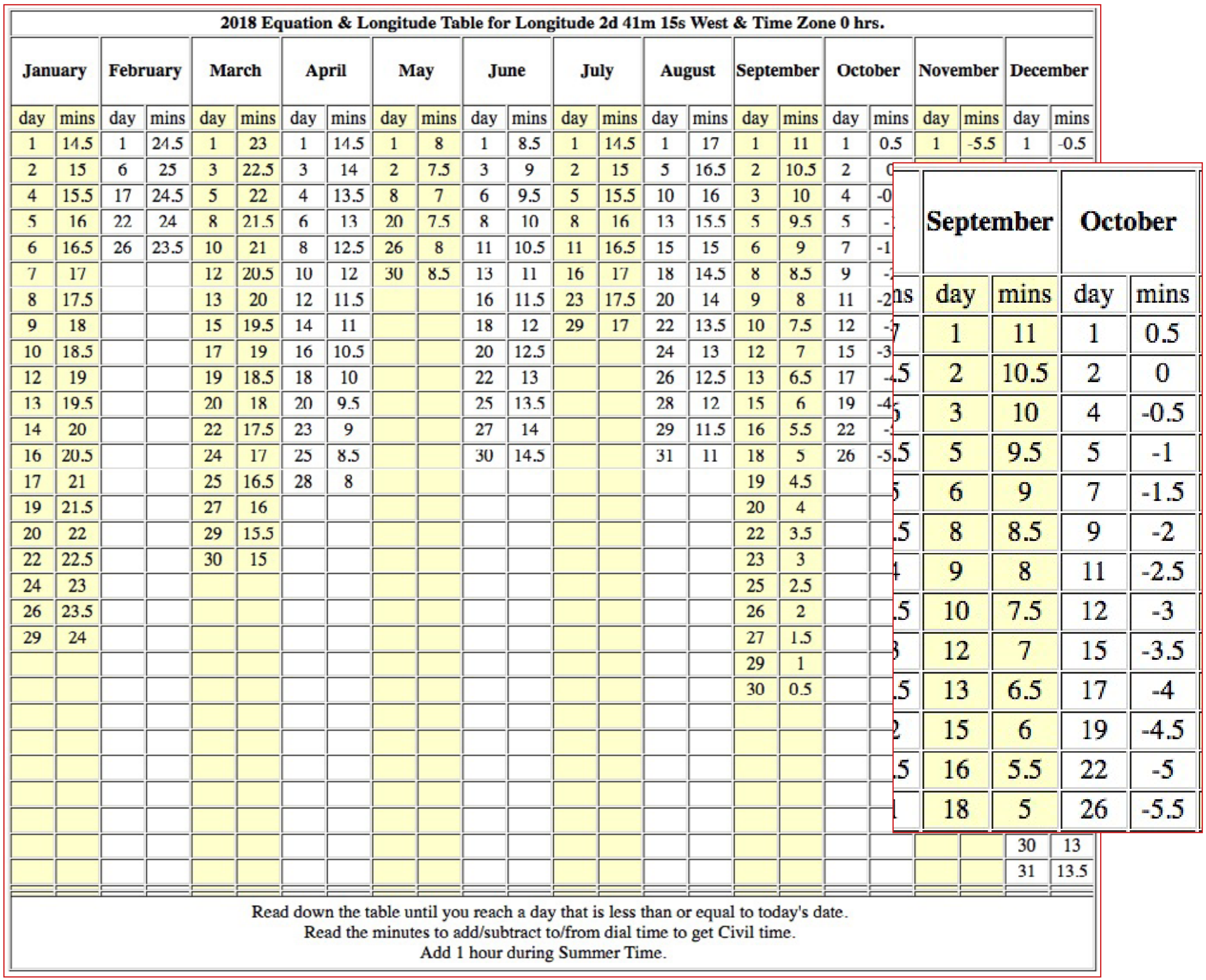
August	
Date	Mins
1	6
7	5
14	4
19	3
23	2
26	1
30	0

September	
Date	Mins
1	0
2	-1
5	-2
8	-3
11	-4
14	-5
17	-6
19	-7
22	-8
25	-9
28	-10

Minute Table on Spot-On dial,
delineated by the Author

return to "Table of Contents"

… As a ¯½ minute 'Victorian' Table



2018 Equation & Longitude Table for Longitude 2d 41m 15s West & Time Zone 0 hrs.

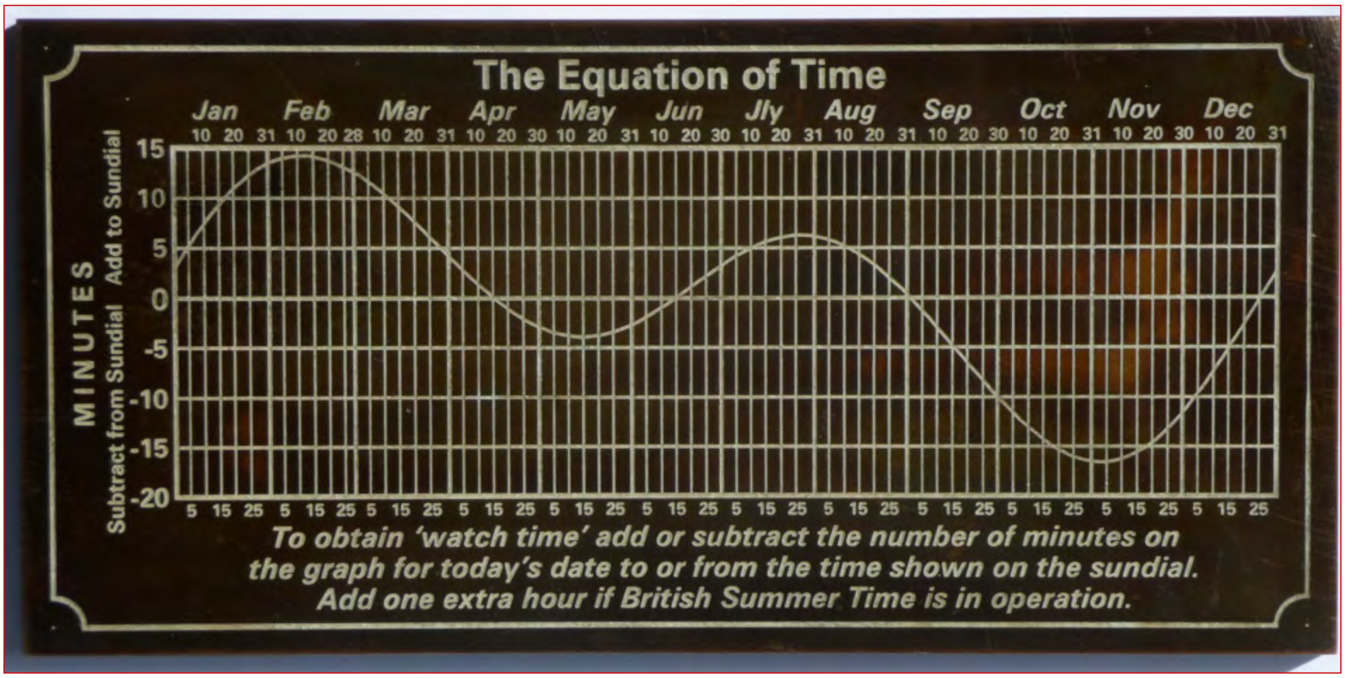
January		February		March		April		May		June		July		August		September		October		November		December	
day	mins	day	mins	day	mins	day	mins	day	mins	day	mins	day	mins	day	mins	day	mins	day	mins	day	mins	day	mins
1	14.5	1	24.5	1	23	1	14.5	1	8	1	8.5	1	14.5	1	17	1	11	1	0.5	1	-5.5	1	-0.5
2	15	6	25	3	22.5	3	14	2	7.5	3	9	2	15	5	16.5	2	10.5	2	0				
4	15.5	17	24.5	5	22	4	13.5	8	7	6	9.5	5	15.5	10	16	3	10	4	-0				
5	16	22	24	8	21.5	6	13	20	7.5	8	10	8	16	13	15.5	5	9.5	5	-				
6	16.5	26	23.5	10	21	8	12.5	26	8	11	10.5	11	16.5	15	15	6	9	7	-1				
7	17			12	20.5	10	12	30	8.5	13	11	16	17	18	14.5	8	8.5	9	-				
8	17.5			13	20	12	11.5			16	11.5	23	17.5	20	14	9	8	11	-2				
9	18			15	19.5	14	11			18	12	29	17	22	13.5	10	7.5	12	-				
10	18.5			17	19	16	10.5			20	12.5			24	13	12	7	15	-3				
12	19			19	18.5	18	10			22	13			26	12.5	13	6.5	17	-				
13	19.5			20	18	20	9.5			25	13.5			28	12	15	6	19	-4				
14	20			22	17.5	23	9			27	14			29	11.5	16	5.5	22	-				
16	20.5			24	17	25	8.5			30	14.5			31	11	18	5	26	-5				
17	21			25	16.5	28	8									19	4.5						
19	21.5			27	16											20	4						
20	22			29	15.5											22	3.5						
22	22.5			30	15											23	3						
24	23															25	2.5						
26	23.5															26	2						
29	24															27	1.5						
																29	1						
																30	0.5						
																						30	13
																						31	13.5

September		October	
day	mins	day	mins
1	11	1	0.5
2	10.5	2	0
3	10	4	-0.5
5	9.5	5	-1
6	9	7	-1.5
8	8.5	9	-2
9	8	11	-2.5
10	7.5	12	-3
12	7	15	-3.5
13	6.5	17	-4
15	6	19	-4.5
16	5.5	22	-5
18	5	26	-5.5

Read down the table until you reach a day that is less than or equal to today's date.
Read the minutes to add/subtract to/from dial time to get Civil time.
Add 1 hour during Summer Time.

Half Minute Table by the Author.

… As a Cartesian Graph



To obtain 'watch time' add or subtract the number of minutes on the graph for today's date to or from the time shown on the sundial. Add one extra hour if British Summer Time is in operation.

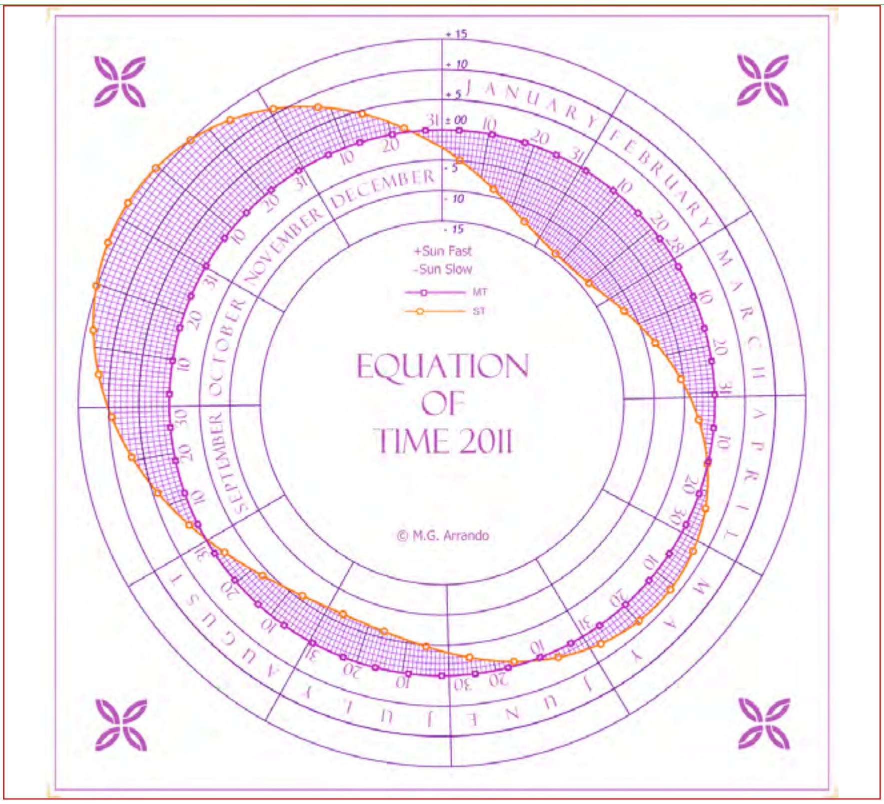
'Normal' Rectilinear Graph by Tony Moss

Joanna Migdal - 2002 - Haslemere Museum - Back of Equatorial Ring.

Roser Raluy & Alexandr Boldyrev - 2012 - Roses, Catalonia

return to "Table of Contents"

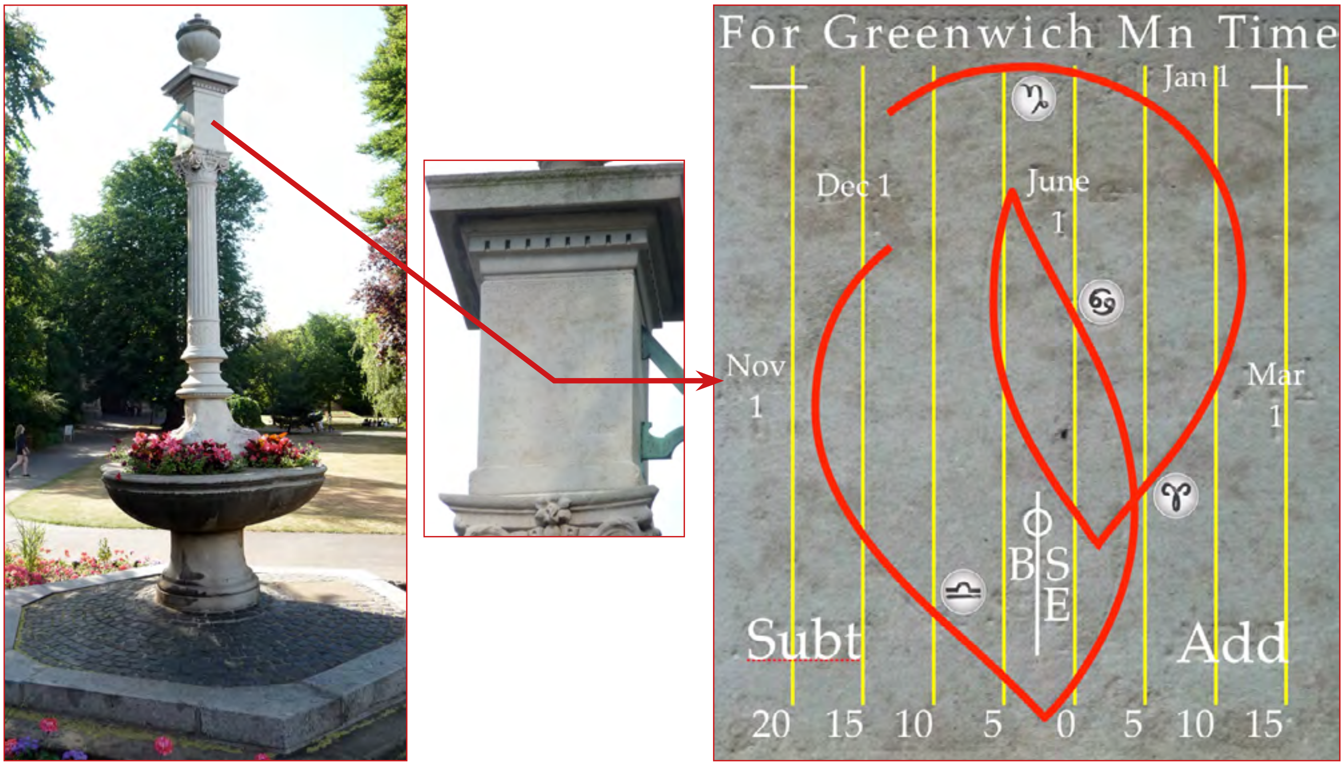
... As a Polar Graph



Polar Graph by M.G. Arrando

... As an Intrinsic Graph

See over for explanation



Cube Dial with its Intrinsic EoT Curve - 1871 - Bury St Edmunds Abbey Gardens, by Francis Penrose.
It is believed to be the first sundial to acknowledge National Mean Time - rather than Local Mean Time.
The BSE line on the dial recognizes the longitudinal difference between Greenwich and Bury St Edmunds.
See 'Footnote on the Bury St Edmunds dial' on page 98.
See 'Graphical Presentations' on page 69. by John Davis,
Fred Sawyer in The Compendium 12(3), September 2005, pp.29-31

There is an extentsive video detailing the extraordinary background - technically, historically and culturally - of this dial.
Instructions how to find and view this may be found on 'Available Videos' on page 234.

In a Cartesian plot,

- while the independent value varies linearly along the one axis, the corresponding dependent value moves along the other axis.

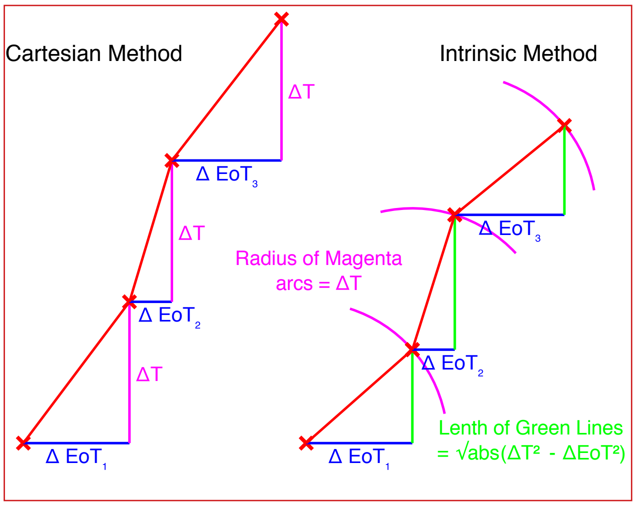
In a Polar plot,

- while the independent value varies around the circumference of the circle, while dependent value moves radially.

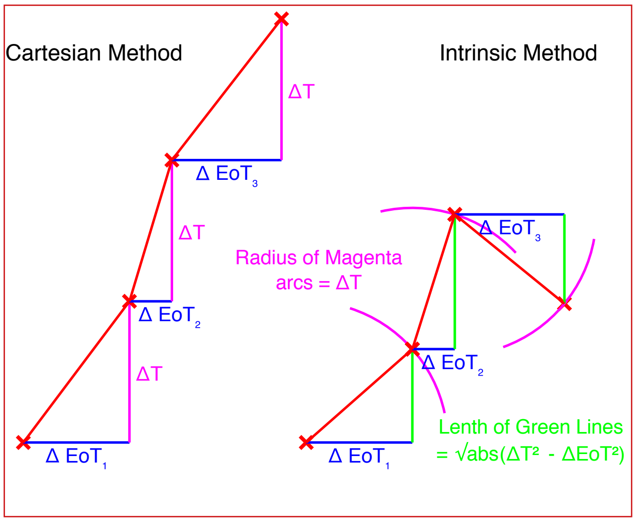
The family of Intrinsic plot, discovered by the Cambridge mathematician William Whewell in 1849, are not common. The type of intrinsic curve of interest in this context, is defined thus:

> the independent variable varies along the graph itself, while the dependent variable moves along either of the axes. This allows the graph to fold upon itself. This in turn allows a very compact visualization of many data points

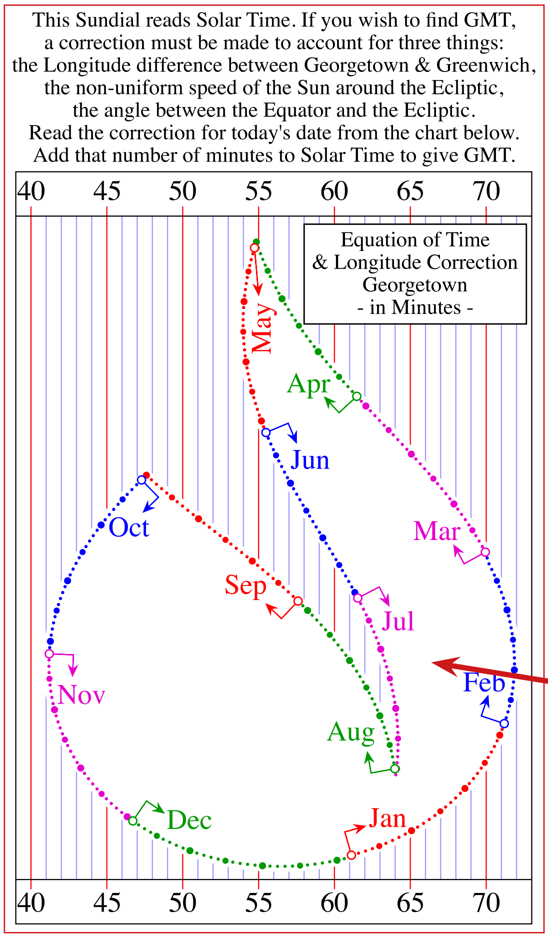
The Bury curve above is open. But, depending on the folding points, a closed version is possible, as shown in the Georgetown dial, below...



Cartesian-v- Intrinsic - 4th Intrinsic Point **UNFOLDED**



Cartesian-v- Intrinsic - 4th Intrinsic Point **FOLDED**



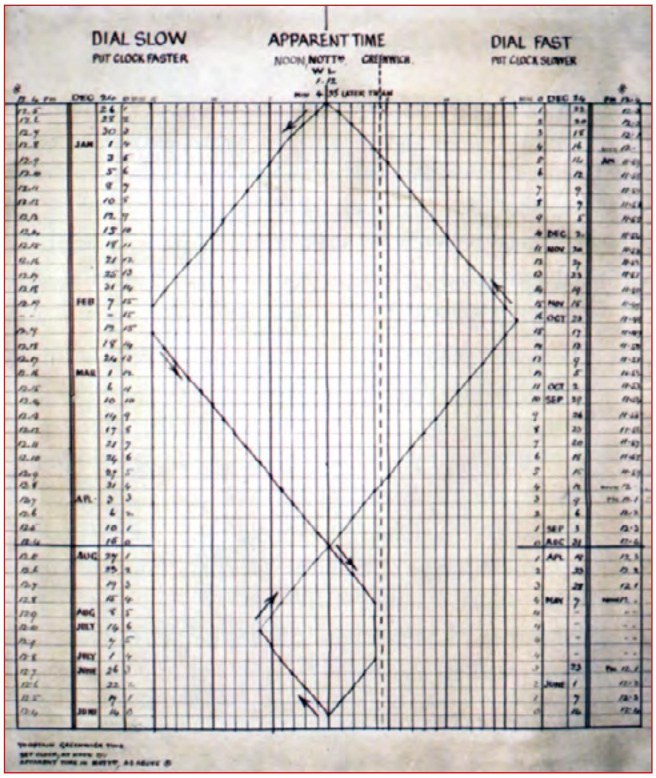
This Sundial reads Solar Time. If you wish to find GMT,
a correction must be made to account for three things:
the Longitude difference between Georgetown & Greenwich,
the non-uniform speed of the Sun around the Ecliptic,
the angle between the Equator and the Ecliptic.
Read the correction for today's date from the chart below.
Add that number of minutes to Solar Time to give GMT.

Georgetown, Ascension Island - Latitude 8° South.
Delineated by the author, made by Nick Tayler

Footnote on the Bury St Edmunds dial

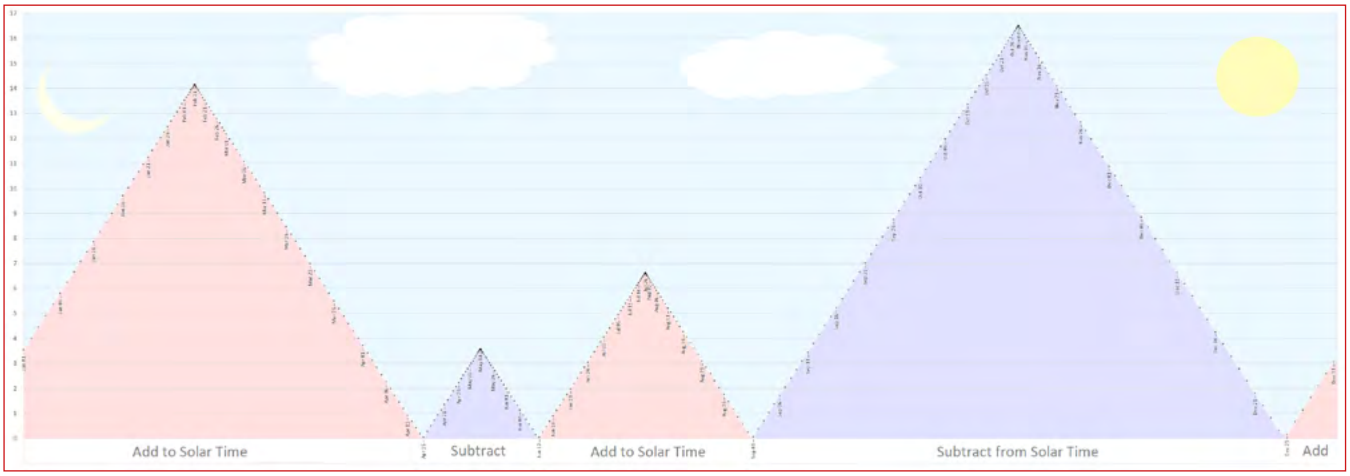
The Bury St Edmunds dial is of great intellectual, social and historical interest. It involves a coterie of Cambridge intellectuals & friends, all of whom had close connections with Bury St Edmonds, some 25 miles away: Rev Professor Whewell (Cambridge mathematician, polymath & 'inventor' of intrinsic curves), George Biddell Airey (professor of mathematics and astronomer royal), both 2nd & 3rd Marquesses of Bristol (local aristocrats), Bishop Lord Arthur Hervey (linguist & antiquarian), Francis Penrose (architect, archaeologist & astronomer). Whewell coined the following words for the first time 'anode', 'cathode', 'ion' and 'scientist'. Under the influence of the coterie above, it is believed that Francis Penrose, designed the sundial for the Marquess of Bristol, who gave it to the town..

return to "Table of Contents"



Figurative EoT Document
from Bromley House Library. Nottingham

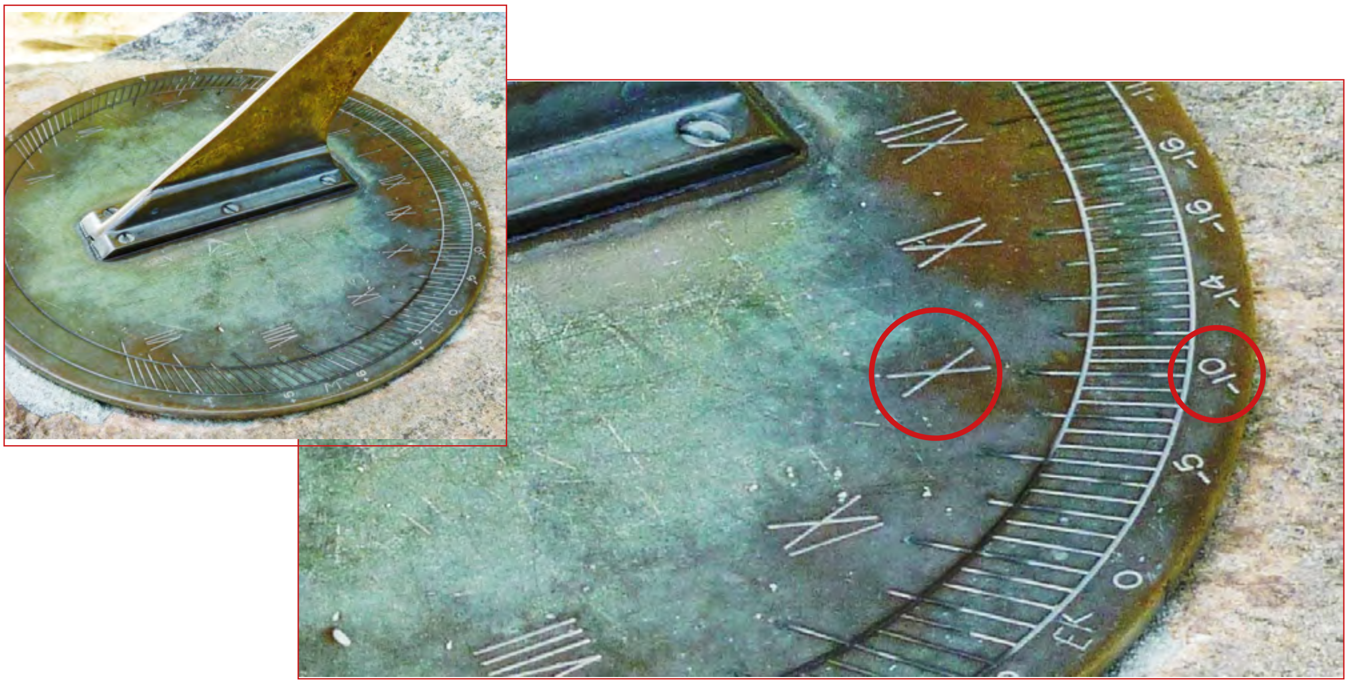
Knot EoT - 1974 - James Richard



Rockies EoT by Steve Lelievre

return to "Table of Contents"

Bob Hampton & Martin Webster - 2010 - Quilt Block Sundial, Burnsville NC.
The purple outline of the Blue Mountains outlines the EoT.



Kirstenbosch Botanical Gardens = 1920 - by J.R. Miller. EoT is marked around the rim of the dial in positions corresponding to the date in the month. Thus at the beginning of the 10th month (X) is -10 mins

EoT Incidentals

ON WHICH DATE IS THE LATEST SUNSET?

One might suppose that this would be on the Winter Solstice. But no - the time of sunset is when the sun is at zero altitude. This time in Civil Hours is given by the formula:

$$\text{Sunrise/set}^{\text{hrs}} = 12^{\text{hrs}} \pm \frac{\cos^{-1}\left(-\tan\left(\text{Latitude}^{\circ}\right)\times\tan\left(\text{Declination}^{\circ}\right)\right)^{\circ}}{15} + \frac{\text{EoT}^{\text{mins}}_{\text{noon}}}{60}$$

The formula shows that the duration of morning and afternoon are not equally dispersed around civil noon. Around the Winter Solstice, the value of the Sun's Declination is only changing marginally each day, while the Equation of Time is changing relatively rapidly. The effect of the changing EoT outweighs that of the changing declination. This leads to the day on which sunset is latest moving away from the winter solstice and towards January (in the Northern hemisphere).

As one moves towards Equatorial Latitudes, the tangent of the Latitude trends towards zero and the timing of sunrise and sunset varies little throughout the year, but the EoT effect becomes significant, as shown in the Fig 1 below.

Fig 2 & 3 show the time of Sunrise in Solar Time (Green) and in Standard Time = Local Mean Time (Cyan & Red)

In the Solar Time Graph, note...

i) the graph is totally symmetrical

ii) at the Arctic circles, there is a linear daily change, midnight to noon, from the summer to winter solstices;

iii) at the Equator, dawn is at 6:00 a.m. throughout the year.

In the Mean Time Graph, note...

iv) at the Arctic circles, there is no effect of the equation, so the linear daily change, midnight to noon, from the summer to winter solstices remains;

v) at the Equator, the time of sunrise exactly reflects the Equation of Time - see the waves of the black Equator line. This is what one would expect bearing in mind that the solar time sunrise is the same throughout the year;

vi) at the marked example latitude (Greenwich), the earliest/latest sunrises are a few days away from the Solstices. While nearer the Equator, the difference from the solstices rise to almost 2 months.

Another view of the effect of EoT on sunrise and sunset times can be seen in Fig 4, overleaf. Here the effect of the EoT can be seen as the twist in the curves around the Equator.

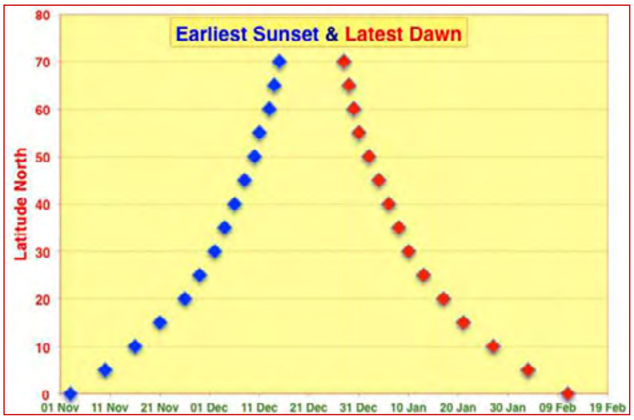


Fig 1- Earliest Sunset and Latest Dawn

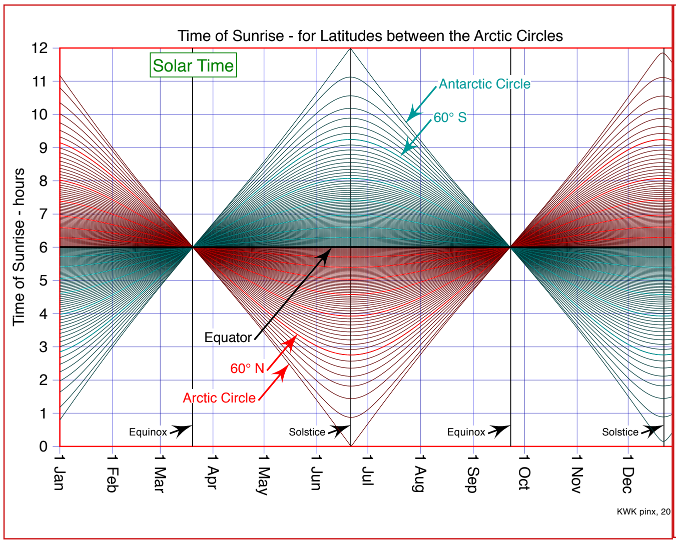


Fig 2 - Sunrise & Sunset for different latitudes in Solar Time

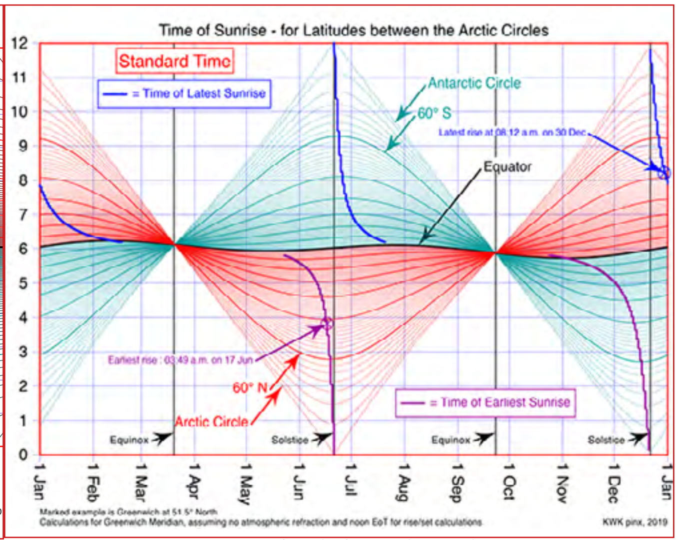


Fig 3 - Sunrise & Sunset for different latitudes in Mean Time

return to "Table of Contents"

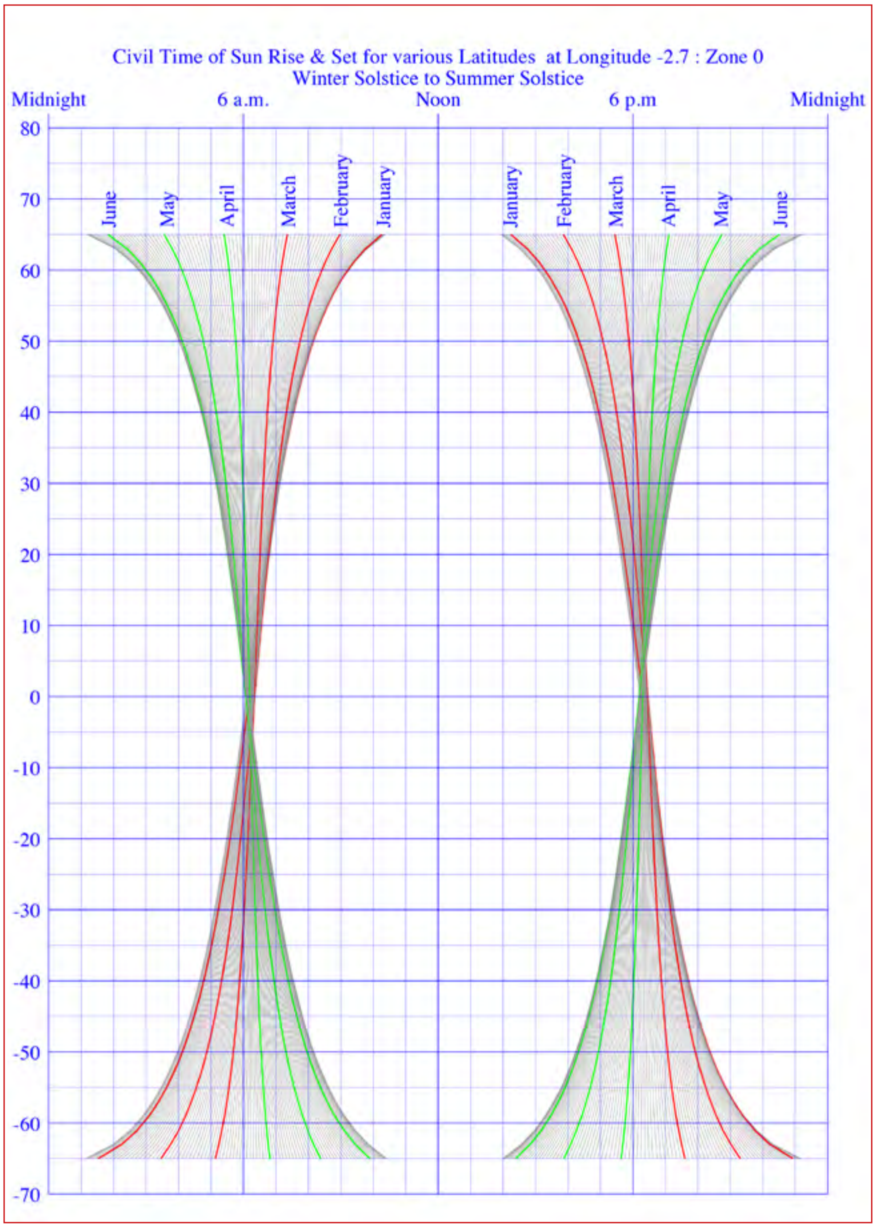


Fig 4 - Sunrise and Sunset over the months and the latitudes, showing the twisting effect of the EoT

A GNOMONIST'S RIDDLE

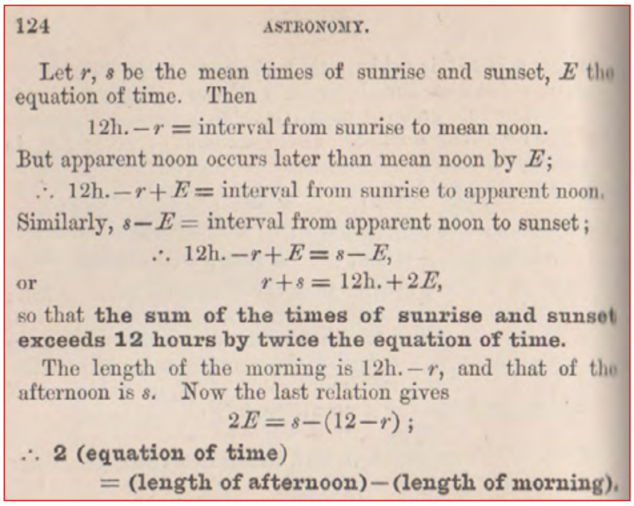
It is a fine winter's day and the sun is shining, you are sitting on a park bench reading the local newspaper. You have no timepiece, but there is a Sundial nearby. How do you find the Standard Time?

Answer

Your local newspaper usually has the time of Sunrise and Sunset. Add together the time of Sunrise and Sunset, subtract 24 and multiply by 30 and you have the EoT and the longitude correction in minutes. Read the Sundial, apply the correction and you have the standard time.

A glance at the formula in the previous section will show why this is so, since adding sunrise to sunset cancels out the +/- trigonometrical part.

This method was well known in the 19th C, as shown below.



124 ASTRONOMY.

Let r, s be the mean times of sunrise and sunset, E the equation of time. Then

$12\text{h.} - r =$ interval from sunrise to mean noon.

But apparent noon occurs later than mean noon by E;

$\therefore\ 12\text{h.} - r + E =$ interval from sunrise to apparent noon.

Similarly, $s - E =$ interval from apparent noon to sunset;

$$\therefore\ 12\text{h.} - r + E = s - E,$$

or $$r + s = 12\text{h.} + 2E,$$

so that **the sum of the times of sunrise and sunset exceeds 12 hours by twice the equation of time.**

The length of the morning is $12\text{h.} - r$, and that of the afternoon is s. Now the last relation gives

$$2E = s - (12 - r);$$

$\therefore$ **2 (equation of time) = (length of afternoon) − (length of morning).**

Text from Elementary Mathematical Astronomy by Barlow & Bryan 1893

ISLINGTON'S EQUATION

There was a restaurant in London - now closed - called 'Islington's Equation'. Therein, a dish was served called...

The Equation of Time

Foie gras, pan-seared with wild forest berries & a port sauce "a la minute" that'll leave you wanting "seconds".

return to "Table of Contents"

EQUATION OF TIME NOMOGRAMS

A nomogram ...

> *a diagram representing the relations between three or more variable quantities by means of a number of scales, so arranged that the value of one variable can be found by a simple geometrical construction.*

The slide rule is a typical example of a nomogram.

The author was looking to improve on the EoT device used in Pilkington & Gibbs Sol Horometer where the Equation correction was incorporated by an ingenious nomographic rotation and lineation.

The outer ring is rotated to line up 2 dates (e.g. 5th June), which provides the correct rotation to correct for the Equation of Time and Longitude correction.

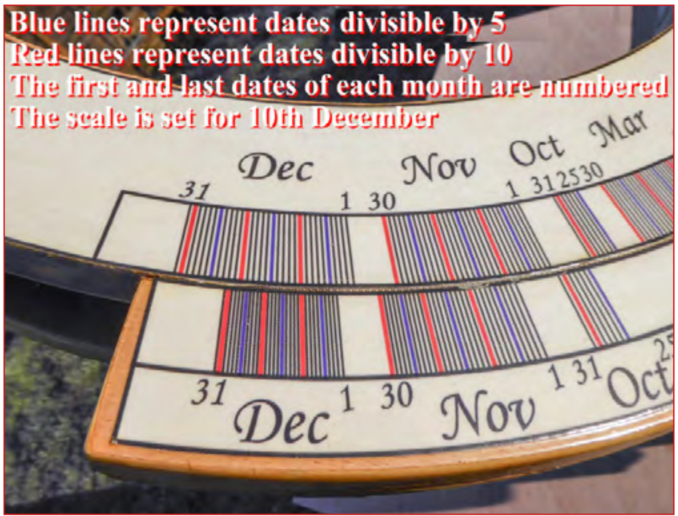
A further improvement was introduced by Brain Hugget, who split the two scales to cover both normal and daylight saving time.

The two nomograms are presented here, GUN & GIN. The names, (GUN = Gnomonic Universal Nomogram and GIN = Gnomonic Instrument Nomographic) are poor!

They were developed as an exercise in laser engraving/cutting using cheap MDF and acrylic. They provide the Longitude corrected Equation of Time & time of Solar Noon correct to a within a few seconds - subject to the accuracy of the fabrication .

Pilkinson & Gibbs Sol Horometer - ca 1910 showing the two marks for 5th June.

The outer ring must be rotated to align the two dates



Blue lines represent dates divisible by 5
Red lines represent dates divisible by 10
The first and last dates of each month are numbered
The scale is set for 10th December

Brian Huggett's Mark II Heliochronometer - ca 2016, showing 10th December selected

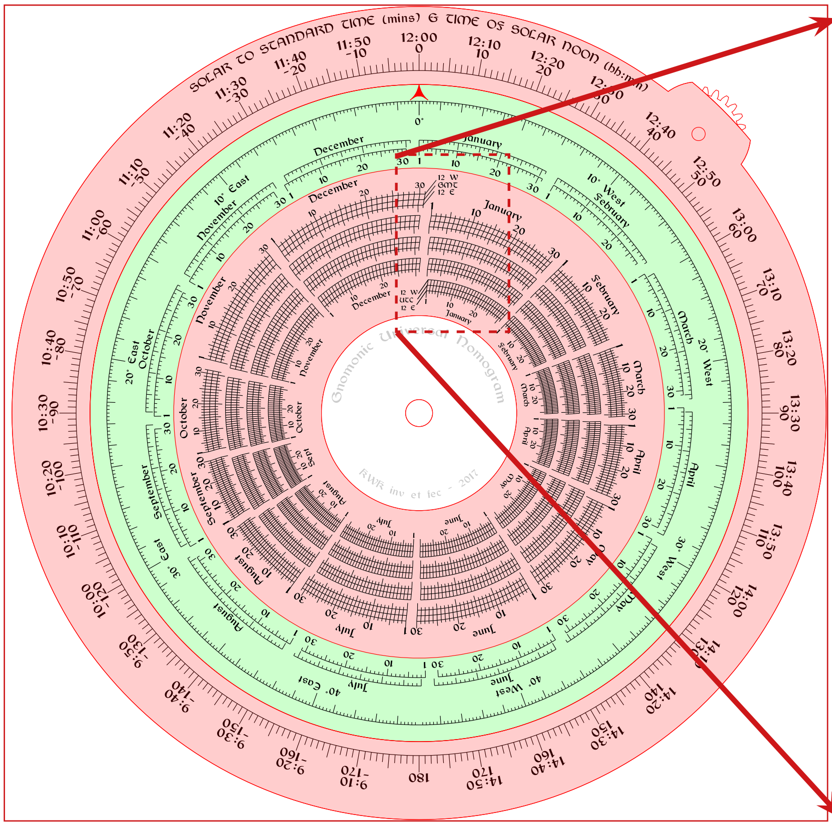
GUN : Gnomonic Universal Nomogram

A large impractical device. It works for any day in the 4-year leap cycle and for any Time Zone. It provides the Equation of Time, Longitude Corrected Equation and Time of Solar Noon.

The GUN is made of three layers of 4mm high quality MDF. Each layer is separated by 0.5mm PTFE, providing lubrication to the rotating central section. The centre section is rotated by a little cog (not visible in this image)

Assembled GUN, as laser engraved and cut.

return to "Table of Contents"

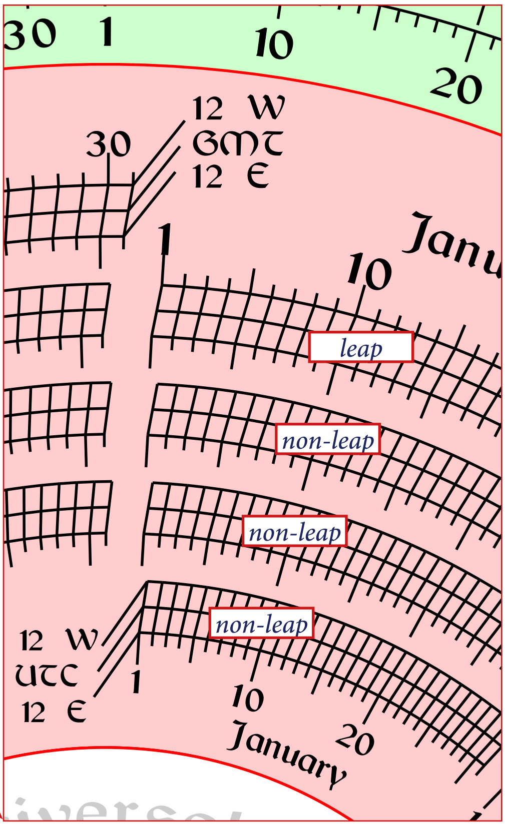


GUN showing :

(i) the inner pink stationary section with the a spiral EoT scale, covering the 4 year leap cycle and all the time zones (see detail)

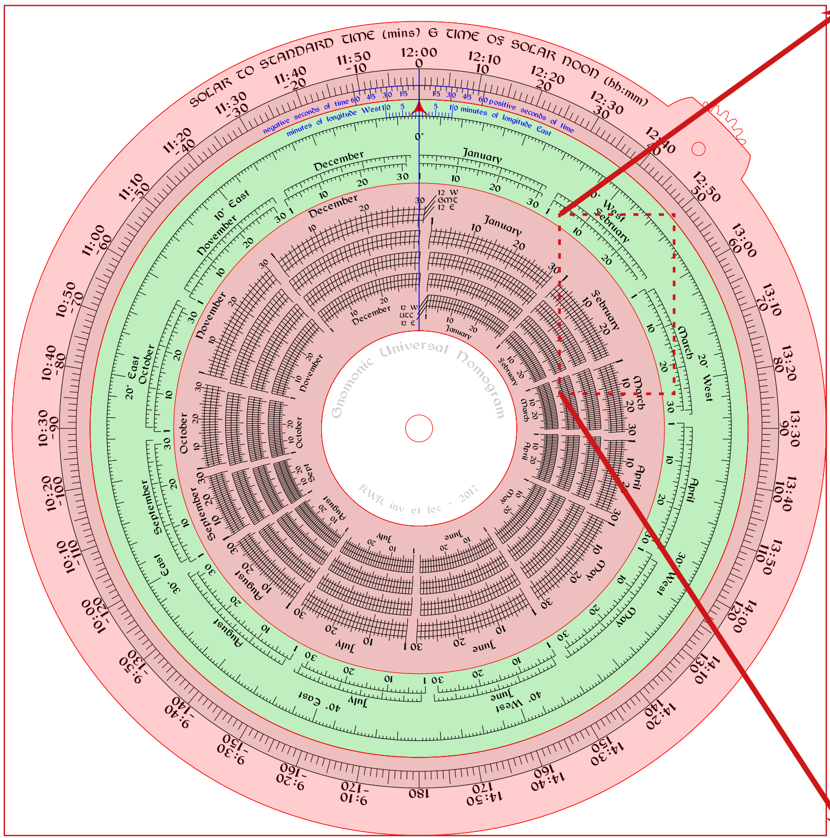
(ii) the green rotating section also with an two EoT scales (the outer for the leap years, the inner for other years, a scale with the longitude offset and a red index pointer

(iii) the outer pink stationary section giving the results - EoT, EoT + longitude correction & time of Solar Noon)

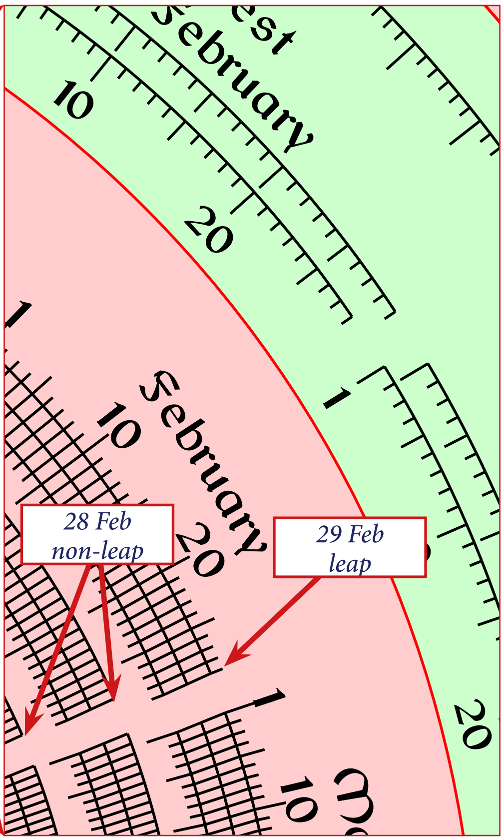


Detail of the inner annual EoT spiral. The bottom of the spiral represents the year following the leap year and outwards to the leap year.

The spiral contains 3 scales, covering all the various time zones .

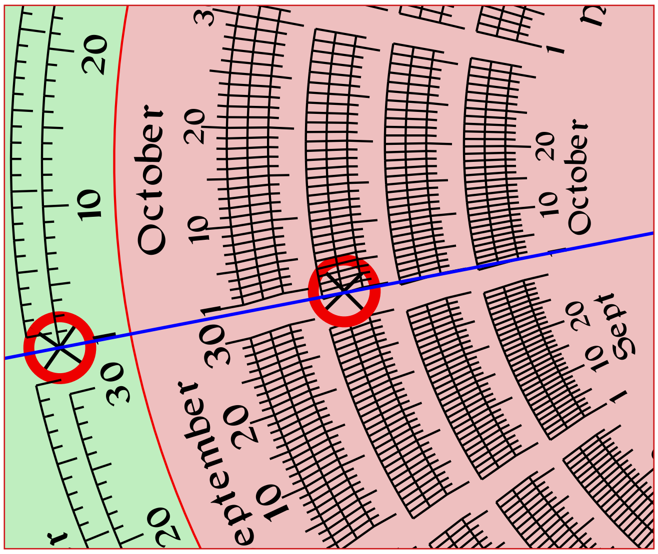


As above but showing the perspex Alidade, with a blue index line & two vernier scales



Detail showing February 29th on the outer edge of the spiral

return to "Table of Contents"

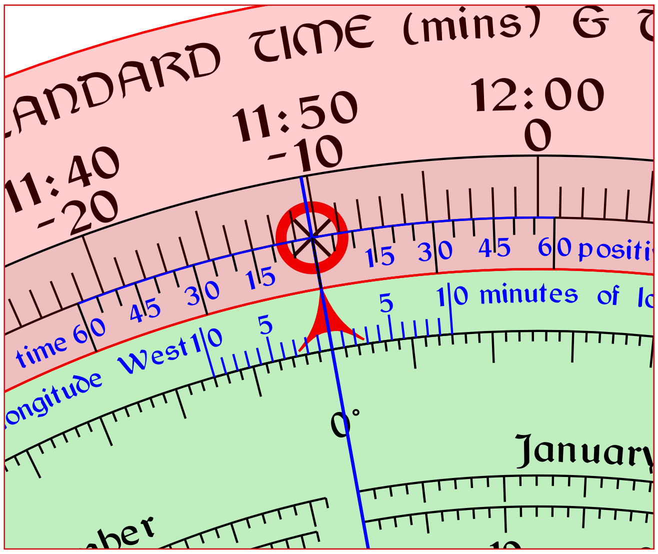


Using GIN - 1st Step.

Date : 1st October 2019.

Blue alidade cursor line is used to match the date on :

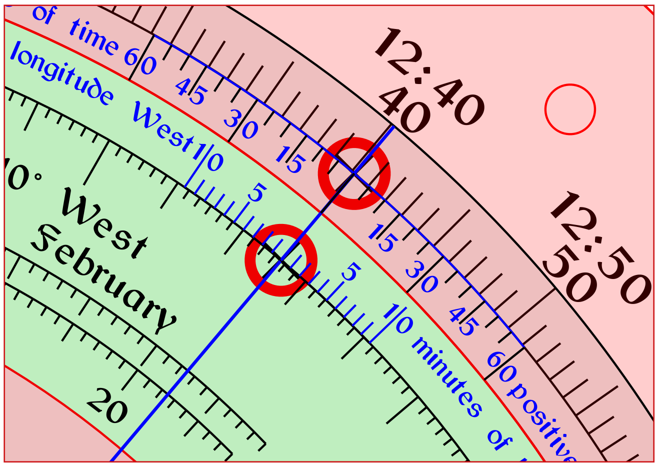
- *Red fixed scale, 3rd spiral outward from centre ;*
- *Green rotating scale, inner (non-leap) scale*



Using GIN - 2nd Step.

Blue alidade cursor line is turned to meet the Red index arrow head

Use the Blue cursor to read the EoT - using the outer vernier - to give a value of -10:15 min:sec



Using GIN - 3rd Step.

Blue Cursor line is turned so that the outer green scale - the location offset from the time zone meridian. The Longitude corrected EoT and the time of solar noon

GIN : Gnomonic Instrument Nomographic

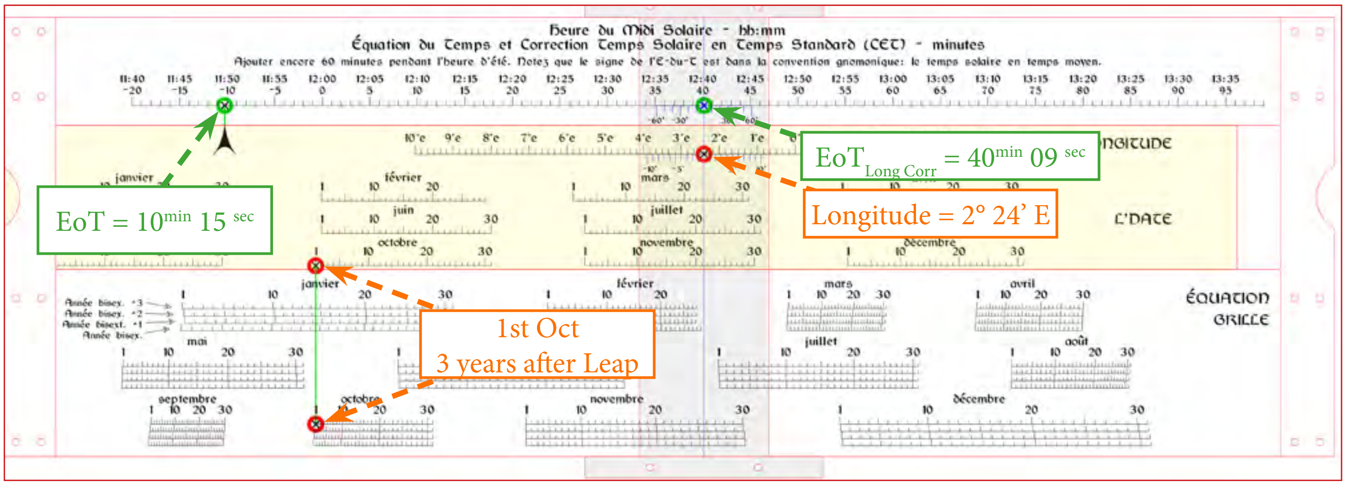
Another poor name, but a practical Device - set up to cover any limited longitude range. The one below was made for France Metropolitan.

This slide rule was built of (a) base of 4mm MDF, (b) a layer of 1mm PTFE and (c) the top & slide of 4mm MDF. The alidade was 3mm Acrylic.

There are two designs available for this device to be (a) cut on a laser (b) printed on card - suitable to use as a training aid.

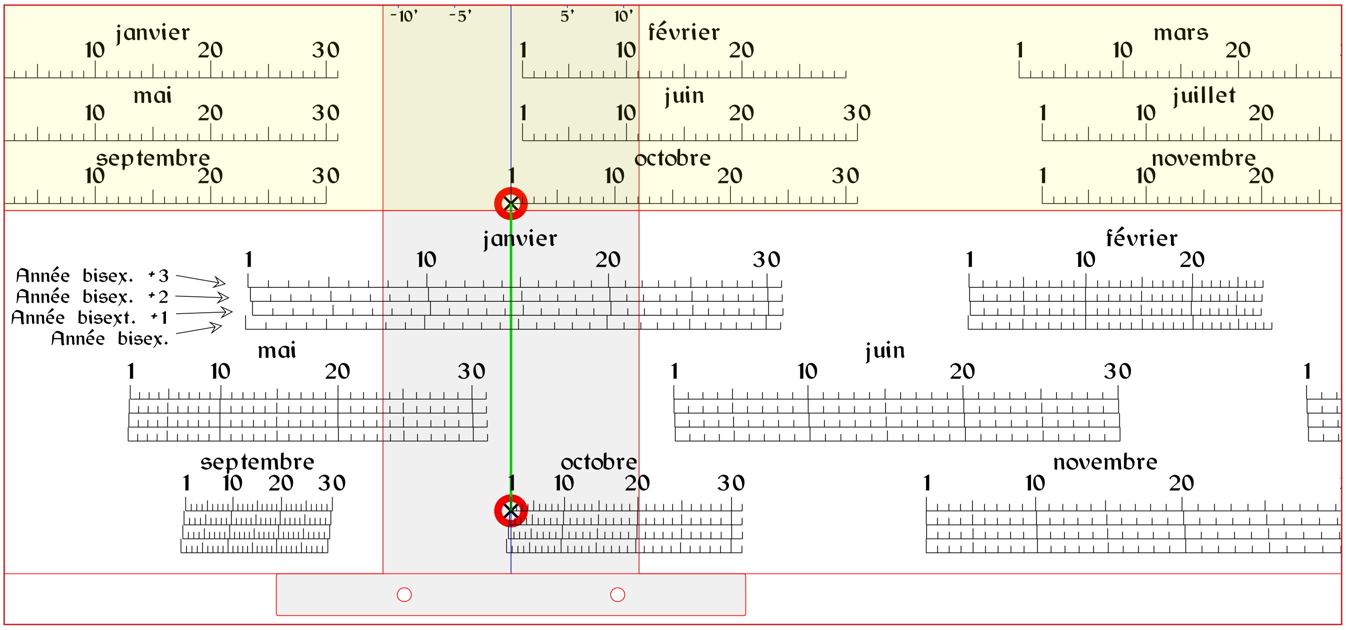
The algorithms used provide the EoT are within +/- 1 second precision. If the device were perfectly made, the results can be read to within almost that precision, using the provided Verniers

If anyone wants the graphics to make GIN or GUN, the author (if alive) would be happy to provide them with the art-work (for their specific time zone, in the case of GIN). .

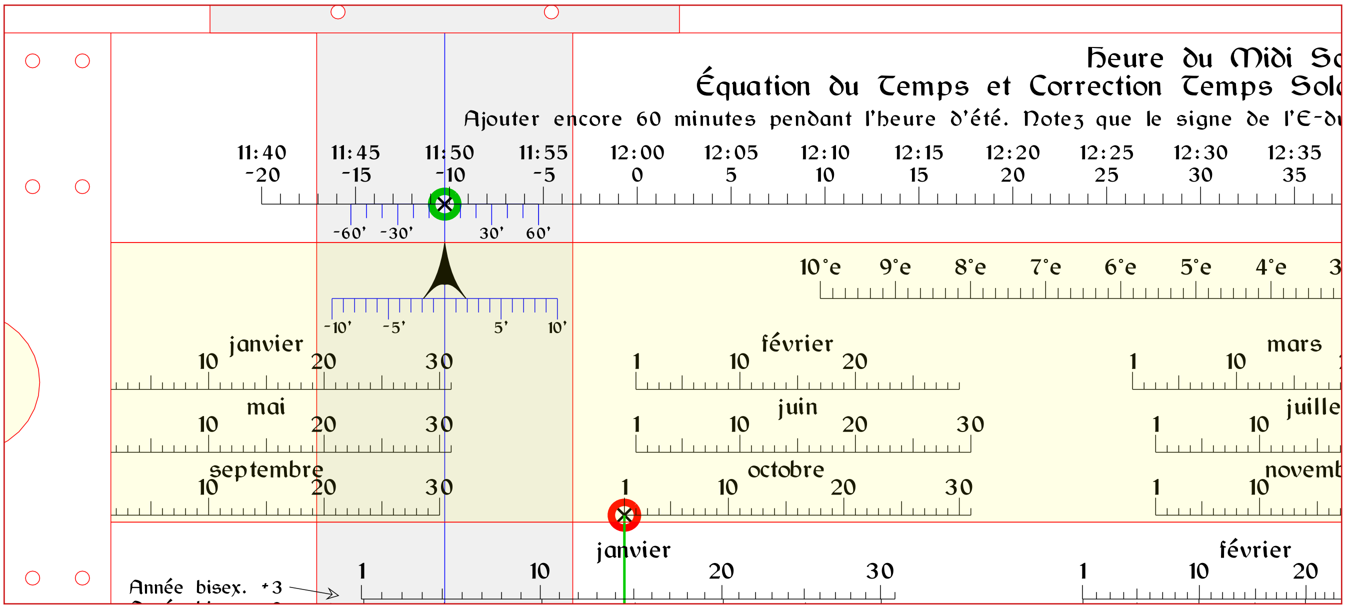


The Gnomonic Instrument Gnomic (GIN) - a slide rule. This version covers the longitudes of France Métropolitaine

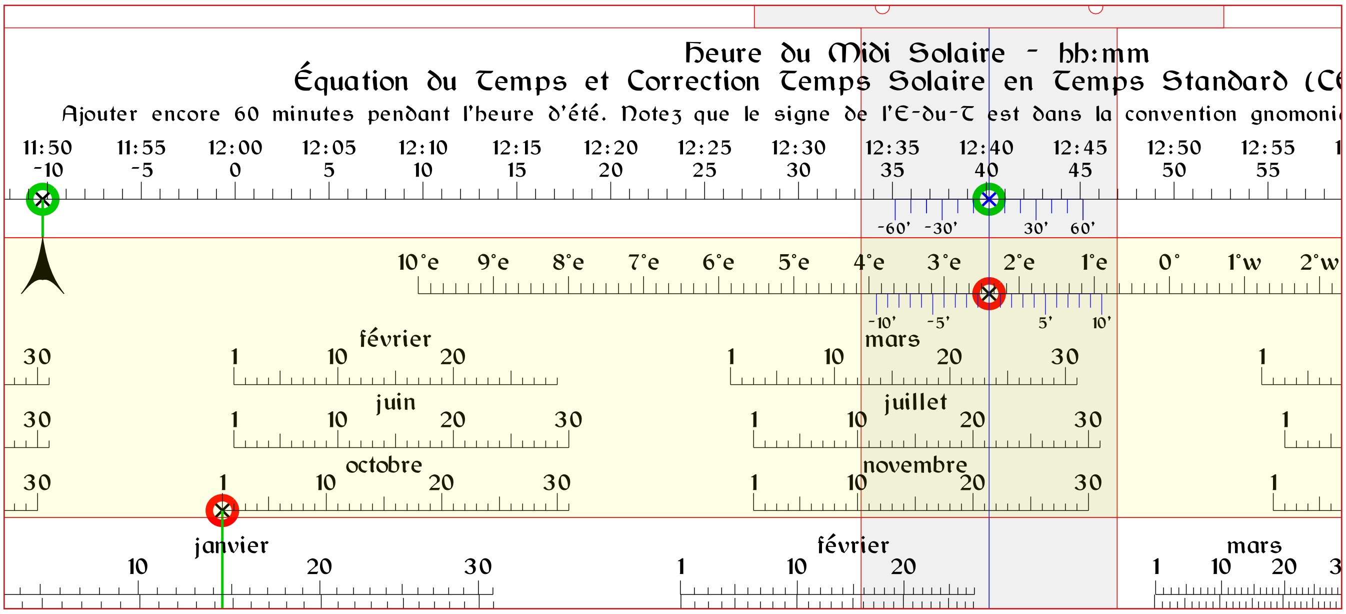
return to "Table of Contents"



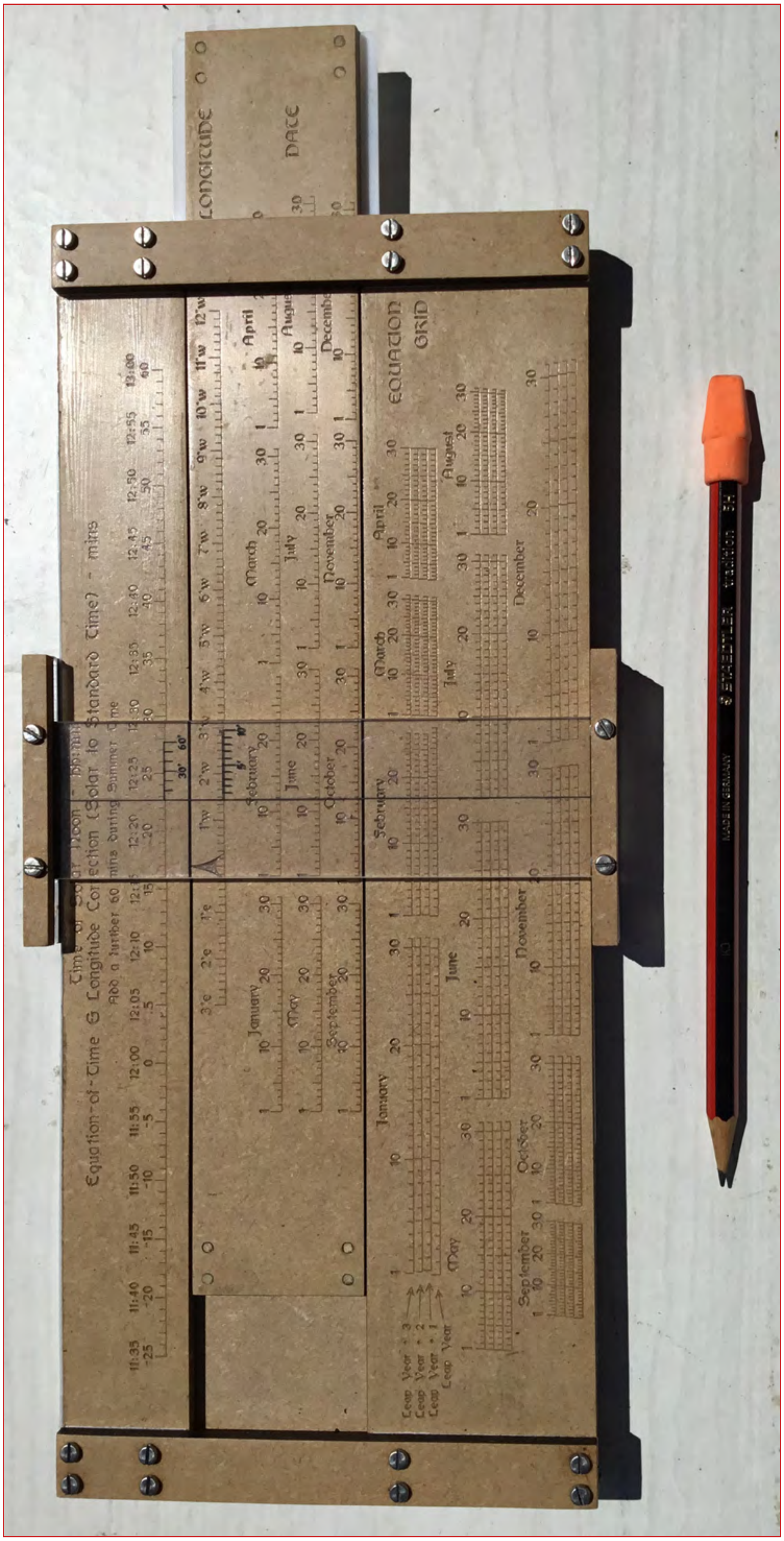
Using GIN - 1st Step: Using the cursor, adjust the yellow slide to match the date (1st Oct 2019 - 3rd Year in the Leap Cycle.)



Using GIN - 2nd Step: Without moving the cursor, read the EoT the black arrow head and the vernier scale -= -10:15 mm:ss



Using GIN - 3rd Step: Longitude is 2.4° E in Time Zone 1 (15° E). Moving the cursor to the local longitude offset (the red circle), the total correction from solar to mean time is seen (the green circle) -= +40:09 mm:ss



The Gnomonic Instrument Gnomic (GIN).- as made for the British Isles

return to "Table of Contents"

return to "Table of Contents"

Mechanically Generating the Equation of Time

MECHANICAL POSSIBILITIES

Cams appropriately shaped provide the easiest and most compact method - typically used in Equation Clocks and Watches. Usually only a single cam is used, but two cams can also be employed.

Gears are more complex - but provide a more accurate, more interesting, visual and educational method - at the expense of complexity. EoT generation with gears is typical for Astronomical Clocks.

Mechanical methods with gears universally use the fact that the Equation of Time can be broken down astronomically or by Fourier Analysis into a series of sine curves. A sine curve is easy to generate with a gear. Usually the accuracy demands of the mechanism mean that only the first two harmonics of the Fourier series are used.

GENERATING A SINE CURVE MECHANICALLY

A rolling wheel can produce a sine curve, but we generally need a linear output. This is best produced by an epicyclic gear using the Tusi Coupling Method (invented by Nasir al-Din al-Tusi, the great Persian astronomer and polymath, in 1247). This requires an outer gear of twice the pitch diameter of the inner. The pitch diameter of the outer being the magnitude of the signal one wishes to produce. In the illustration, As the Green gear is rotated by the purple arm, the blue fixed point on the green gear will move linearly up and down with sinusoidal motion. The Tusi Couple is a special case of hypocycloid theory.

An example of a Tusi couple in a sundial maay be found in Bill May's work. See 'Tusi Mechanism' on page 158.

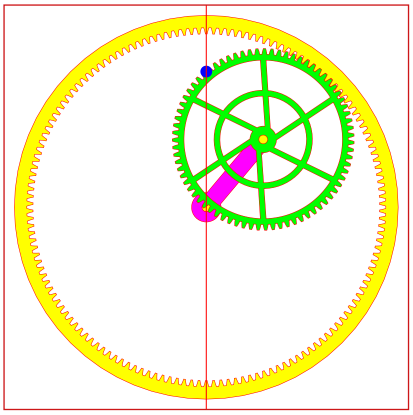
John Goodman has developed modified Tusi couples using simple spur gears, removing the need of the complex and expensive ring gear.

In most cases of gears and double cams, it is necessary to add the different sine components together. This can be done in three ways.

صورة الدائرتين في المبدأ ولتتحرك الصغيرة إلى جهة يمين الناظر والكبيرة إلى جهة يساره	صورتهما بعد أن قطعت الصغيرة نصفًا والكبيرة ربعًا	صورتهما بعد أن تمت الصغيرة دورةً والكبيرة نصفها	صورتهما بعد أن قطعت الصغيرة دورة ونصفًا والكبيرة ثلثة ارباع دورة

ولبيان أن النقطة لا تزول عن الخط أصلًا وإن لم نكن نقصد إيراد البراهين الهندسية في هذا المختصر فلتكن الكبيرة دائرة ا ب ج وقطرها ا ج ومركزها

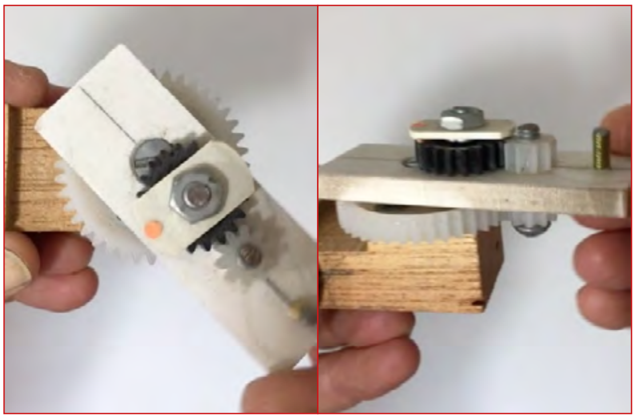
Description of the Tusi Couple - 13th C



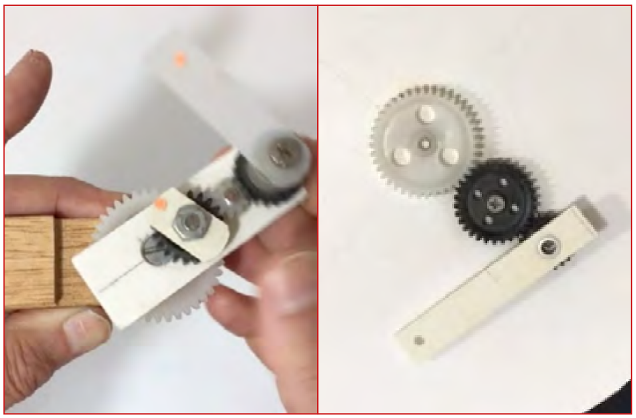
The Tusi Couple. As the Green gear is rotated by the purple arm, the Blue fixed point on the Green gear will move up and down along the Red line with sinusoidal motion.

There are animations available of these mechanisms. See 'Available Videos' on page 234.

Commonly used Tusi Couple called a 'Scotch Yoke', as used in steam trains



Goodman's Tusi method without a ring gear. On rotation, the orange spot moves linearly with sinusoidal motion



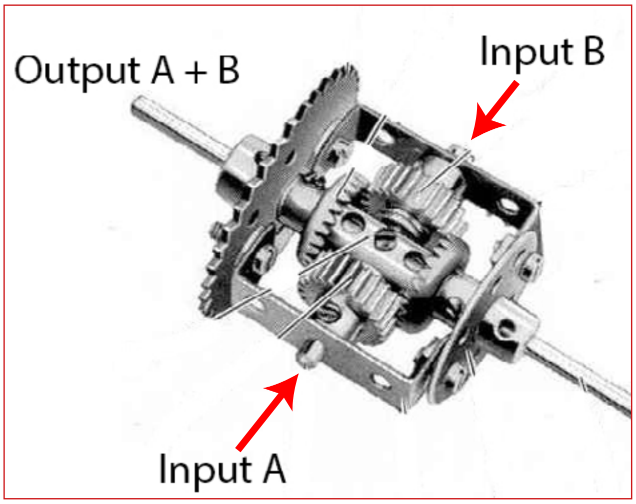
Goodman's Tusi method using small gears to produce a greater movement (left). Using just three spur gears (right)

THE DIFFERENTIAL ADDER

The differential is most commonly seen on the back axle of a car. A single inner rotation from the engine is divided into two equal outer rotations that are fed to the wheels when the car is travelling in a straight line. But, when cornering, the same inner rotation is divided so the outer wheel turns faster.

In reverse, the two outer rotations can act as input and the inner rotations is the sum of the two.

Typically in Equation clocks, the EoT from a cam is added to pendulum driven mean time r provide a solar time hand .



Basic Differential Adder - commonly used to add Mean Time - EoT = Solar Time

A differential adder from the 1930's Astronomical Clock by Paul Pouvillon

THE PULLEY ADDER

Pulleys can be used as shown to add two or more linear movements. By changing the diameter of the pulleys, any combination of

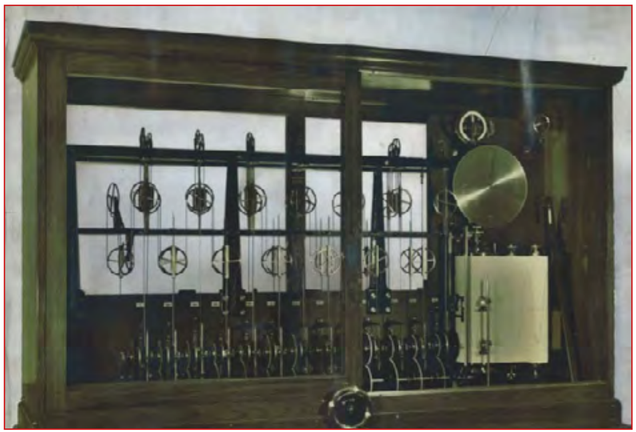
$$y = a.x + b.y + c.z$$

can be derived.

A Pulley Adder

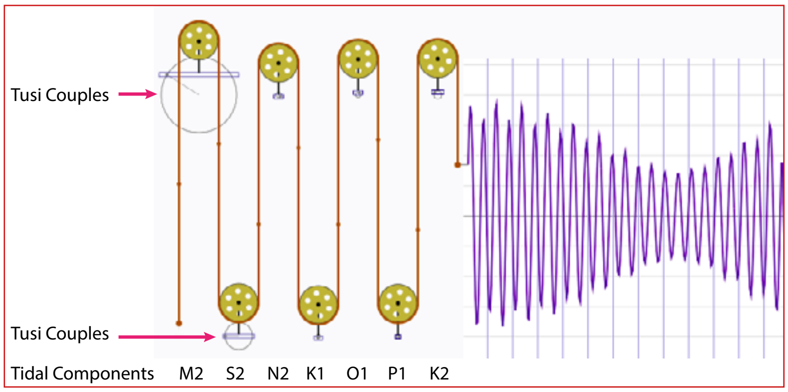
An example of 2 Tusi Couples and a Pulley adder used to compute the EoT can be seen in the Jens Olsen astronomical clock. See 'Moon & Equation Works' on page 180.

Pulley adders have been used extensively in tide prediction machines, where multiple Tusi couples are added. Invented in 1872 by Lord Kelvin, some 34 of such machines were made for various parts of the world. They were, of course, succeeded by digital methods in the 1960s. The most complex, with 62 couples, was made in Germany in the 1930s.



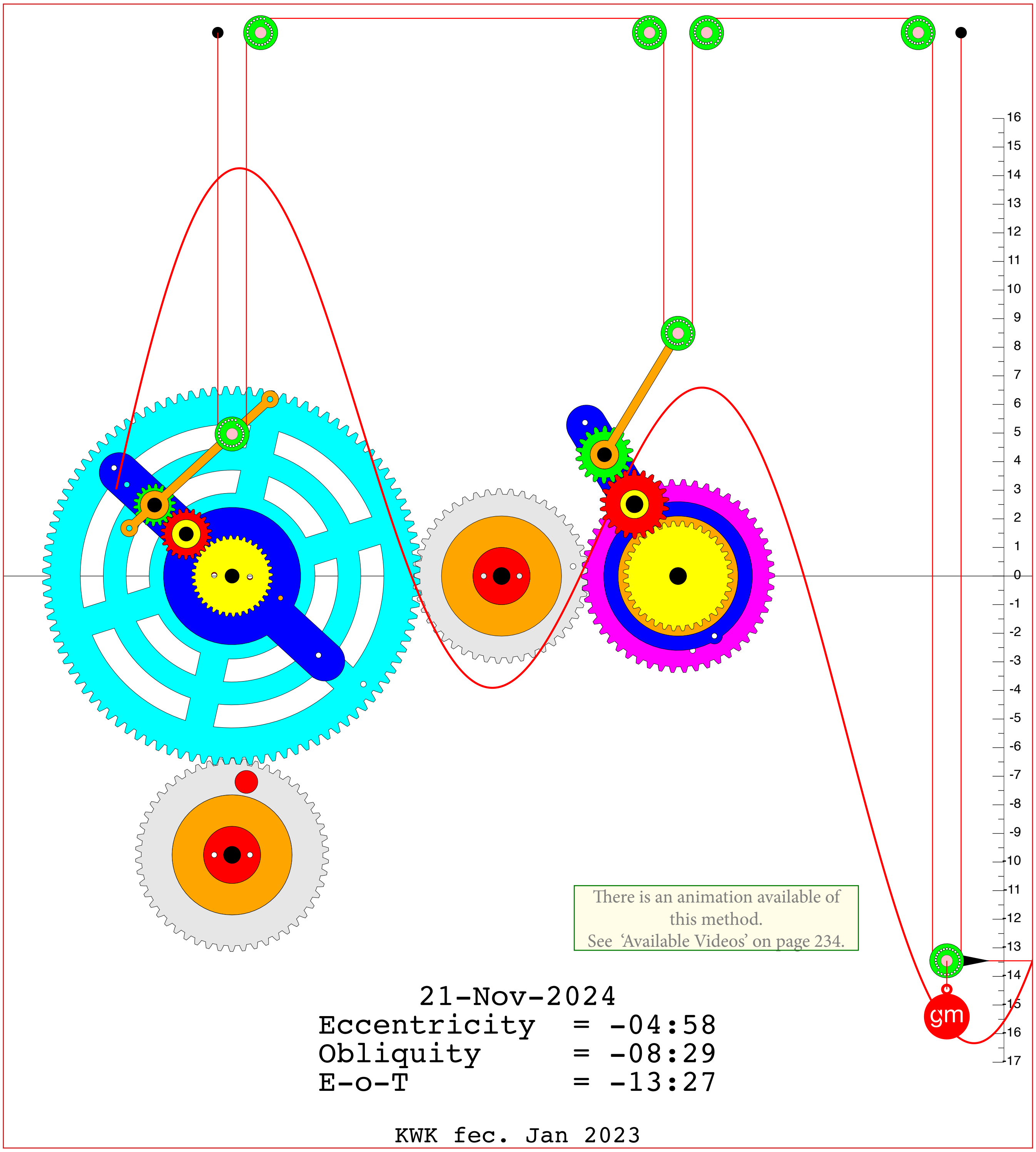
Lord Kelvin's 1872 Tide Machine

return to "Table of Contents"



Illustrative Tidal Simulation for Recife, Brazil using Scotch Yoke Tusi Couples. The size of the Couple reflects the amplitude the specific signal.

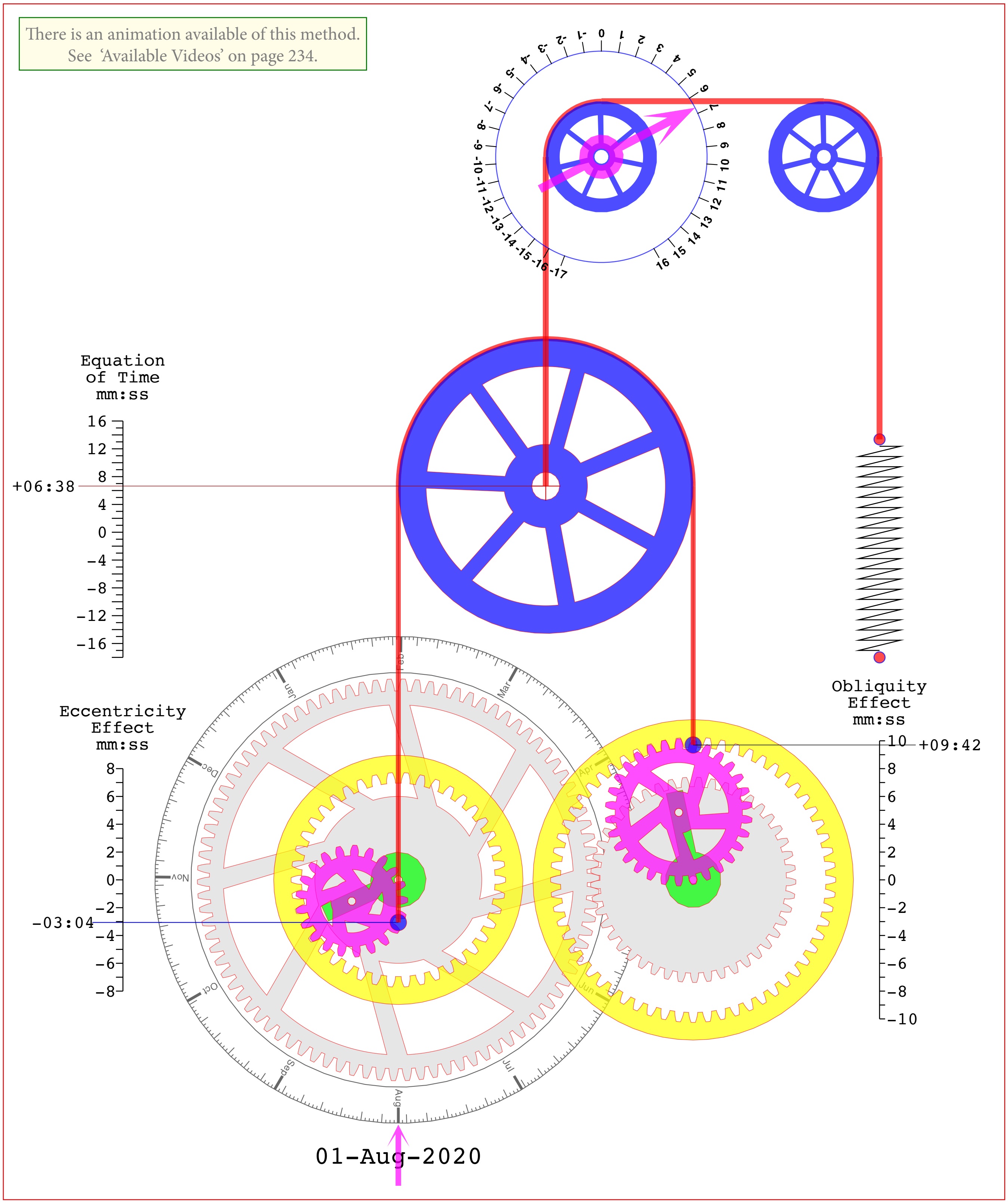
https://www.ams.org/publicoutreach/feature-column/fcarc-tidesiii3



There is an animation available of this method.
See 'Available Videos' on page 234.

One of the Author's Equation Machines using pulley adders and Goodman Tusi Couples.
The Cyan and Magenta gears represent the Eccentricity & Obliquity components.
The Eccentricity carrier has twice the number of teeth of the Obliquity carrier
The lower gear is simply a driver and the central grey gear is an idler

return to "Table of Contents"


There is an animation available of this method.
See 'Available Videos' on page 234.

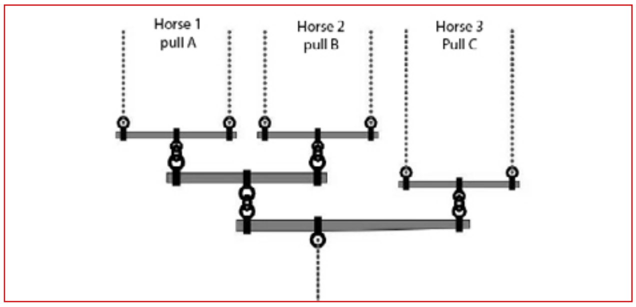
Another of the Author's Equation Machines using a pulley adder and Tusi Couples.

The Whiffletree Adder

Originally Whippletrees were used in tension to distribute forces from a point load to the traces of draught animals. But they can be used for any additional functions. Once again, any combination of

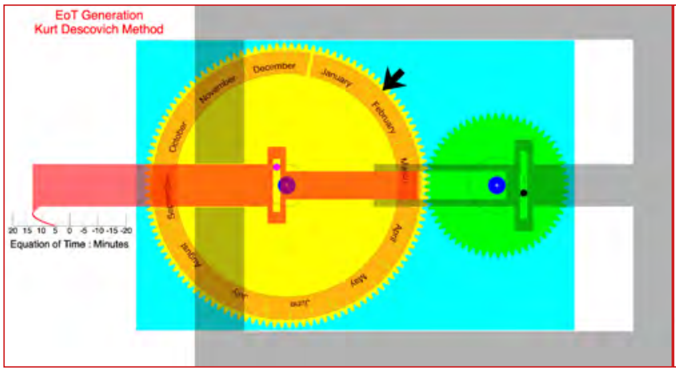
$y = a.x + b.y + c.z + \ldots$

can be derived, by changing the position of the anchor points. Thus the pull of e.g. two weaker horses can be balanced against one stronger one.

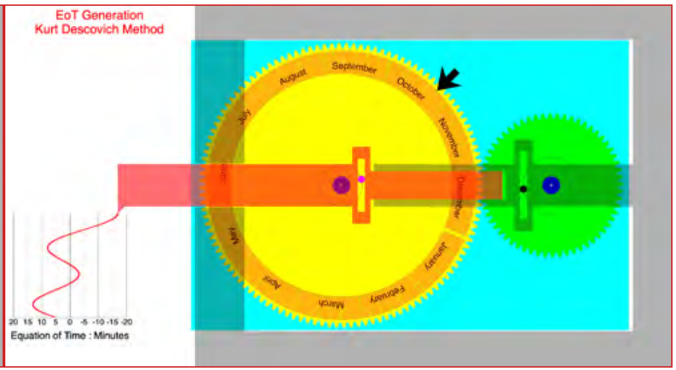


The Whiffletree (or Whippletree)

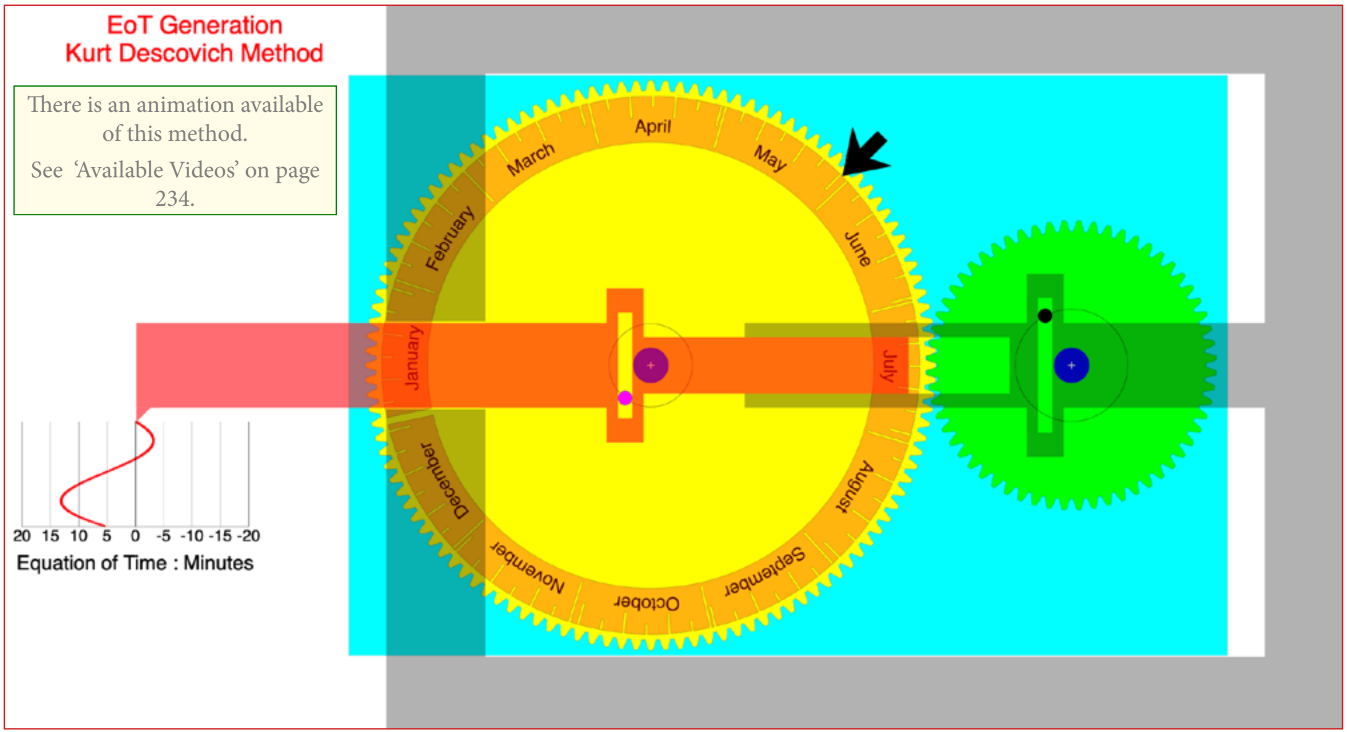
return to "Table of Contents"



Descovitch Method - 5th Feb. Maximum EoT.



Descovitch Method - 25th October. Minimum EoT.



There is an animation available of this method.

See 'Available Videos' on page 234.

Descovitch Method - 30th May. EoT = zero.

DESCOVITCH METHOD

Ref. NASS Compendium Vol 23 no 1 - March 2016

This elegant means to generate the Equation of Time was applied by Kurt Descovitch in his 2015 Schwarzenau heliochronometer. In the above illustrations, the ...

- the green gear rotating twice a year and the yellow gear rotating once a year are attached to the cyan floating piston within the grey fixed frame.
- the blue piston is moved by the pin in the scotch yoke on the green gear, creating the Obliquity component.
- the orange piston is moved by the pin in scotch yoke on the yellow gear, creating the Eccentricity component.
- the two components are added to provide the EoT, since the yellow gear rests on the blue piston.
- the purple and black pins are at a distance from the centre of their respective gear proportional to the eccentricity and obliquity errors.

In the installation further refinements include compensation for longitude and daylight saving.

LUSBY TAYLOR METHOD

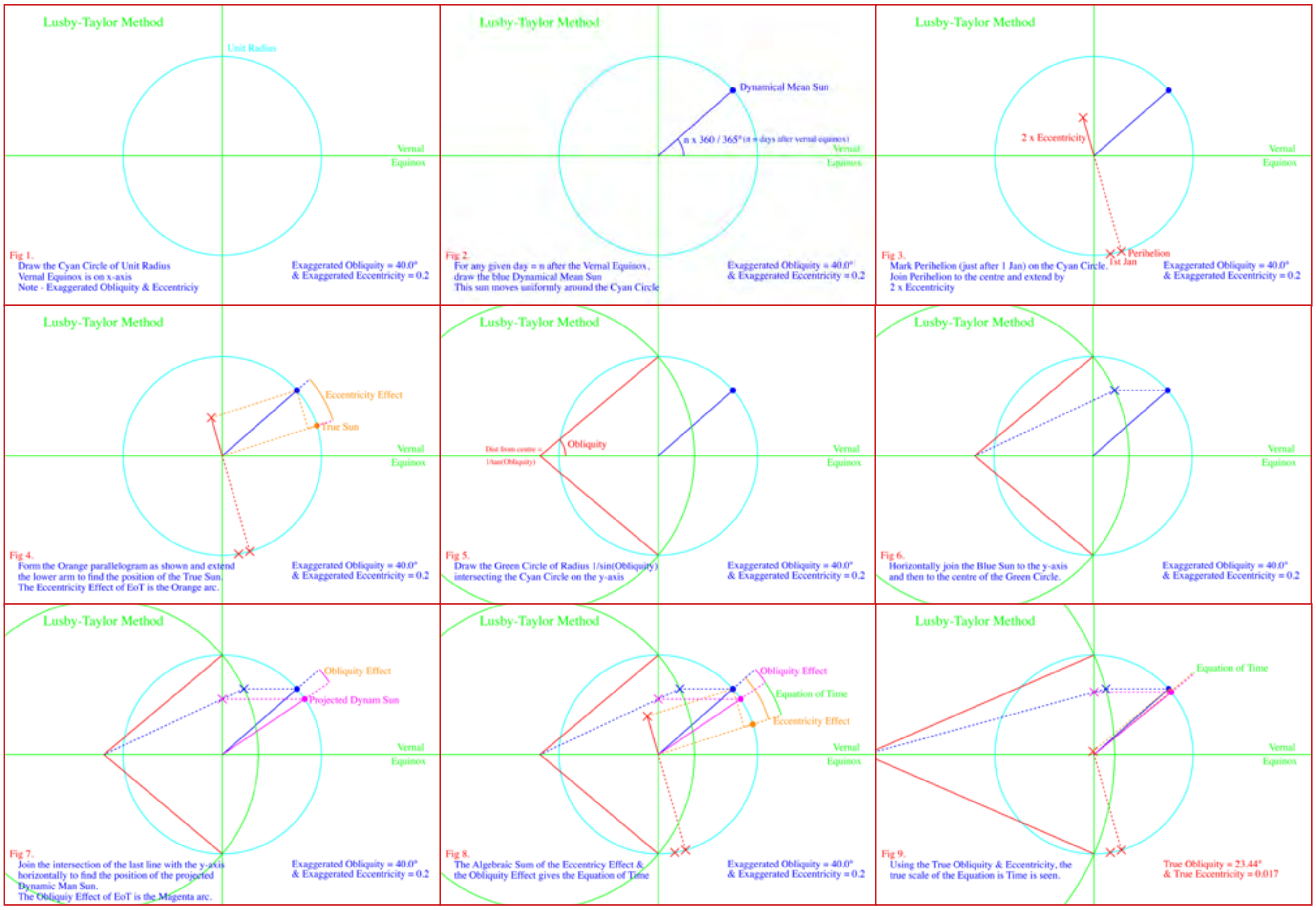
Ref. Personal Communication

This method, devised by Chris Lusby Taylor, uses the ancient Greek method in order to calculate the Sun's true longitude and thus the Eccentricity Effect. It then uses a geometrically correct method to find the Obliquity Effect.

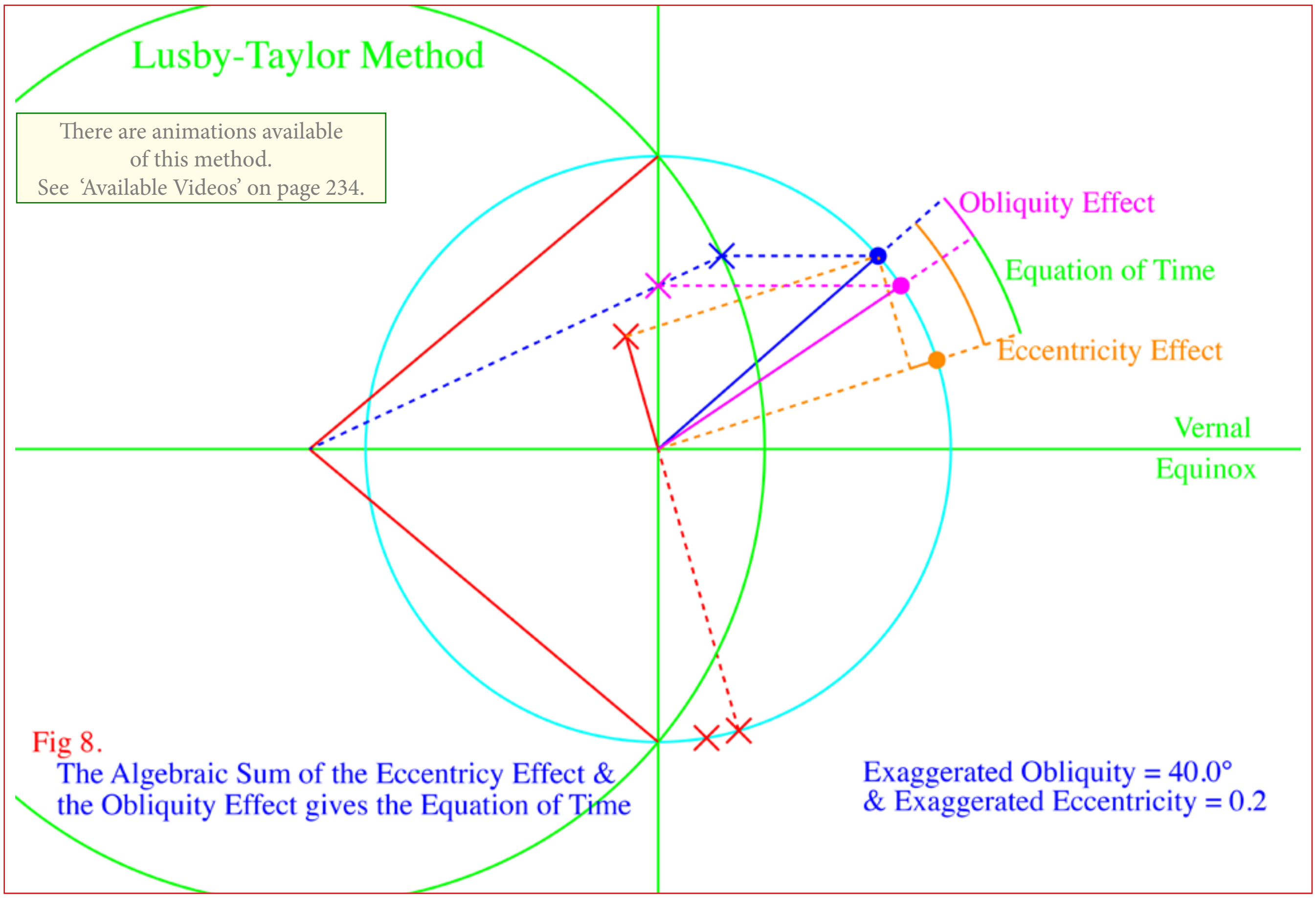
Although the method very closely produces an elliptical path for the sun, it does not correctly emulate Kepler's 3rd Law that equal areas are swept out in equal times.

The errors produced are small.

All that is wanted is a mechanical method to simulate the geometry. This would be useful since it produces its result on an arc of a Circle - easily translated to rotation of a dial plate. Other methods of mechanically simulating the EoT produce results in a linear fashion - requiring a rack and pinion or a chain system to convert to rotary motion.



The Geometrical Build-up of the Lusby Taylor Method, using an Exaggerated Obliquity & Eccentricity.
The last image uses the true eccentricity and obliquity.
Note that in Fig 3 above, Lusby Taylor has recently found that better accuracy is obtained if the distance '2xEccentricity' is replaced by some smaller figure. Further research ia required.



There are animations available
of this method.
See 'Available Videos' on page 234.

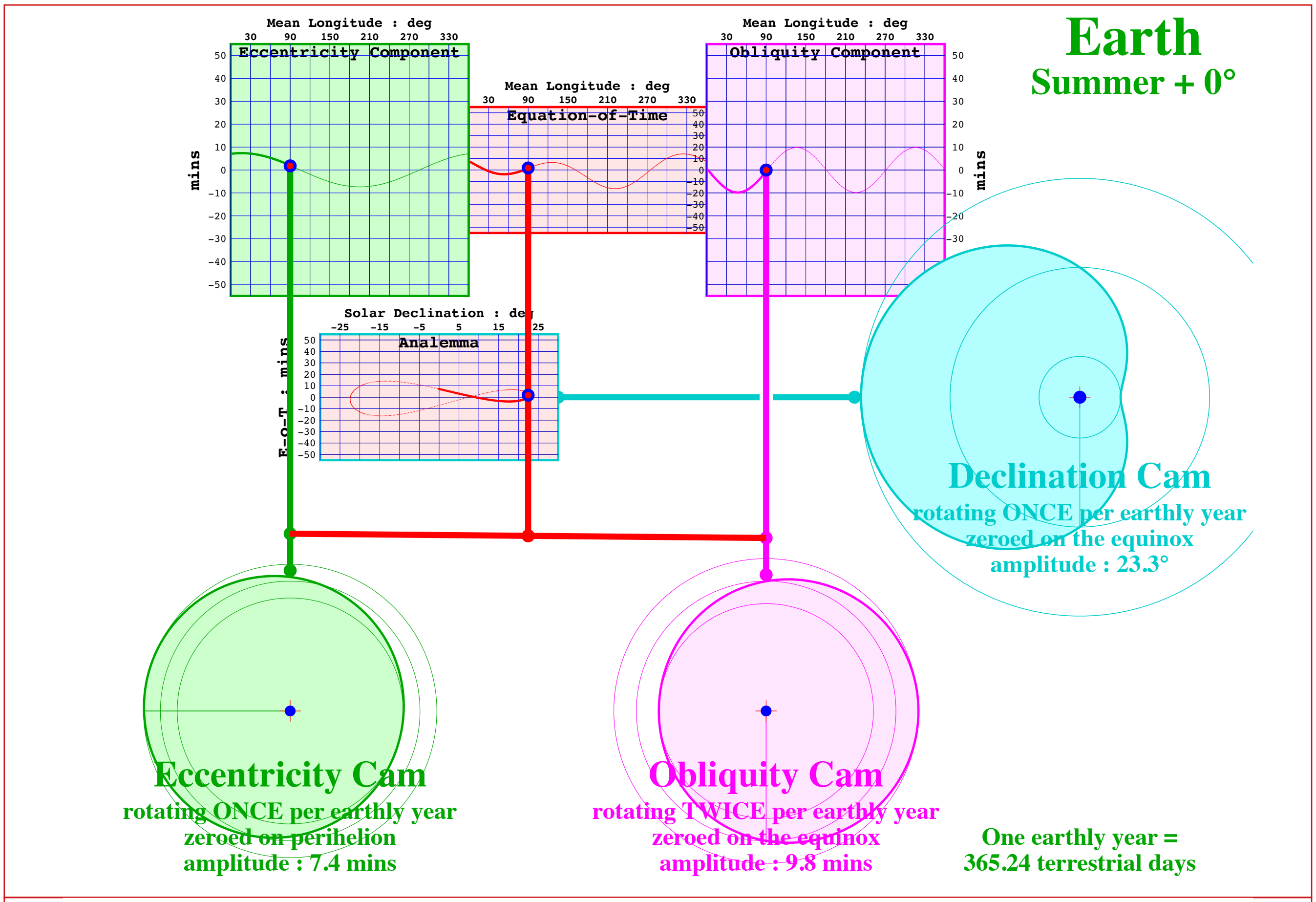
Fig. 8 - Enlarged for clarity

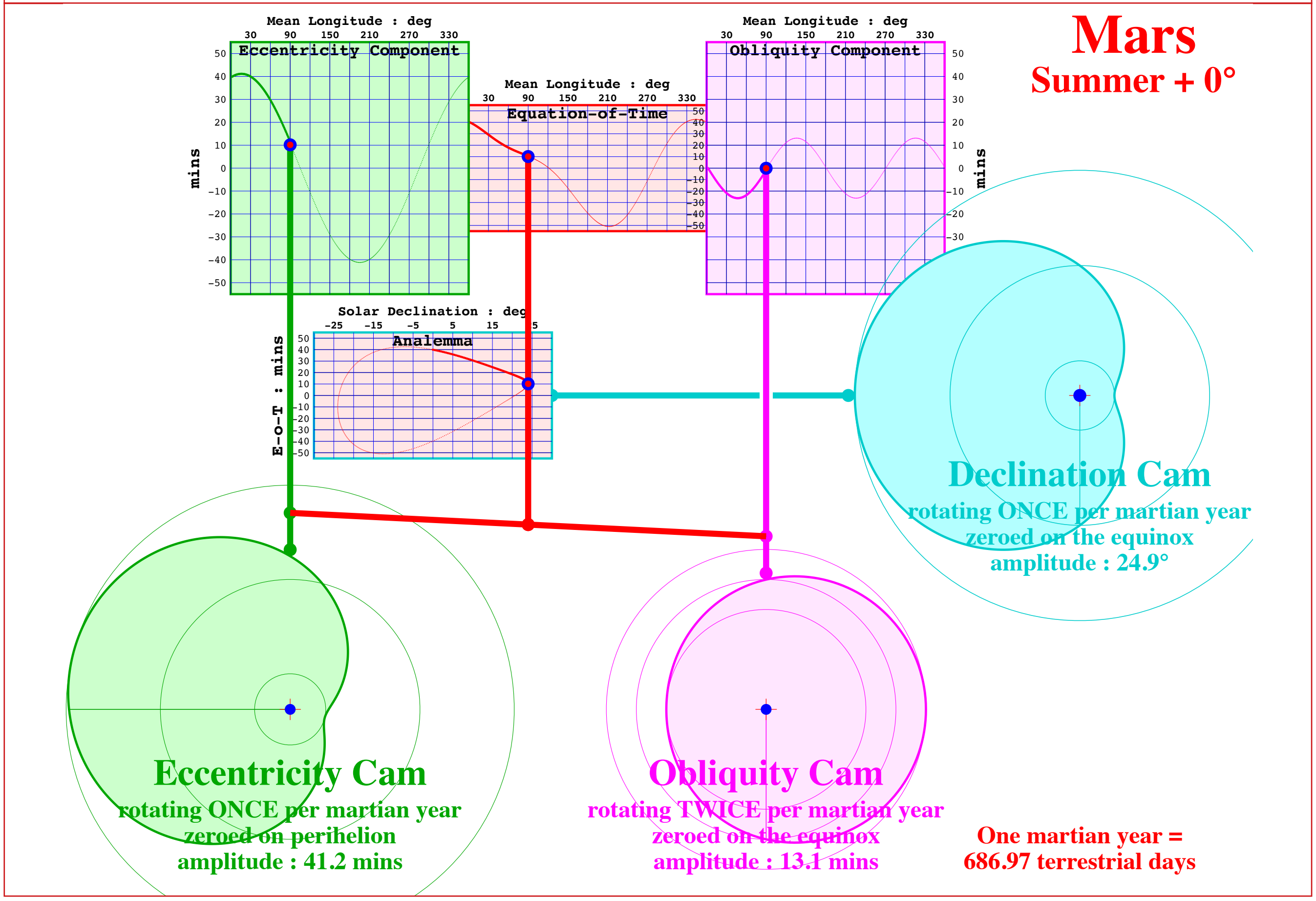
return to "Table of Contents"

COMBINING CAMS & ADDERS

The examples, below, show the combination of Cams & Waffletrees to produce the Equation of Time of both the Earth and Mars

There are animations available of this method.
See 'Available Videos' on page 234.





return to "Table of Contents"

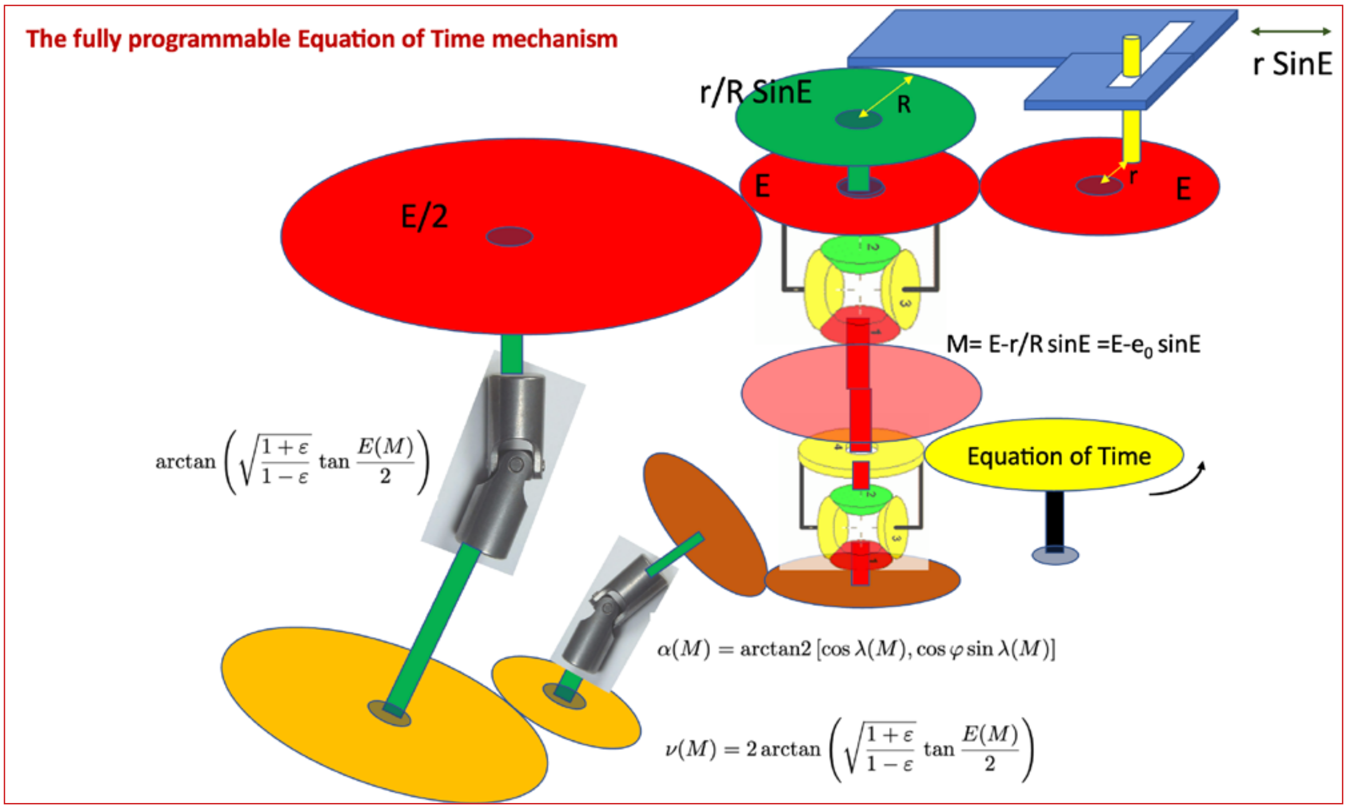
Riegler's Methods

British Sundial Conference Exeter 2023

To my knowledge, there is currently no clock, that incorporates the relation of the mean solar time to the physical motion of the earth around the sun.

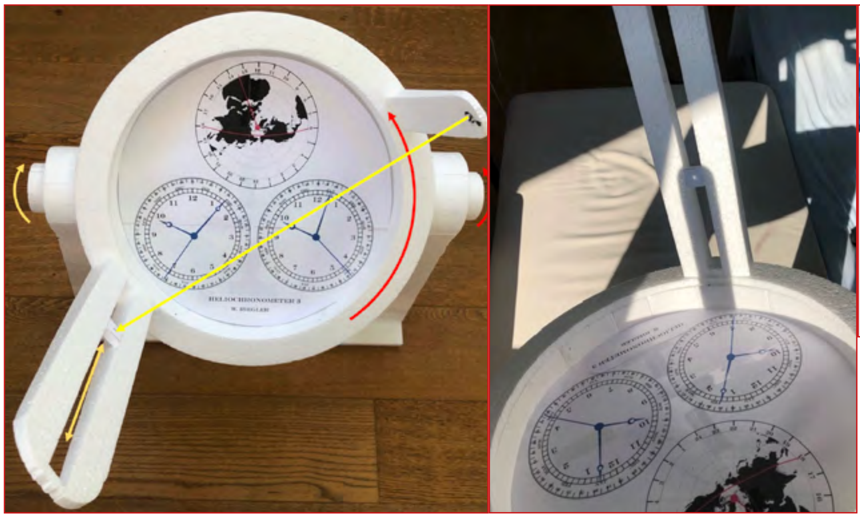
The complex device proposed by Werner Riegler uses a Tusi Couple and Cardan Joints to mechanically generate sine and tangent functions, together with two differentials to add/subtract the relevant components to mechanically generate the EoT.

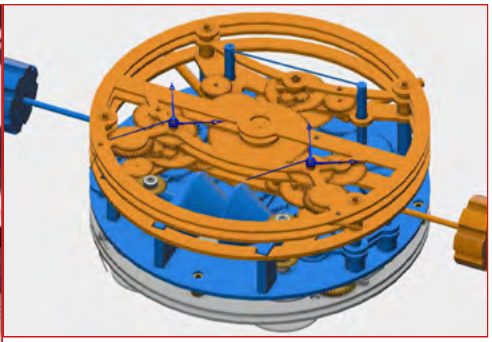
The geometry of the device is such that the three main long-term variables involved in the computation of EoT over the longer term, Eccentricity, Declination and Perihelion Longitude, can all be adjusted.


The fully programmable Equation of Time mechanism

Riegler's Universal Equation Machine

Riegler's Heliochronometer 3 - 'Grand Complication'





Both knobs are turned until the sun coincides with the point on the display.

The two clocks show the date and time in the two half-years. The world map shows the local solar time, and world time.

PART 2 - PICTURES OF AND REFERENCES FOR EQUATION CORRECTED SUNDIALS

Nota bene: In this part of the book, there are a number of the references beginning 'BSS', 'NASS' and 'Sci' : these refer to the BSS Journals, NASS Compendia and the Sciatherics books of Frederick W. Sawyer III, see 'Available Videos' on page 234.

Why Bother to Correct a Sundial

.... there are two schools of thought. Those who think that solar time is true time. And those who think a sundial should aim to supply some indication of civil time.

DO NOT BOTHER

There is a simplicity and historical purity about the approach that thinks that solar time is 'true time' or 'God's time'. The delight in such dials comes their decorative and geometrical design.

But there may also be a degree of political irritation! Santiago de Compostela is 8.5° West, yet its time is based on a meridian of 15° East (Central European Time). Civil midday in the city is more than an hour and a half before solar noon. In Kashgar in Western China, civil noon is at 9 am solar time.

This is perhaps why the beautiful bifilar dial on a beach in Barcelona, below, made in 1996 by Rafel Solar i Gaya, is emboldened with 'Temps Vertader' or 'True Time'.

Interestingly in 2016, The Spanish government was discussing moving its clocks back 1 hour to help the work-family balance in the country - and come closer to True Time.

DO BOTHER.

The alternate approach - that is to say to organise a dial to read standard time - is based on one of 3 possible premises....

i) a dial is meant to read time, then it should read the same time that is shown on one's watch.

ii) it's just fun to engineer something that forces nature's time to match mankind's time.

iii) it's sad for a gnomonist to see someone (especially a child) dismiss a sundial as old-fashioned or useless because "it doesn't tell the right time".

However, it is acknowledged that few people bother to read the instructions that usually accompany a mean time dial. Thus, the education that one is trying to accomplish may be in vain.

CONCLUSION

It is just a matter of taste whether to correct a sundial or not!

Rafel Solar i Gaya - 'True Time' - 1996 Bifilar Dial on the seafront in Barcelona,

return to "Table of Contents"

Equiangular Sun Dials

INTRODUCTION

Any equatorial dial or other construction that gives equiangular hour lines can easily be converted to a standard time dial simply by rotating the hour and minute graduations by an amount proportional to the Equation of Time and Longitude correction on the day in question. This is commonly done using Pilkington's method.

PILKINGTON'S METHOD

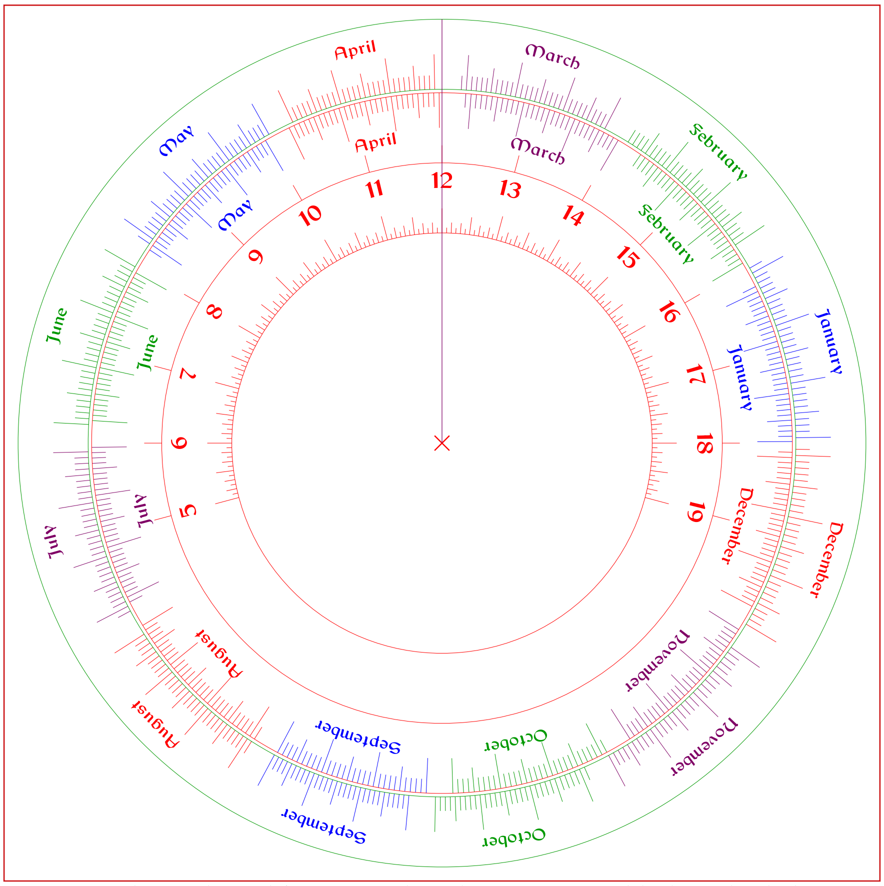
In the illustrations,

1) the outer calendar ring is fixed and contains a year calendar with evenly spaced days.
2) the inner calendar ring where the date markings are squeezed or expanded to cater for the EoT. On any particular day, the date on inner and outer rings are aligned for that day
3) the inner hour ring can rotate and contains the hours (evenly spaced). For dial's particular longitude, the inner 2 rings are locked together. This caters for the fixed longitudinal correction.

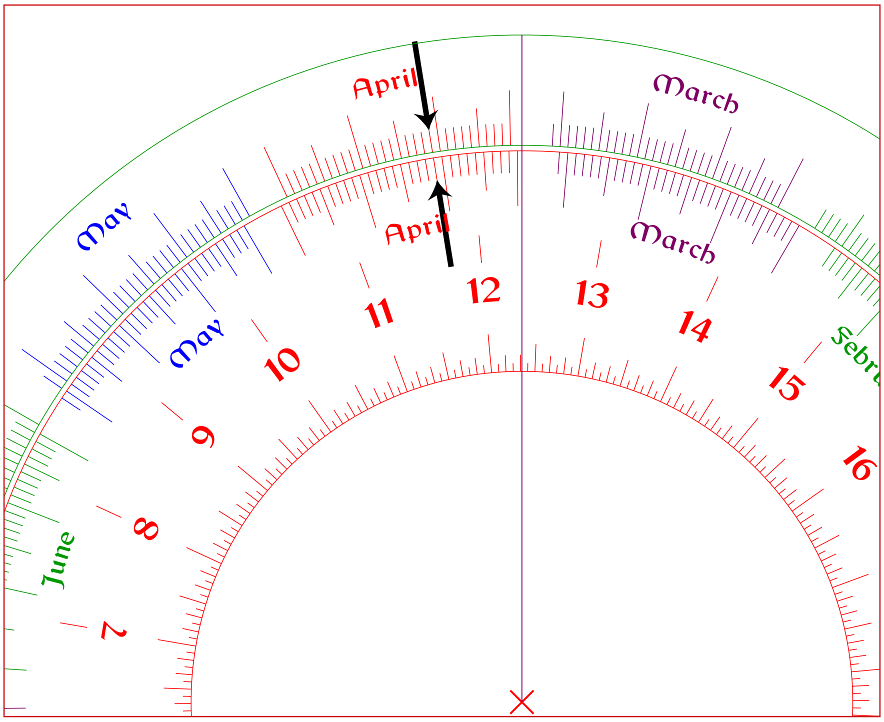
Notes

1) The Longitude Correction ^{hrs} = (Time Zone meridian's latitude° - local latitude°) / 15 $^{mins/deg}$
2) For a background paper on Equiangular theory, see

Gordon Taylor on Equiangular Sundials
British Astronomical Association 1975.
This is available at
https://v/full/1975JBAA...86....7T



Dial not adjusted for Longitude. Solar Noon at 12 o'clock on the dial

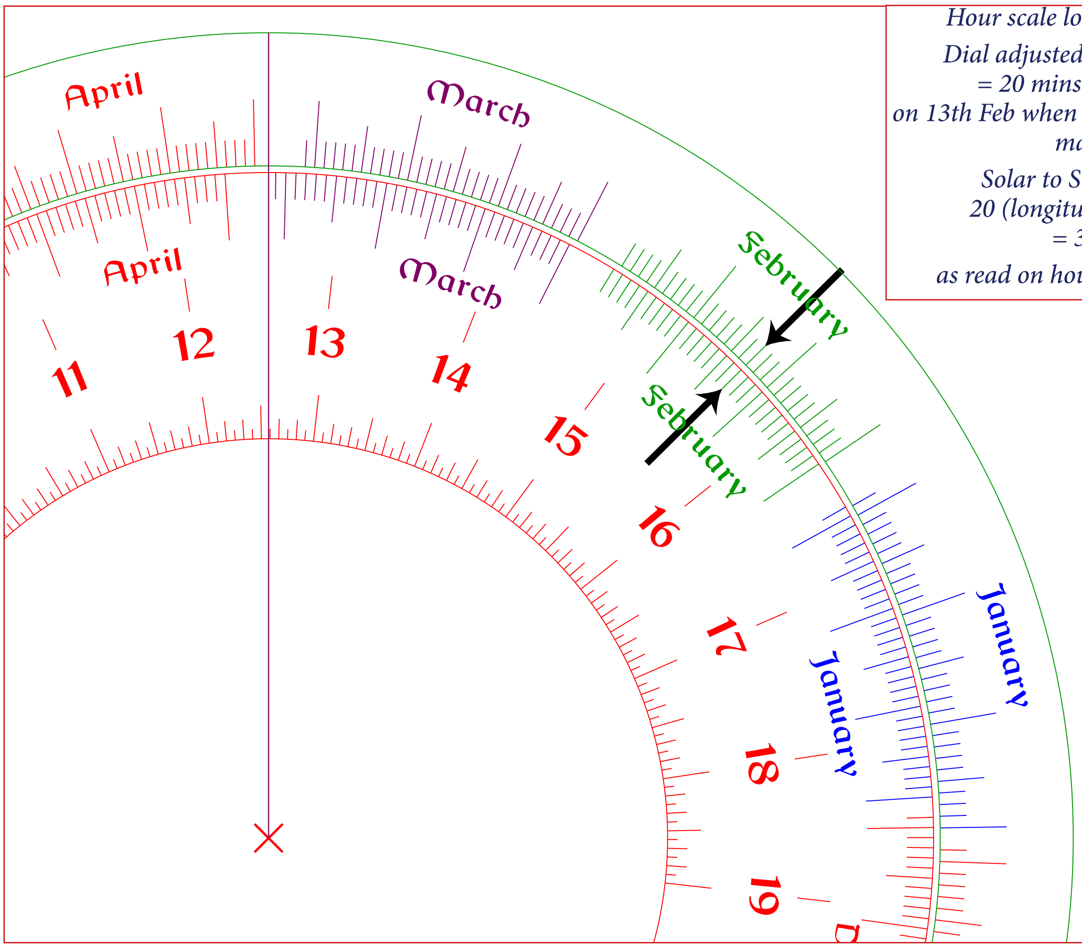


Hour scale locked to central ring

Dial adjusted for Longitude 5°W = 20 mins (360° in 24 hrs)
on 15th April when the EoT is zero

Solar to Standard Time = 20 (longitude) + 0 (EoT)
= 20 mins
as read on hour ring at solar noon

return to "Table of Contents"

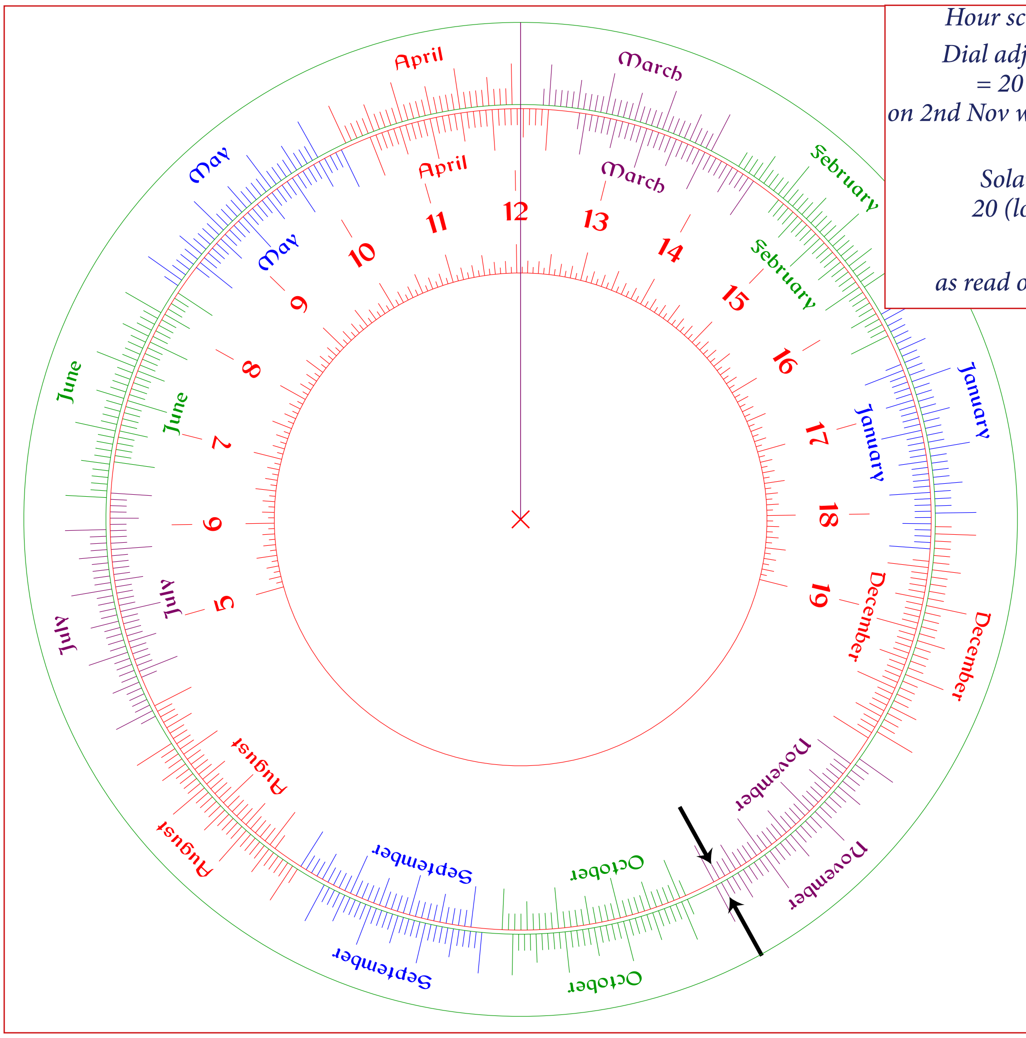


Hour scale locked to central ring

Dial adjusted for Longitude 5°W = 20 mins (360° in 24 hrs) on 13th Feb when the EoT = 14.2 mins (its max value)

Solar to Standard Time = 20 (longitude) + 14.2 (EoT) = 34.2 mins

as read on hour ring at solar noon



Hour scale locked to central ring

Dial adjusted for Longitude 5°W = 20 mins (360° in 24 hrs) on 2nd Nov when the EoT = -16.4 mins (its min value)

Solar to Standard Time = 20 (longitude) - 16.3 (EoT)

= 3.5 mins

as read on hour ring at solar noon

return to "Table of Contents"

THE HUGGETT METHOD

A feature of Pilkington's method is that - for clarity - the month may be space out, as seen in Brian Huggett's dial. This dial also separates the winter time months on the left from the summer time (DST) months on the right. There is some overlap to account varying dates of start/end of Summer Time.

Brian Huggett - 2018 - Equatorial Heliochronometer

THE HOUSEWIFE'S TRICK REVISED - THE SCOUT'S TRICK

If, at 8'oclock standard time in the morning, a horizontal dial is rotated about its gnomon root, A.P. Herbeer opined some 70 years ago that the dial will read mean time correctly throughout the day. He called this the Housewife's Trick. Such a name would now be considered insulting. Equally, it is also an insult to gnomonical science as discussed by H.R. Mills and analysed by Fred Sawyer.

Notes to the Editor
H.R. Mills - BSS Oct 1991, 91(3)
and Herbert's Correction
Fred Sawyer - NASS June 1998, 5(2):24-27 & Sci 2

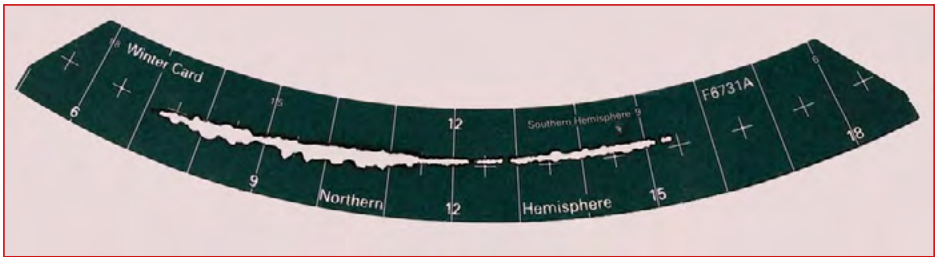
The Campbell-Stokes Sunshine Recorder more-or-less provides an example of how the Housewife's trick can work. This device uses a glass globe to focus the sun, such that it singes a strip of cardboard, which is changed every day, At the end of the day, the length of the burn is measured to give the hours of sunlight during the day, But if, at any moment early in the day, the cardboard strip is be moved so that the burning spot reads civil time. Throughout the rest of day. The burning spot will give civil time. Hardly a good sundial!

So I have revised Housewife's Trick to the Scout's Trick, whereby if any equatorial dial in rotated to read Standard Time, it will then be correct for that day and probably for the next few days, if the EoT diurnal change is small.

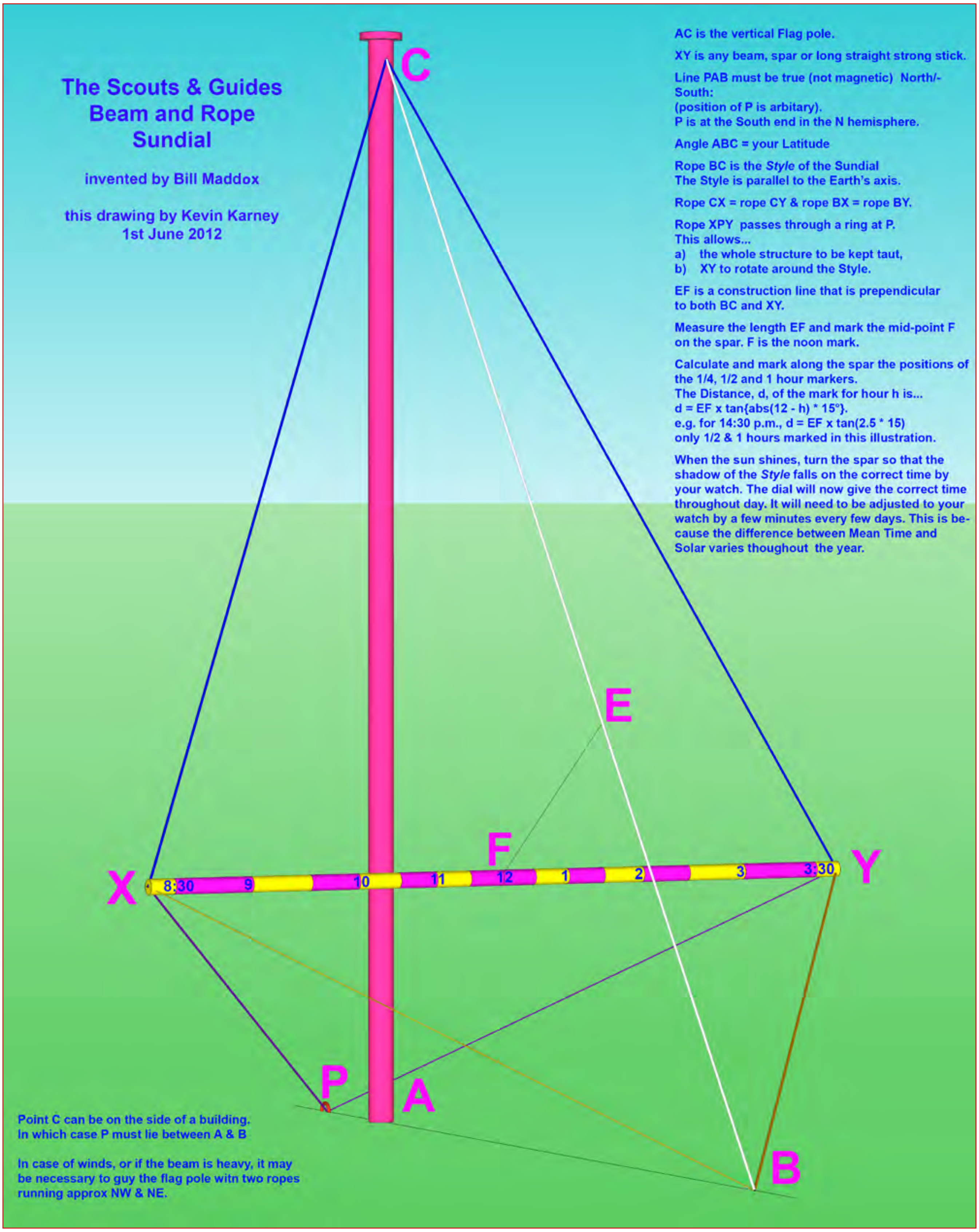
The name is in honour of Bill Maddox's Beam and Spar dial. See overleaf overleaf.

This was designed to provide an example of rope work and construction skills for boy scouts or girl guides. The pole in the centre represents the flag pole in a their camping ground.

Campbell Stokes Sunshine Recorder.
The end of the white back of the cardboard strip sticks out of groves in the instrument.



An example of a daily burnt strip.
The length of the burnt strip indicated the hours of sunshine during the day
In this example, the instrument is not truly set, since the burnt line is not parallel to the central set of crosses.



The Scouts & Guides Beam and Rope Sundial

invented by Bill Maddox

this drawing by Kevin Karney
1st June 2012

AC is the vertical Flag pole.

XY is any beam, spar or long straight strong stick.

Line PAB must be true (not magnetic) North/-South:
(position of P is arbitary).
P is at the South end in the N hemisphere.

Angle ABC = your Latitude

Rope BC is the *Style* of the Sundial
The Style is parallel to the Earth's axis.

Rope CX = rope CY & rope BX = rope BY.

Rope XPY passes through a ring at P.
This allows...
a) the whole structure to be kept taut,
b) XY to rotate around the Style.

EF is a construction line that is prependicular to both BC and XY.

Measure the length EF and mark the mid-point F on the spar. F is the noon mark.

Calculate and mark along the spar the positions of the 1/4, 1/2 and 1 hour markers.
The Distance, d, of the mark for hour h is...
d = EF x tan{abs(12 - h) * 15°}.
e.g. for 14:30 p.m., d = EF x tan(2.5 * 15)
only 1/2 & 1 hours marked in this illustration.

When the sun shines, turn the spar so that the shadow of the *Style* falls on the correct time by your watch. The dial will now give the correct time throughout day. It will need to be adjusted to your watch by a few minutes every few days. This is because the difference between Mean Time and Solar varies thoughout the year.

Point C can be on the side of a building.
In which case P must lie between A & B

In case of winds, or if the beam is heavy, it may be necessary to guy the flag pole witn two ropes running approx NW & NE.

Instructions provided to a Scout Leader of the author's acquaintance of Bill Maddox beam and chord dial

Ref NASS Mar 02 9(1) - Cord and Spar Sundials

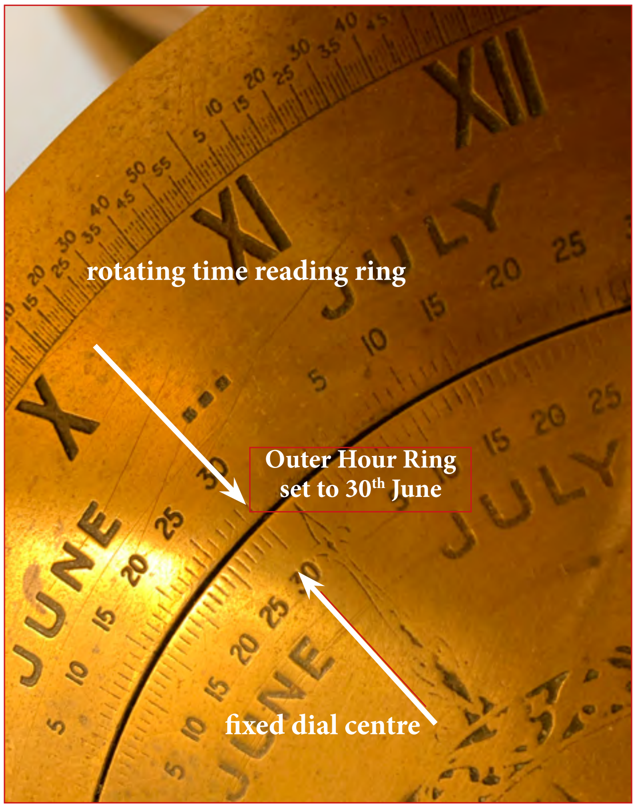
Moving Hour Lines Sundials

INTRODUCTION

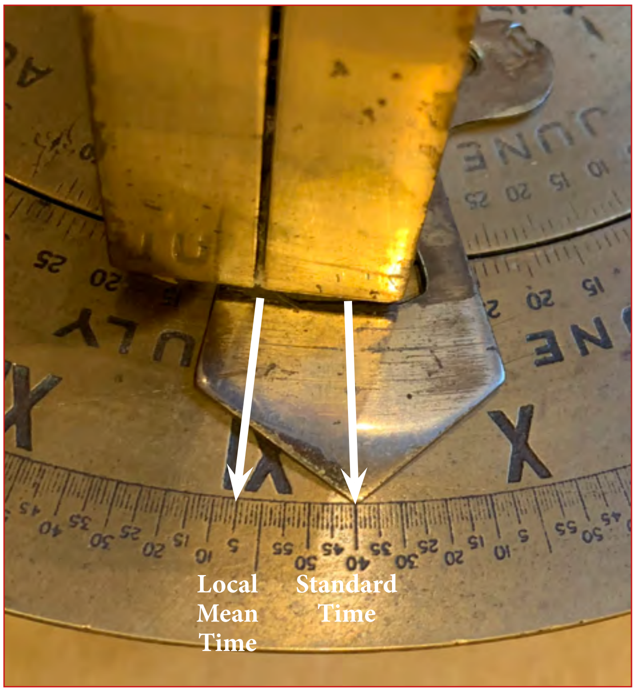
Most Moving Hour Lines Dials are equatorial in nature where the hour lines are necessarily spaced at 15° per hour. Normally one would expect the noon hour line to be due north in the Northern hemisphere. By rotating the hour lines around the polar axis by an amount proportional to the Equation of Time and Longitude correction on the day in question, direct reading of civil time is possible. Commonly this is done using Pilkington's method. 'Equiangular Sun Dials' on page 119.

EQUATORIAL DIALS

Pilkington & Gibbs' 1914 Sol Horometer has a box-like gnomon with a slit at one end and a 'screen' at the other. The whole box is turn until the sun, shining through the slit hits the screen on the other side of the box.



Sol Horometer - EoT alignment scales. Notice the scales for one month is separated from those of the next month to provide clarity. 'Equation of Time Nomograms' on page 103.where a similar technique is used.

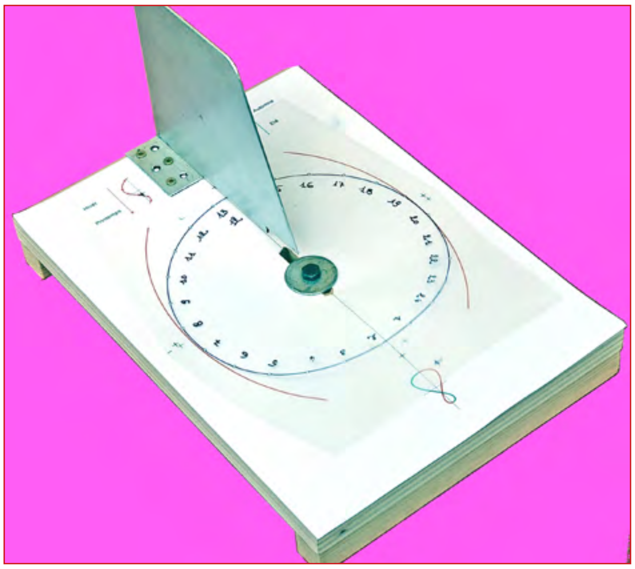


Sol Horometer. The bronze arrow is set for the Longitudinal offset, allowing both local mean time and standard time to be read.

Kurt Descovich - 2016 - Schwartenau Heliochronometer

Ref NASS Compendium 23-1 Mar 2016

Also see ‘Descovitch Method’ on page 113.

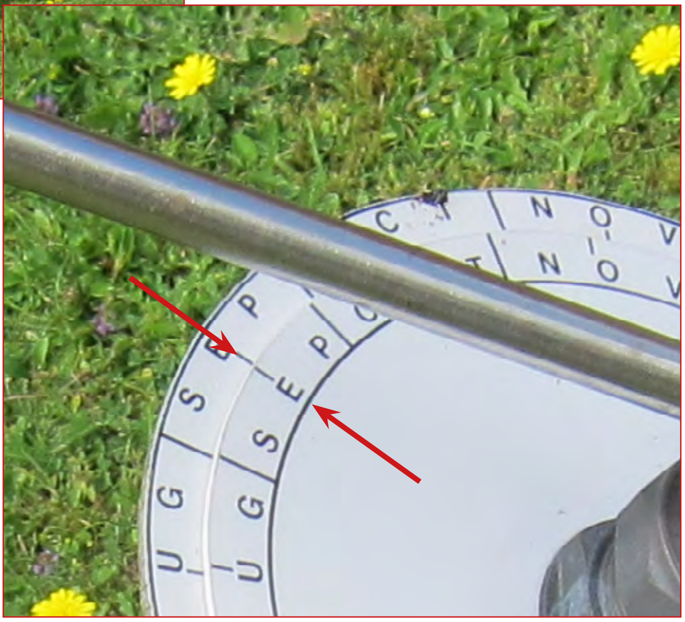


Dasypodius Society Dial, Alsace

The Authur is uncertain how this dial works...

2015 - John Singleton Dial - Stockcross.

The spiral mounted on the stainless steel polar axis that can be rotated according to the EoT



The Equation Table at the base of the Stockcross Dial

The photograph was taken in mid Septembert

return to "Table of Contents"

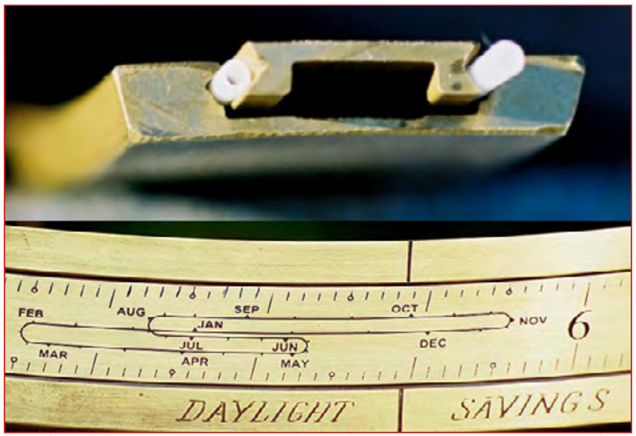
Renaissance dial - Bill Gottesman,

www.precisionsundials.com.jpg

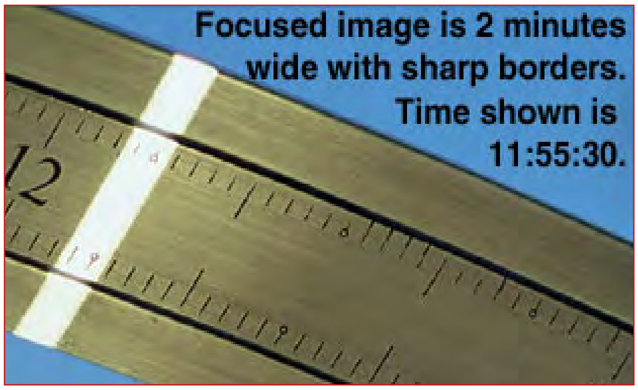
This exquiste dial used special polar-mounted focusing mirrors, cylindrically ground and polished to create the sharp image.

Carlo Heller's Icarus Portable Sundial

www.helios-sonnenuhren.de



The sliding time strip in the helix of a Renaissance Dial, showing the EoT setting for either 29th September or 5th December (when they are the same)



Showing the sharp image on a Renaissance Dial

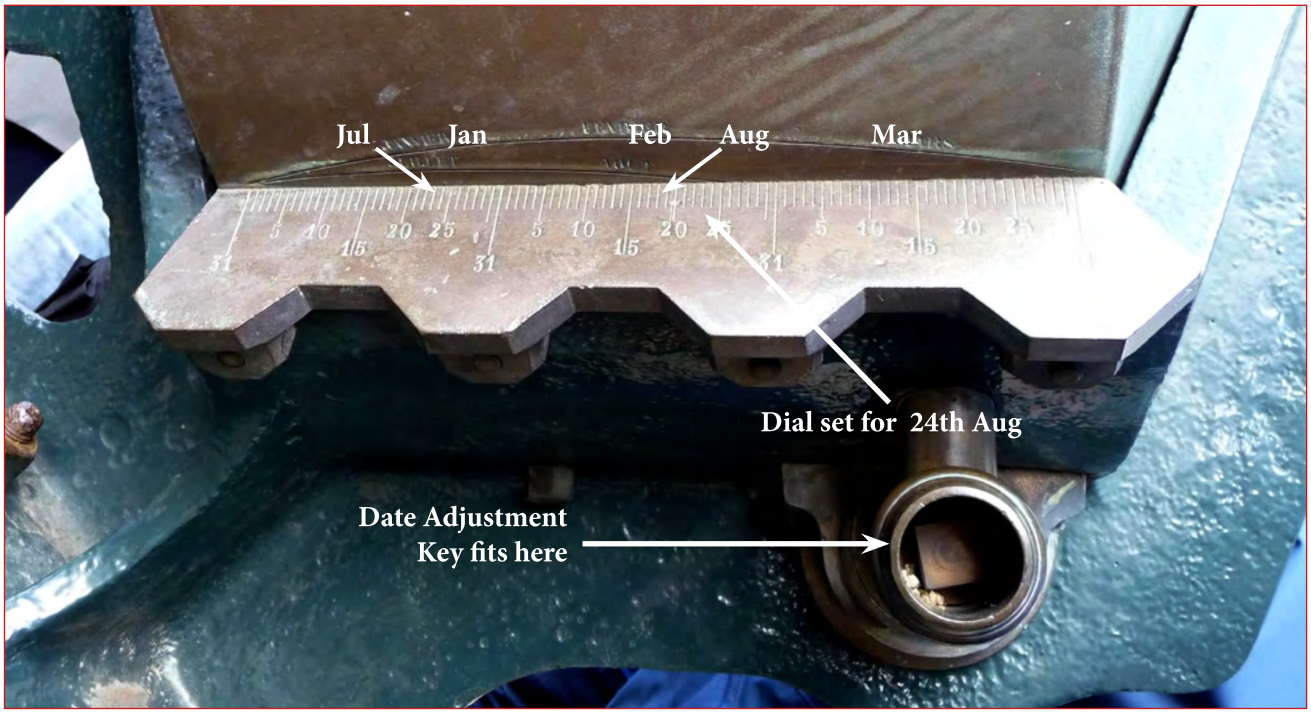
return to "Table of Contents"

1882 - Ernest Bolle - Chartres, France

The sundial of Ernest Bolleé
Told time in an accurate way
Right down to the minute
You just had to spin it
By turning a crank every day

First prize in the Limerick Competition during the BSS field trip to the Loire Region. Author forgotten...

Showing the part moving around the polar axis



Adjusting the Dial for the Equation of Time

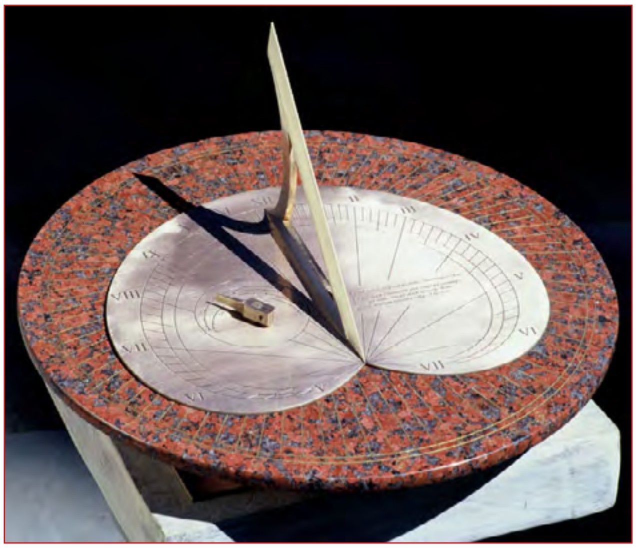
Paris Hours
& Railway Time

return to "Table of Contents"

NON-EQUATORIAL DIALS WITH MOVING HOUR LINES

With clever geometrical means, it is possible to convert conventional hour lines to read on an equiangular scale allowing Equation & Longitude correction.

The Equant Dial below in named after the great Ptolemy who showed how to convert non-uniform motion about a point to uniform motion about his Equant Point. See 'Ptolemy and Unequal Days' on page 74.

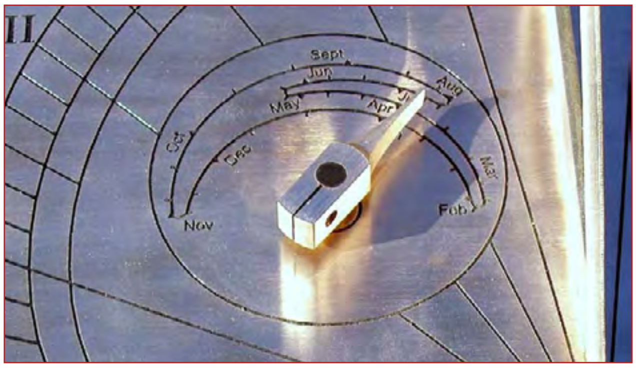


Sawyer Equant Dial by Bill Gottesman

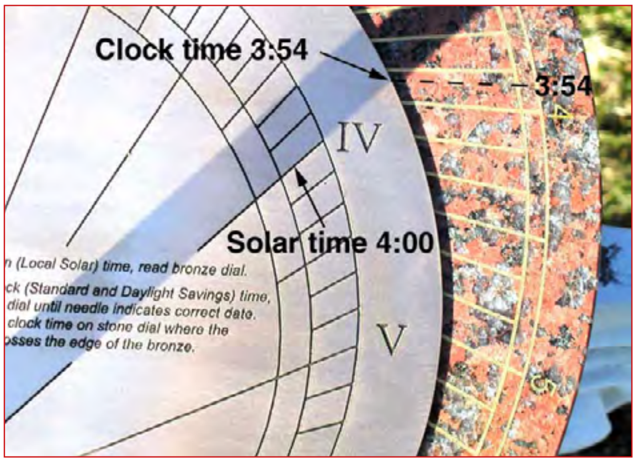
The Equant point is the centre of the granite circle.

The granite equiangular disk can be rotated under the dial, depending on the EoT and longitude

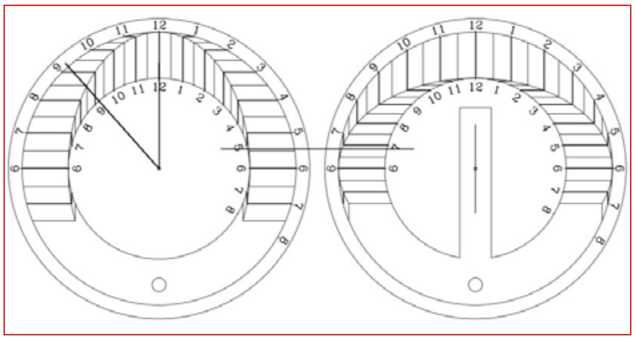
www.precisionsundials.com
BSS 91(2) - Jul 1991 and NASS 7(4) - Dec 2000



Setting the EoT adjustment on the Sawyer Equant Dial



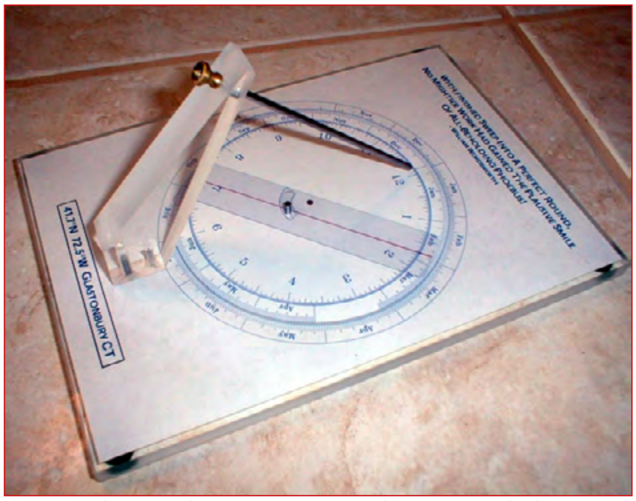
Reading the Time on the Sawyer Equant Dial



Sawyer's Self-orienting latitude-independent analemmatic equant dial

NASS 10(4), Dec 2003

This dial would have been an improvement on the various Solar Compasses developed for desert warfare in World War II. 'Solar Compass Background' on page 86.



Fred Sawyer's Foster-Point Dial by Mac Oglesbury

The Foster Point Sundial Time In A Perfect Round -
Fred Sawyer
NASS Sep 01 8(3): 121 - 128

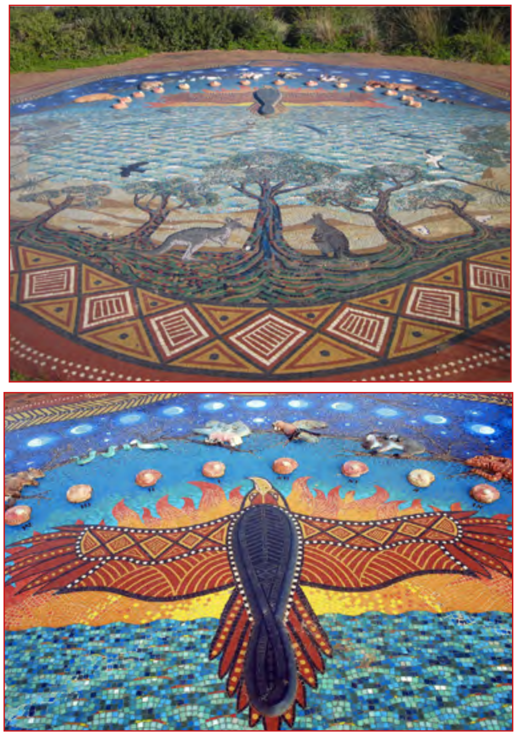
return to "Table of Contents"

Moving Style Sundials

ANALEMMATIC MEAN TIME DIALS

In these dials, one stands on the appropriate point on a month scale and one's shadow falls on a hour point. These points are on a an ellipse, which is the projection of the equatorial circle onto the ground.

In Australia, there are a number of single analemmatic dials (or, as they called there, 'dials of human involvement'). Here the normally straight month-marker line is replace by an analemma with the months marked along it. This method makes attractive dials - but theoretically, they do not give mean time.



1996 : Beautiful analemmatic dial near the Beach at Torquay, Victoria, Australia. It is a mosaic comprising more than 120,000 Italian glass Tesserae tiles. The mosaic represents the traditional dreaming stories of the indigenous Wathaurong people, with images of flora, fauna, marine environments, star constellations and local aboriginal stories.

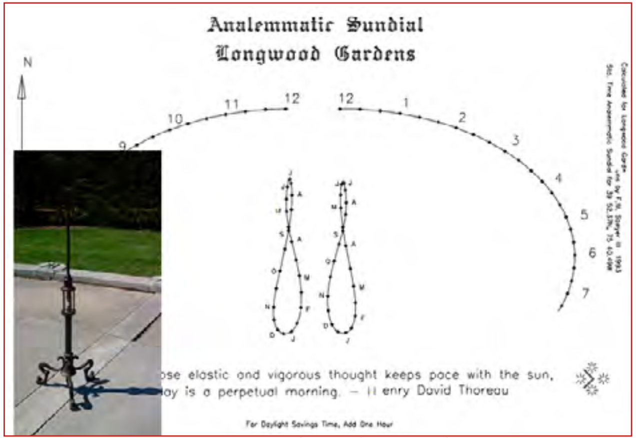
However, by drawing two curves somewhat like an analemma, and spliting the hour-marking ellipse into morning and afternoon segments, one can make a mean time dial. This was first done by Pierre du Pont in 1937, later corrected by Kenneth Seidelman in 1978.The theory was recalculated by Fred Sawyer and documented by Helmut Sonderegger.

Of Analemmas, Mean Time & the Alalematic Sundial - Fred Sawyer, BSS Jun 94, Vol 94-2 & BSS Feb 95 Vol 95-1 : Sci 1
Analemmatic Sundials and Mean Time - Helmut Sonderegger - NASS Sep 2003 10(3)

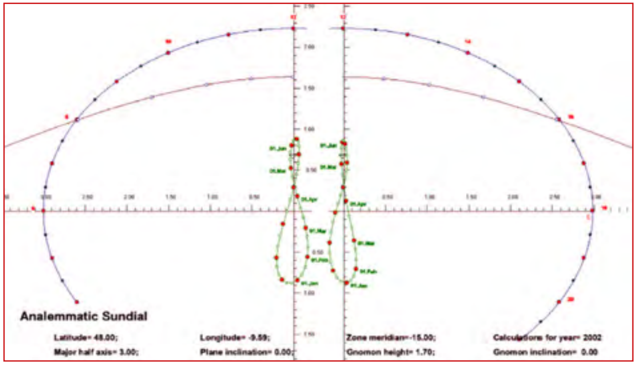
In the first of the above references, Sawyer largely condems Helmuth Egger's Mean Time Polyanalemmic Dials as interesting but wholly impractical.

Results of Sundial Competition - Helmuth Egger, Sky & Telescope, Nov 1966

Pierre du Pont 1939 - Longwood Gardens, Pensylvania



Longwood Gardens Dial,
corrected by Kenneth Seidelman in 1978



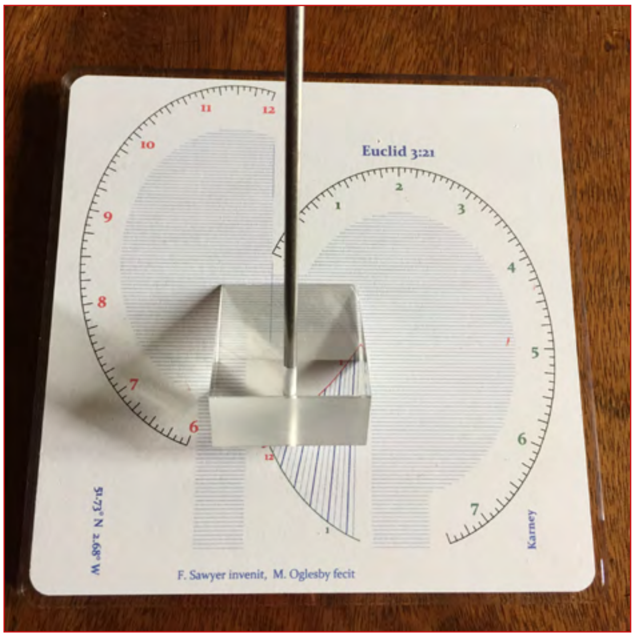
Theory behind Longwood Gardens Dial

This type of dial make an excellent exercise in geometry, trigonometry, measurement and astronomy for senior secondary school students. This teaching aid has been pioneered by Brian Albinson in North Vancouver.

Brian Albinson at Highlands School - 2010

return to "Table of Contents"

Gordon Taylor - 1975 - Herstmonceux Castle, Kent, UK : Forster Lambert Dial



Sawyer's Wandering Gnomon Dial - 2016 :

NASS Jun 14 21(2):5-11 & Sci 5

James Richard, Vertical Equiangular Dial - ca 1995

BSS Bulletin 3(1), June 1991

Mac Oglesby - 2003 - Foster Lambert Vertical Decliner.

NASS Dec 2003 10(4) Dec 2003

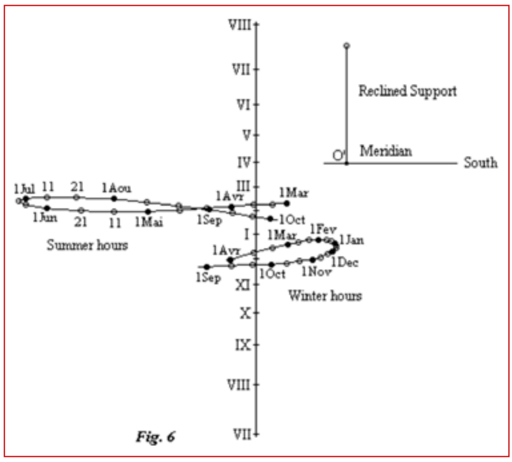


Fig. 6

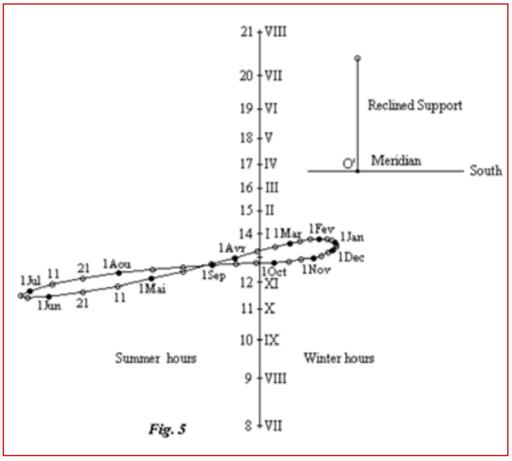
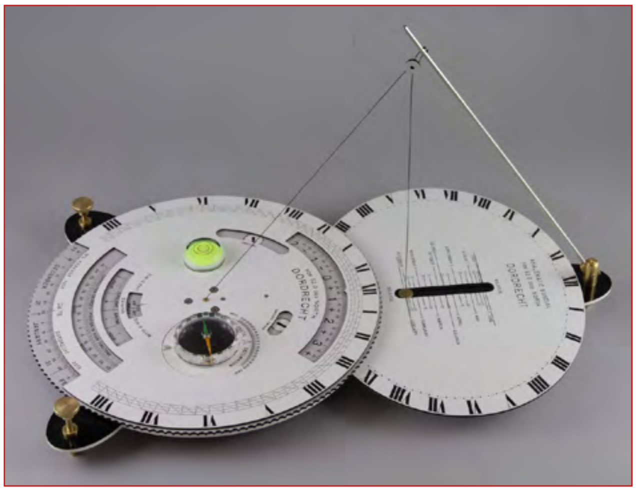


Fig. 5

Yvon Massé - Two mean time analemmatic sundials

NASS Mar 1998 5(1)

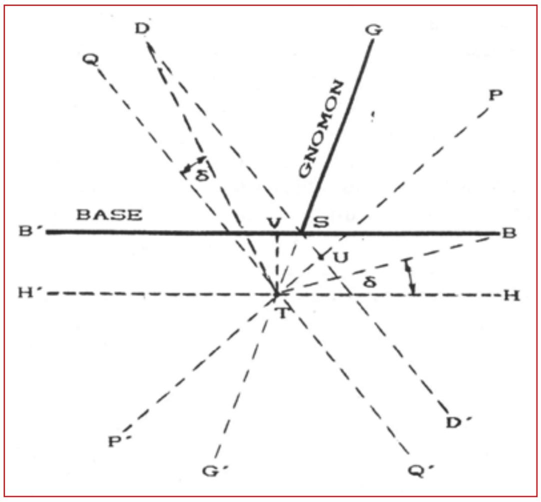
return to "Table of Contents"



Wil Kerkhof - Self Aligning Double Foster-Lambert/
Analemmatic Dial - 2013

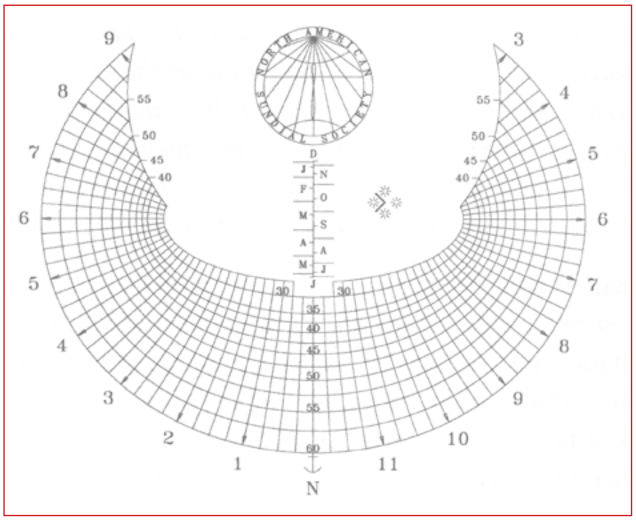
Ref: wkerkhof@antiqueclocks.nl

S.Wetzel, Foster-Lambert Dial



Sawyer's- A Self-Orienting Equiangular Sundial

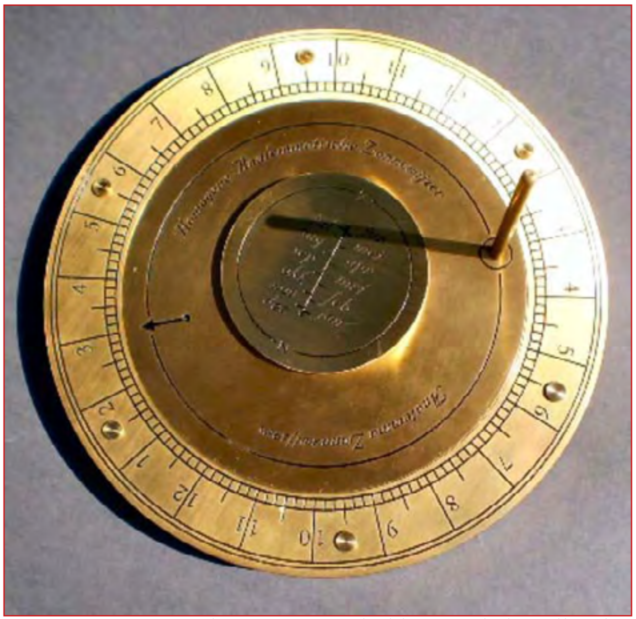
BSS Oct 1991 91(2)



Universal Sun Compass

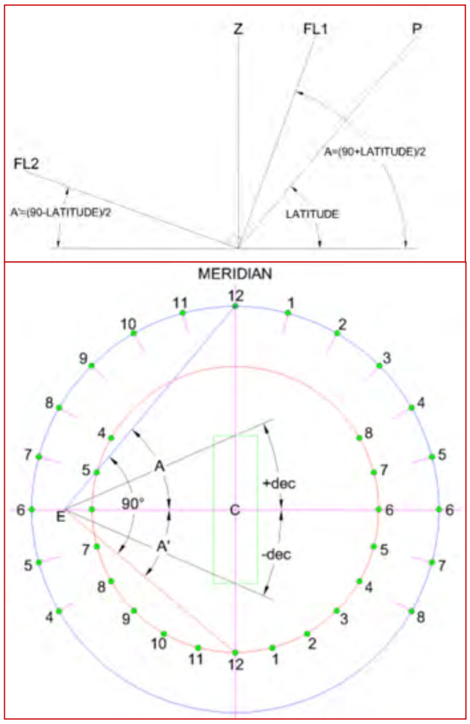
See 'The Abrams & Cole Compasses - WW II' on page 87.

NASS Sep 03 10(3): 18-19 & Sci 3



Homogeneous Analemmatic Sundial by Hendrik Hollander

www.analemma.nl
NASS Jun 08 15(2)



Self-Orienting Equiangular Dial

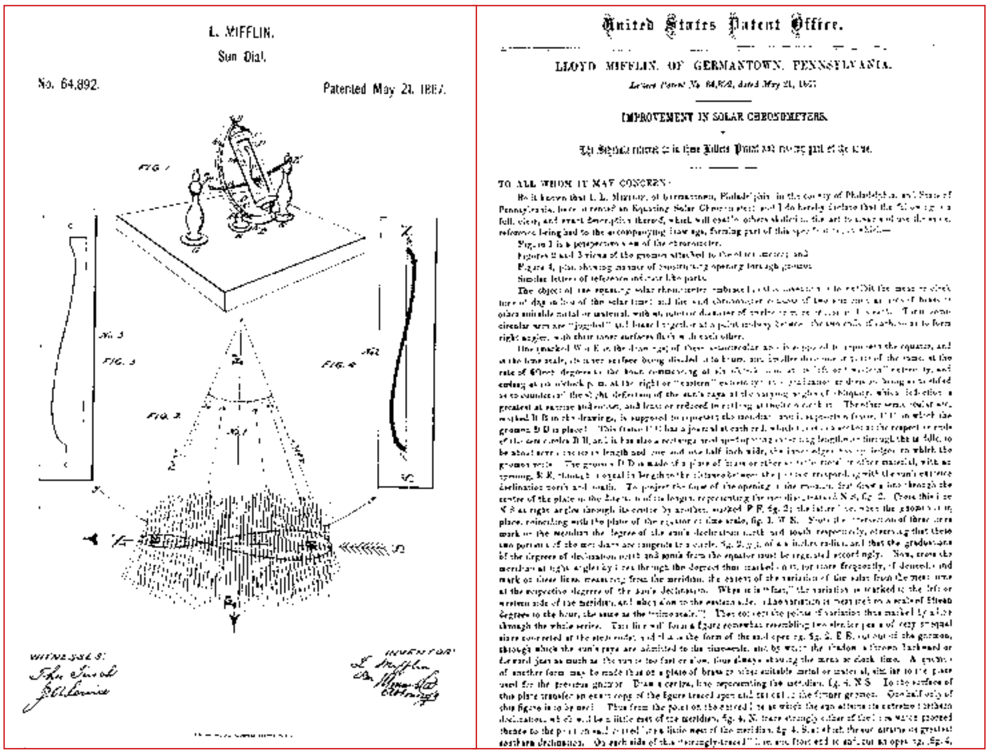
Fred Sawyer - BSS 91(3) - Oct 1991
& Carl Sabanski
www.mysundial.ca/tsp/double_foster_lambert_sundial.html

return to "Table of Contents"

Shaped Style Sundials

THE PATENTS

Patents were in force in the US and UK, coving the basic shaped style dial



United States Patent Office.

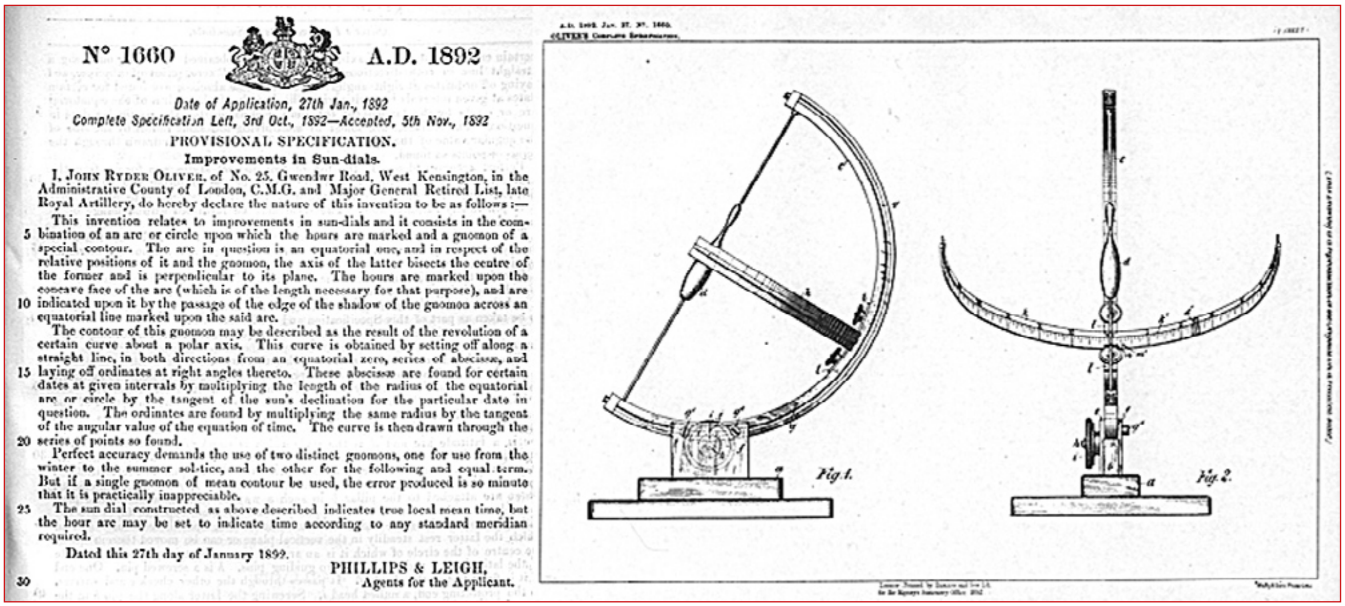
LLOYD MIFFLIN, OF GERMANTOWN, PENNSYLVANIA.

Letters Patent No. 64,892, dated May 21, 1867.

IMPROVEMENT IN SOLAR CHRONOMETERS.

TO ALL WHOM IT MAY CONCERN:

1867 US Patent by L.Miflin for Improvements in Solar Chronometers



N° 1660 A.D. 1892

Date of Application, 27th Jan., 1892

Complete Specification Left, 3rd Oct., 1892—Accepted, 5th Nov., 1892

PROVISIONAL SPECIFICATION.

Improvements in Sun-dials.

I, JOHN RYDER OLIVER, of No. 25, Gwendwr Road, West Kensington, in the Administrative County of London, C.M.G. and Major General Retired List, late Royal Artillery, do hereby declare the nature of this invention to be as follows:—

This invention relates to improvements in sun-dials and it consists in the combination of an arc or circle upon which the hours are marked and a gnomon of a special contour. The arc in question is an equatorial one, and in respect of the relative positions of it and the gnomon, the axis of the latter bisects the centre of the former and is perpendicular to its plane. The hours are marked upon the concave face of the arc (which is of the length necessary for that purpose), and are indicated upon it by the passage of the edge of the shadow of the gnomon across an equatorial line marked upon the said arc.

The contour of this gnomon may be described as the result of the revolution of a certain curve about a polar axis. This curve is obtained by setting off along a straight line, in both directions from an equatorial zero, series of abscissæ, and laying off ordinates at right angles thereto. These abscissæ are found for certain dates at given intervals by multiplying the length of the radius of the equatorial arc or circle by the tangent of the sun's declination for the particular date in question. The ordinates are found by multiplying the same radius by the tangent of the angular value of the equation of time. The curve is then drawn through the series of points so found.

Perfect accuracy demands the use of two distinct gnomons, one for use from the winter to the summer solstice, and the other for the following and equal term. But if a single gnomon of mean contour be used, the error produced is so minute that it is practically inappreciable.

The sun dial constructed as above described indicates true local mean time, but the hour arc may be set to indicate time according to any standard meridian required.

Dated this 27th day of January 1892.

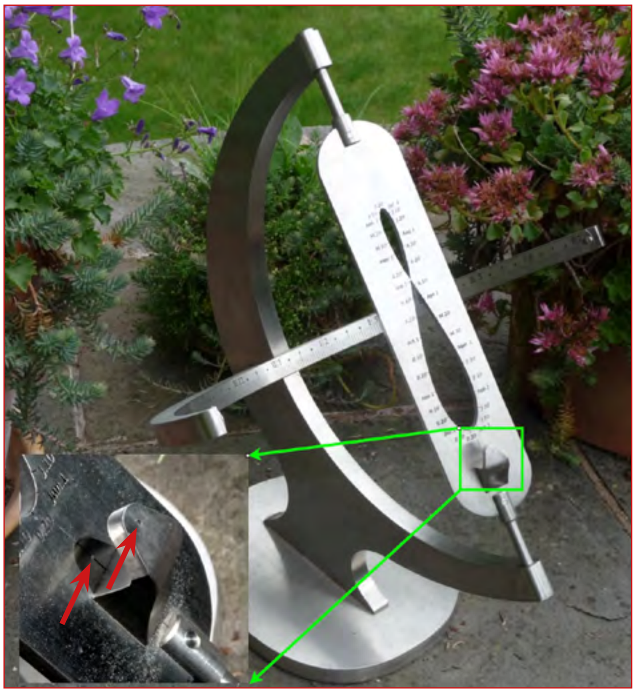
PHILLIPS & LEIGH,
Agents for the Applicant.

1892 UK Patent by Major General (retired) John Ryder Oliver for Improvements in Sun-Dials

Ken Clark - Schmoyer Dial 2018

3-D printed Schmoyer dial by Bill Gottesman

Ref www.precisionsundials.com



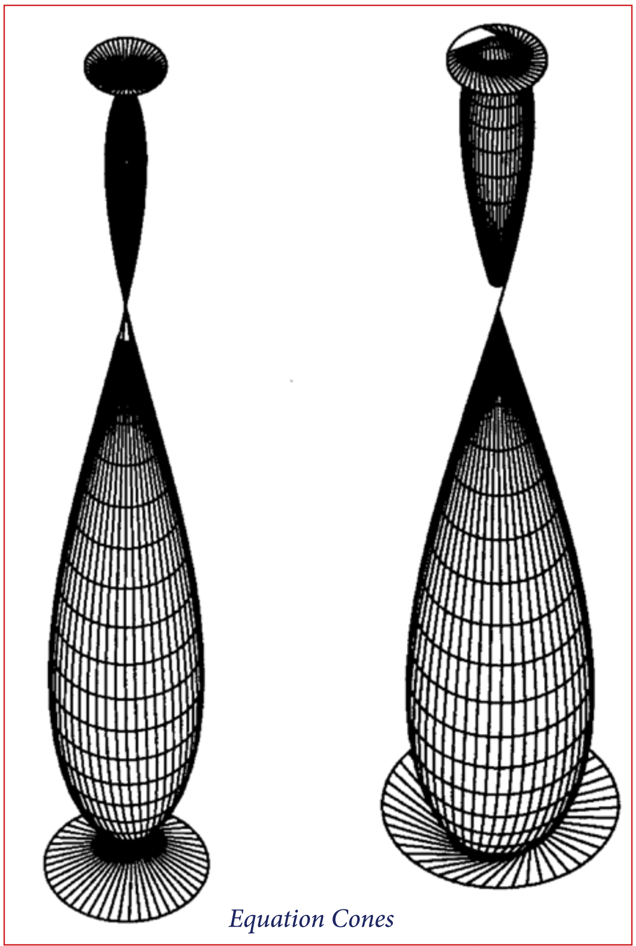
Tony Moss 'Henry Moore' dial - ca 2010 - note the clever arrangement to ensure that the plane of the analemma is perpendicular to the sun's rays

Pete Swanstrom - - New Millennium Dial - 2018

www.swanstrom.net/sundial/

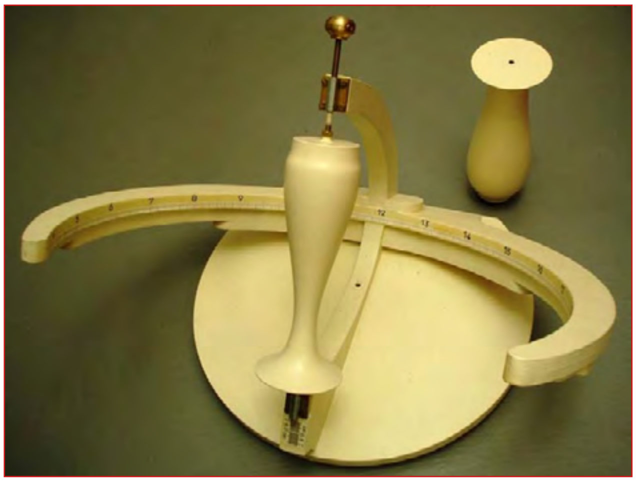
Werner Riegler

Martin Bernhard' s dial at the Stuttgart Planetarium.
Other Gnomon not shown.



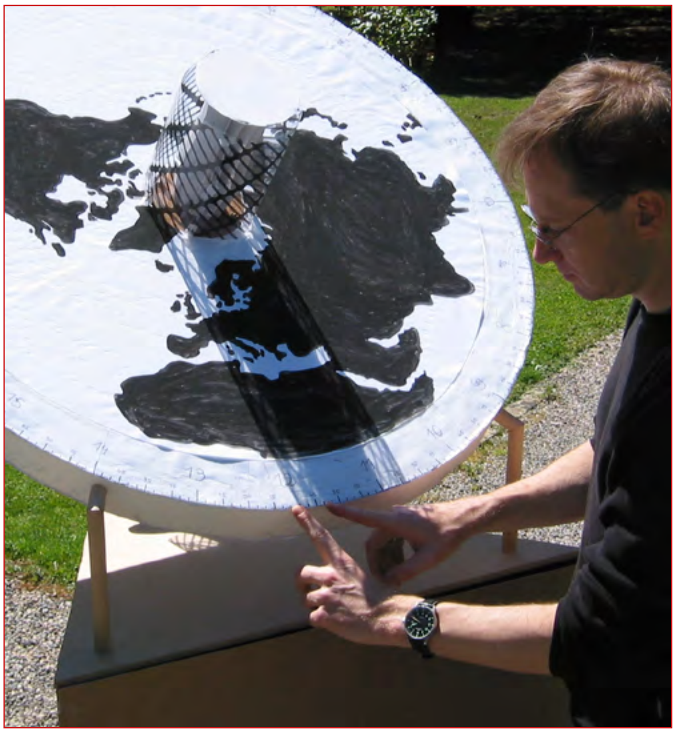
Equation Cones

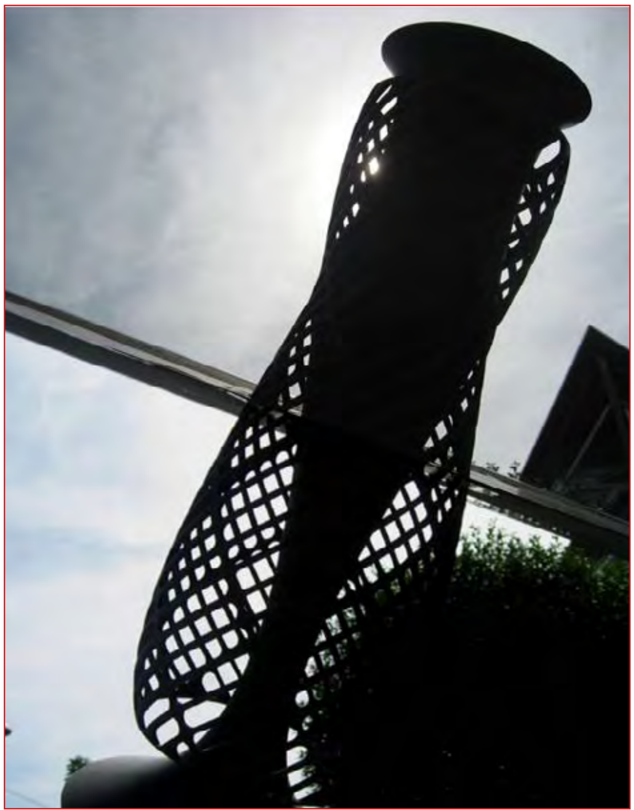
Keijo Ruohonen, Sundials and Mathematical Surfaces,
NASS Mar 2001 8(1)



Rolf Wieland : There are two shaped styles :
each representing half the analemma

NASS June 2011 v18(2)





Composite of two images, along equatorial plane

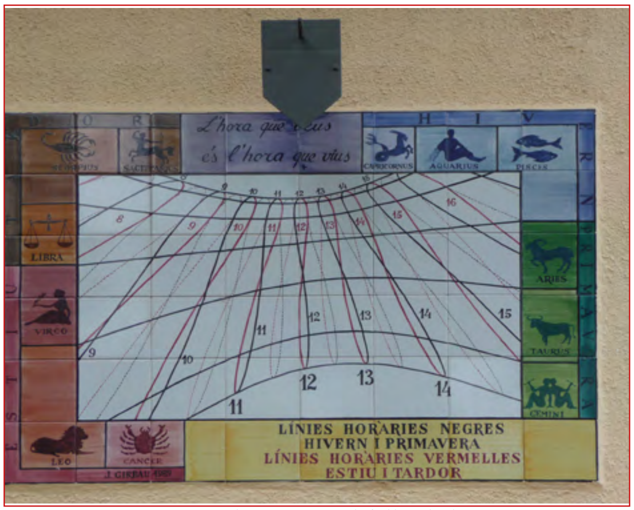
Curved Hour Lines Sundials

This family divides into 4 subclasses.

1) Full Analemmas and Noon Marks
2) Half Analemmas
3) Noon Marks

These first 3 all require a nodus or a point gnomon. They are the commonest and easiest-to-make equation-corrected sundials. The Equation of Time is usually combined together with the longitude correction. The Curve is - in all cases - a representation of the analemma is used (or 2 separate halves of the analemma. (See page 'Displaying the Equation of Time')

4) Spider Dials, which are much more complex, but have the advantage of showing the time of sunrise and sunset throughout the year.



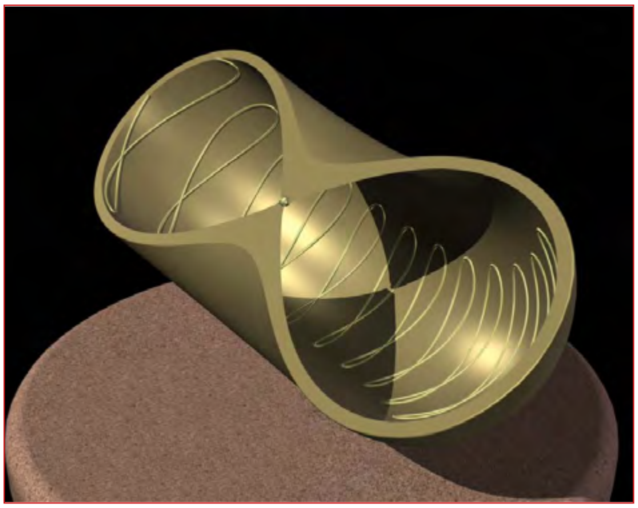
1989 - J Girbau - Castelifollit de la Roca

ca 2000 - Port Augusta Arid Lands Botanic Garden, South Australia

Full Analemmas

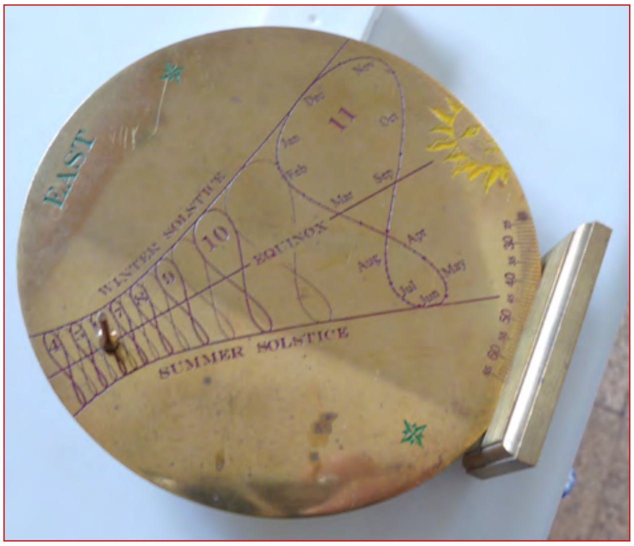
(see also Noon Marks, below)

Rafel Soler i Gayà - 1988 - Port of Tarragona



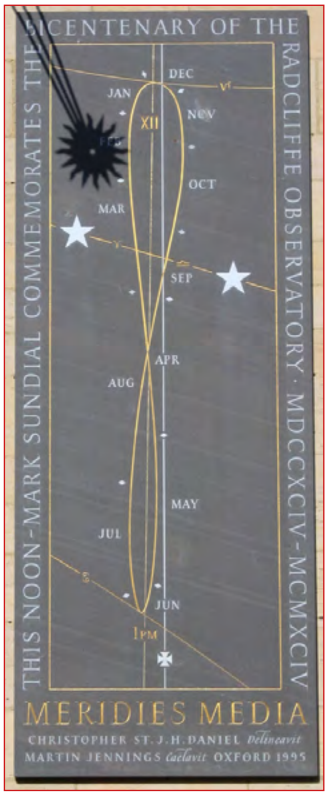
Stephen Luecking - Lemniscate Dial.

NASS Dec 13 20(4)



Recent - Harriet James - Private

19C - Urbain Adam - St Martin, Colmar



1995 - Martin Jennings - Radcliff Observatory, Oxford

2001 - Vic McGrath - Reconciliation, Canberra

2003 - Dr Frank King & Kristi Shea - London Stock Exchange

2007 - Dr Alan Mills - Eye of Time,
Leicester University

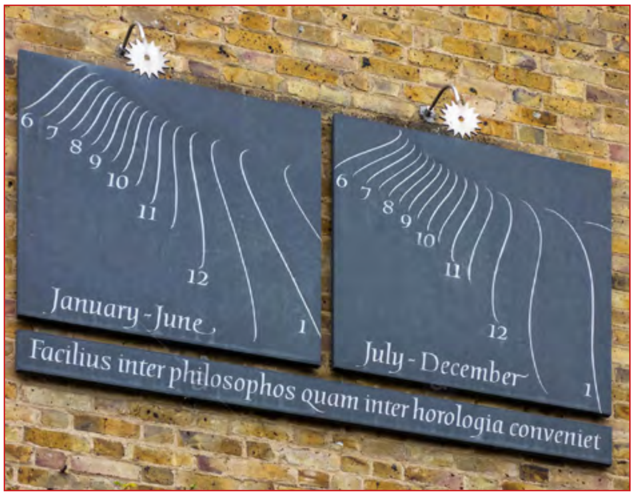
Recent - Alastair Hunter - Private

return to "Table of Contents"

Half Analemmas

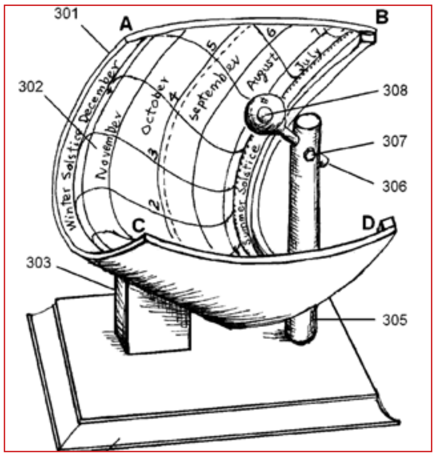
These are easier to read than full analemmas, but either require the biennial change of the dial plate or two separate dials

Early 20C - Ferguson Solar Chronometer, for use by Victorian travellers & explorers - Musée de la Vie Wallonne, Liège



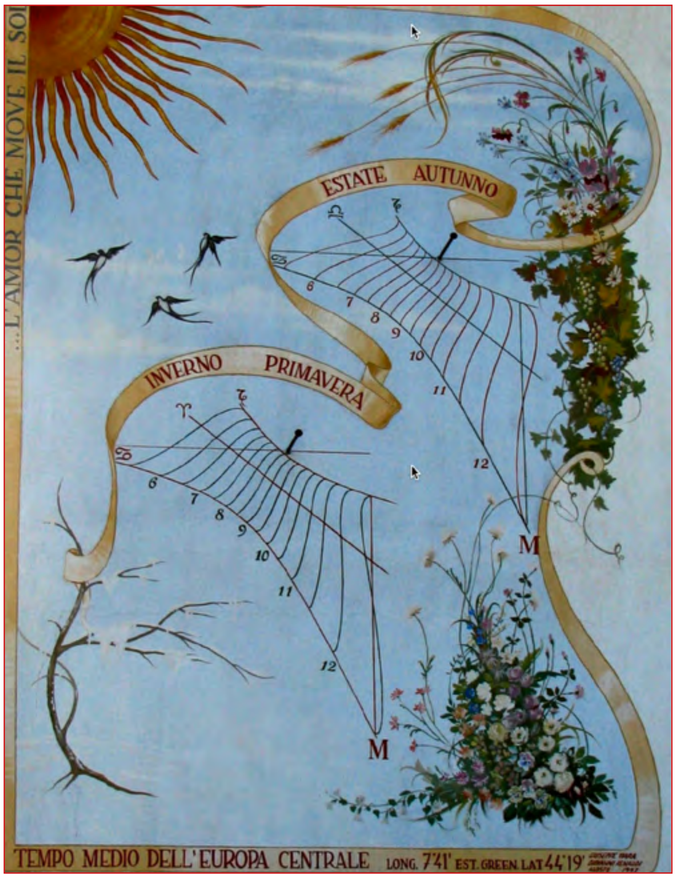
1987 - Will Carter - Magdalene College, Cambridge

2004 - Sir Mark Lennox Boyd & Ben Jones - Rosemoor Garden, Devon



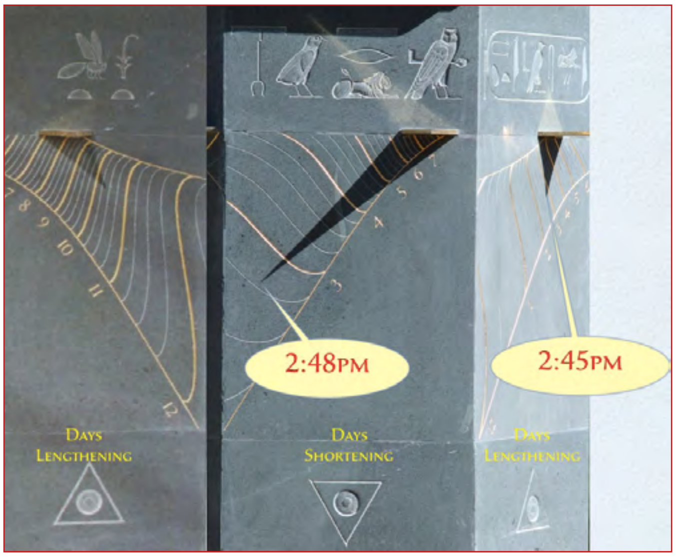
Chengjun Julian Chen - Omni-Directional Lens Dial.
Of interest since a hollow Perspex nodus sphere of very specific dimensions, filled with water, produces a highly focused image of the sun on the dial plate

US Patent 7,555,840 B2



1997 - Giuseppe Viara & Giovani Renaudi - Chiusa di Pesio Nr Turin

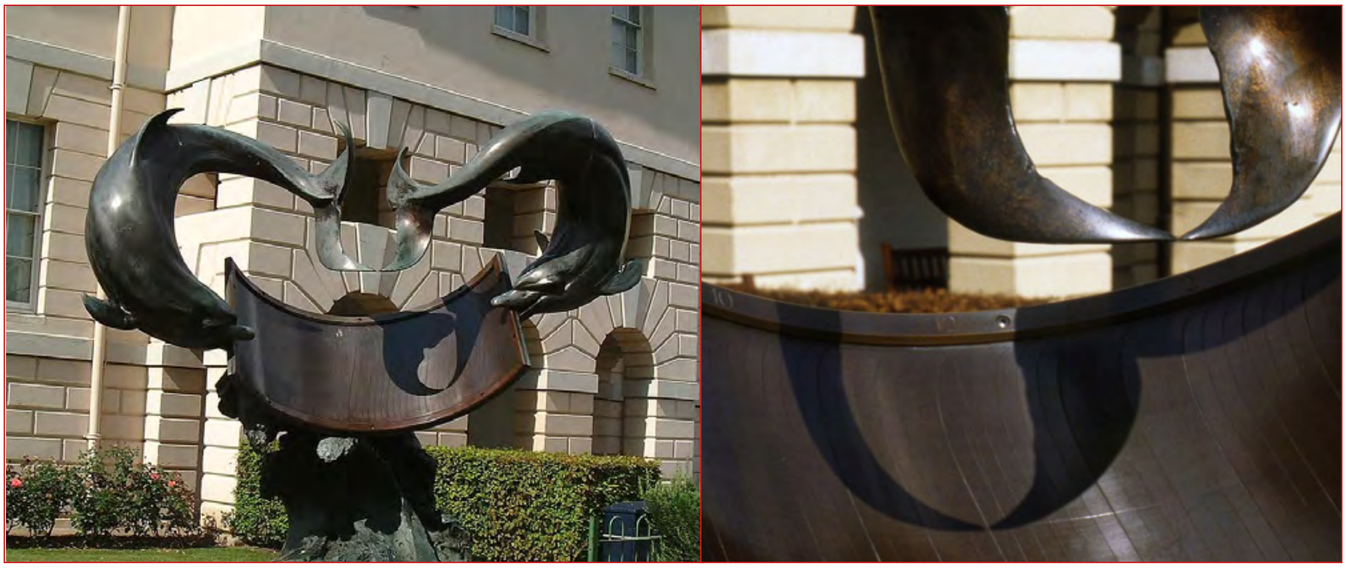
2013 - Sir Mark Lennon Boyd & Fergus Wessel - Buscot Garden, Oxfordshire



Detail of Buscot Dial showing the time in the two Halves of the year

2007 - Hoffmann Albin-

available from www.precisionsundials.eu

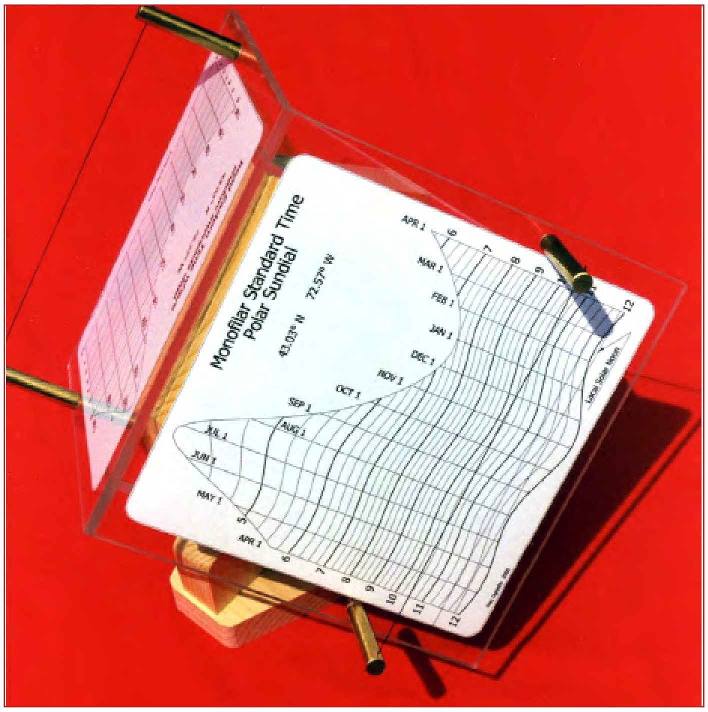


1979 - Christopher St. J.H. Daniel & Edwin Russell - Dolphin Dial, Greenwich Royal Observatory

SPIDER DIALS

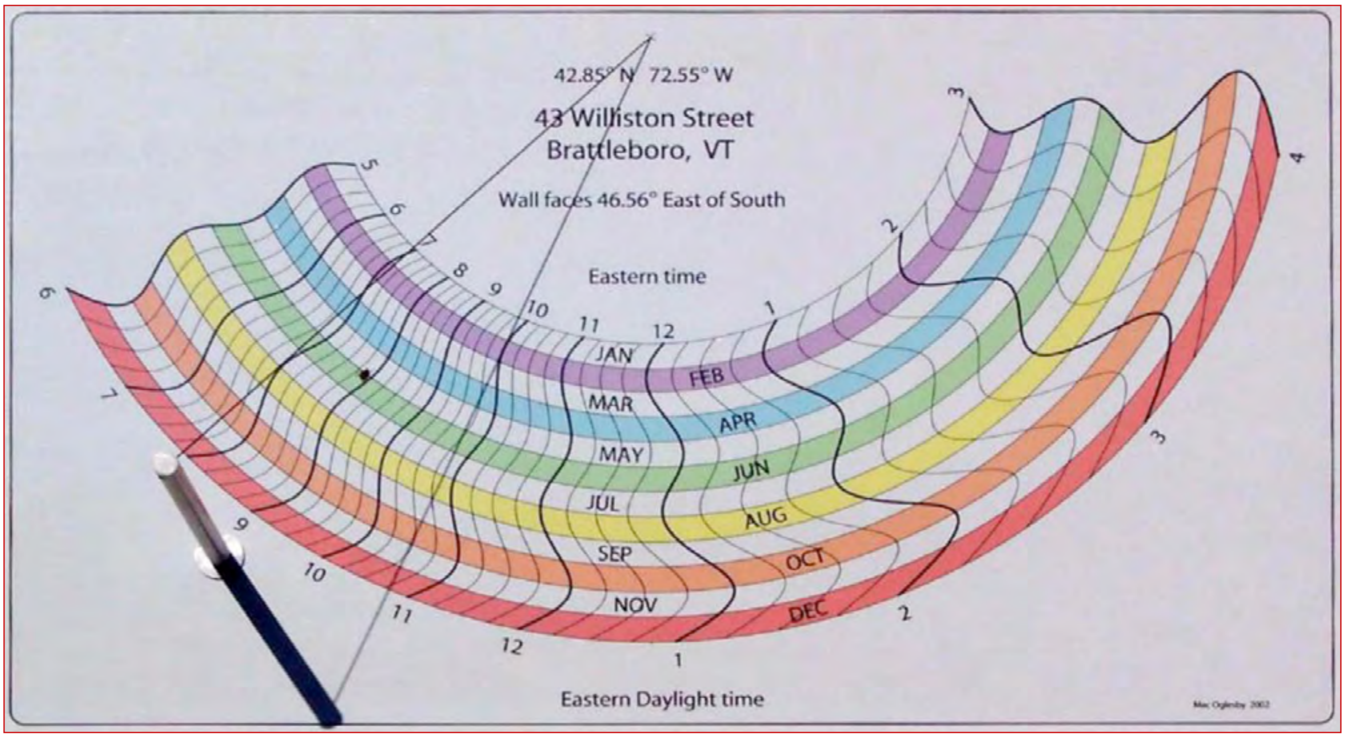
These require a lot more calculation to delineate, but are used with a standard gnomon, rather than a nodus. A nice feature of the spider dial is its ability to indicate sunrise and sunset times (see last figure in this section)

2001 - Rick Twardy - Parkes Observatory, Dubbo, New South Wales



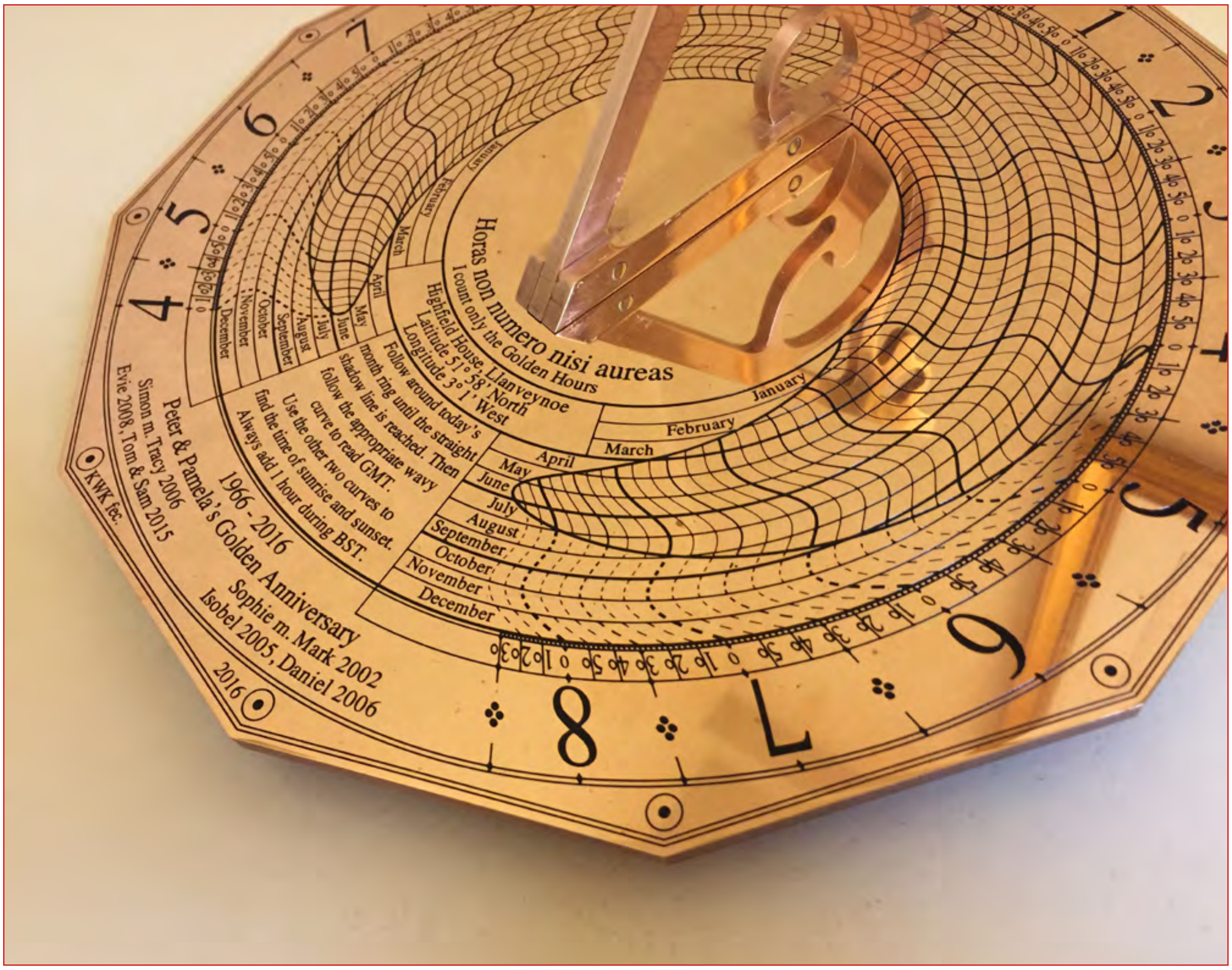
2001 - Mac Oglesby - Monofilar Standard Time Polar Dial

NASS 10(1) - March 2003

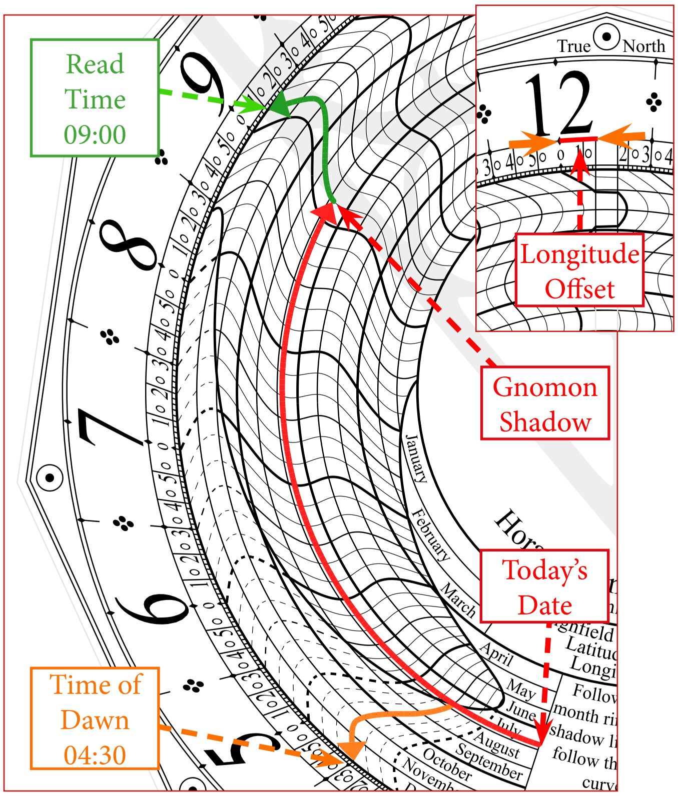


2002 - Mac Oglesby - Vertical Declining "Rainbow" Spider Dial - Brattlesbury, Vermont, USA

return to "Table of Contents"



2016 - Kevin Karney - Bronze Highfield House dial - before being exposed to the atmosphere

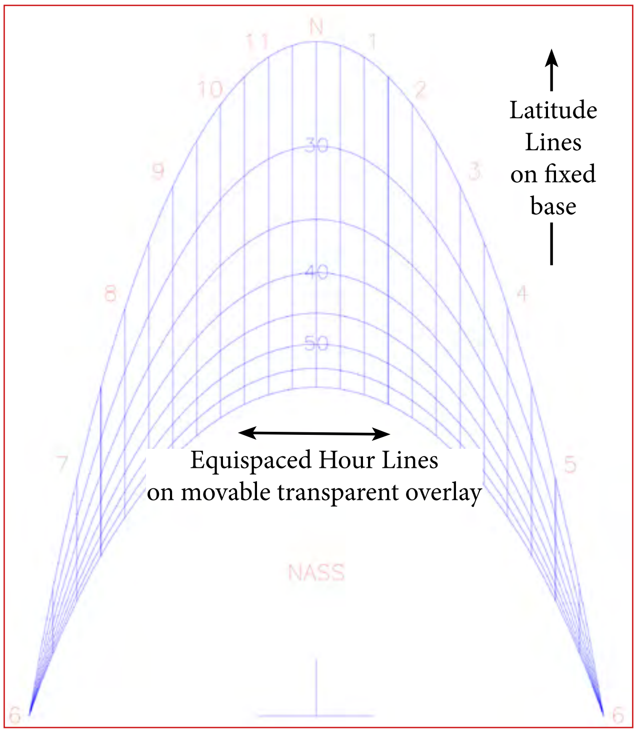


Reading the Highfield House dial

Highfield Dial after two years in the atmosphere

return to "Table of Contents"

Linearly Equispaced Hour Line Dials

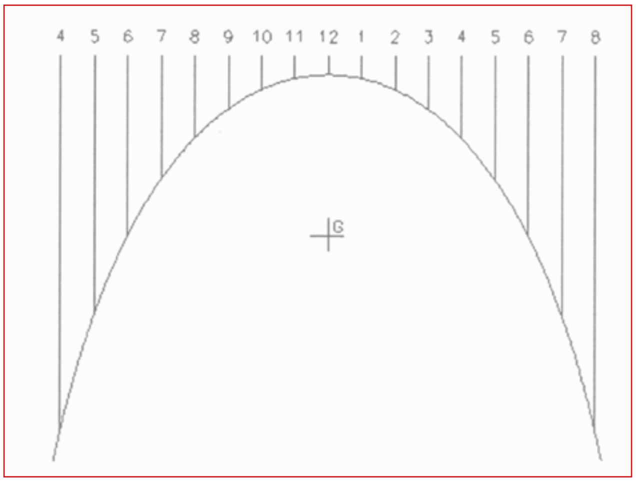


Latitude Lines on fixed base

Equispaced Hour Lines on movable transparent overlay

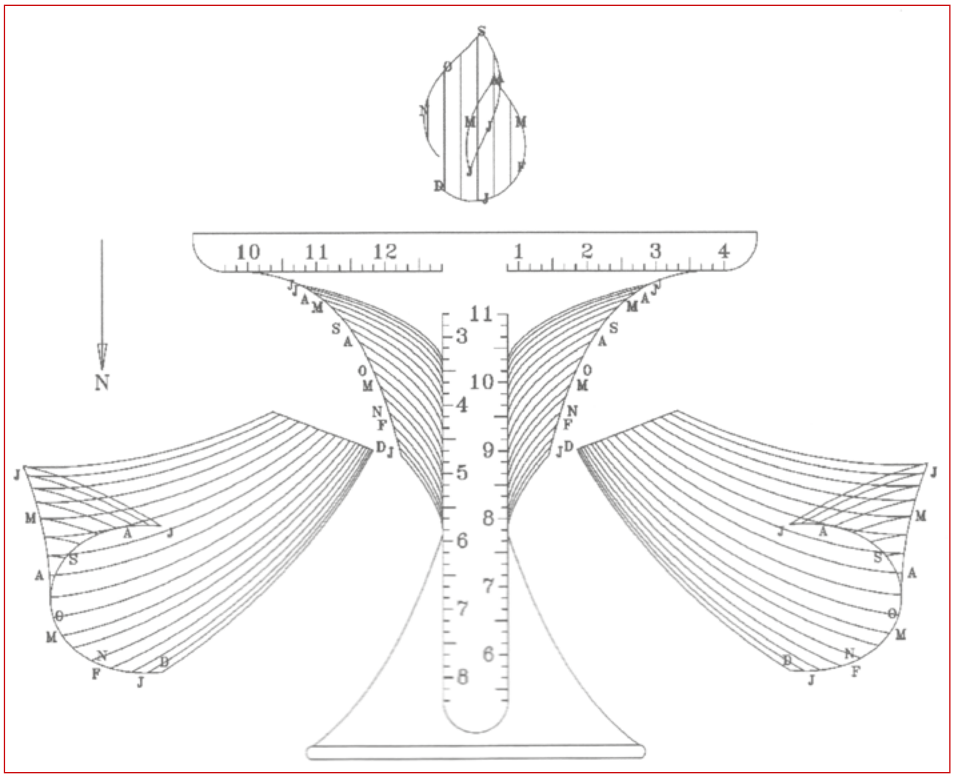
Sawyer's Universal Horizontal Equi-spaced Hours Dial

NASS Sep 95 2(3):11-18 & Sci 2



de Vrijs & Sawyer Linear Equant Sundial

BSS Jul 91 91(3):34 & Sci 1



Sawyer's 'Born of Light' Dial

An Osculatory Sundial NASS Mar 07 14(1) :13-16 & Sci 4

return to "Table of Contents"

Dials with Alidades

An alidade or a turning board is a device that allows one to sight a distant object and use the line of sight to perform a task.

19th C, French Heliochronometer by Radiguet-Massiot, Paris - previously thought to have been used by the French Railways - but this is not the case..

Photo from Musée de la Vie Wallonne, Liège

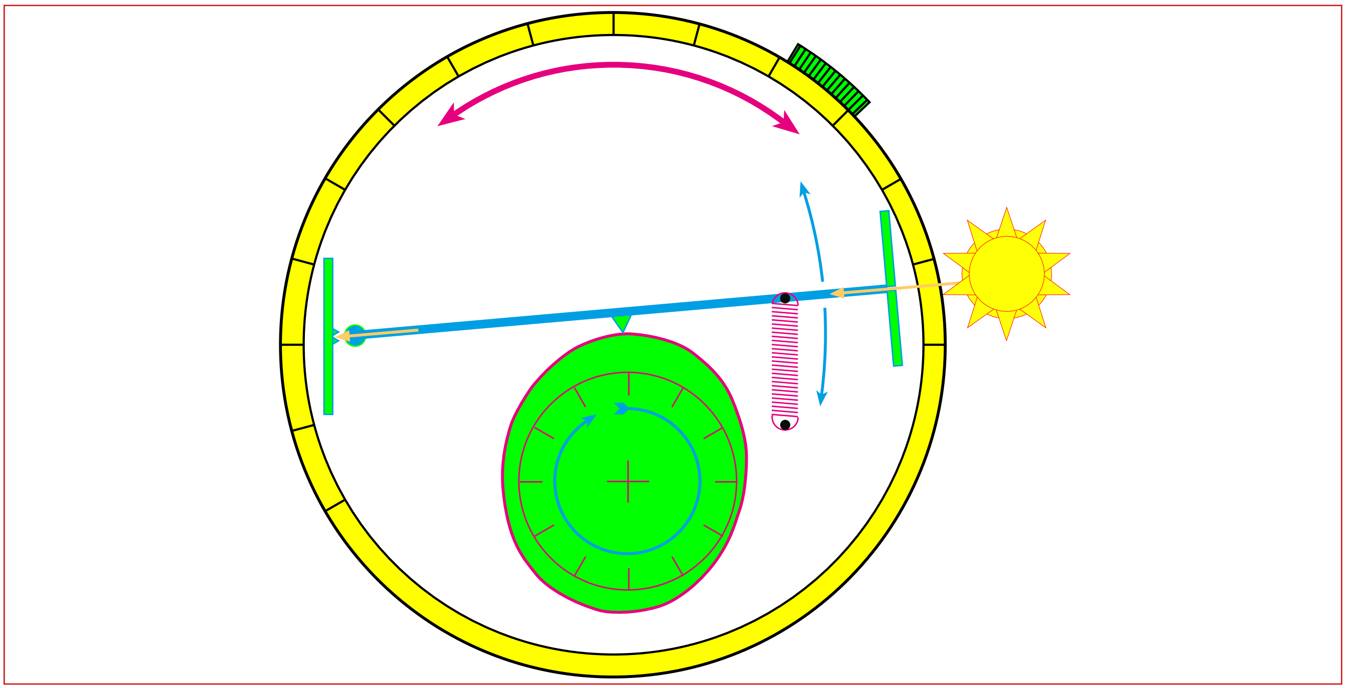
Pilkington Gibbs Heliochronometer

1910 gold-plated Pilkington Gibbs Heliochronometer, originally given to Robert Peary, who first reached the North Pole,by the Lord Chief Justice of England in recognition of the first expedition to the North Pole. The two were friends, having both paricipated in an internation squabble over national maritime boundaries between Russia, Canada and the U.S.

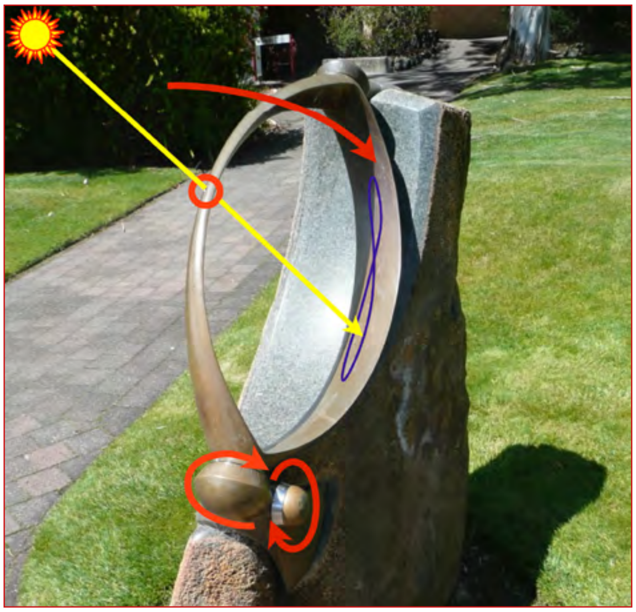
Now owned by the author and still in mint condition

Detail of Radiguet-Massiot analemma

return to "Table of Contents"



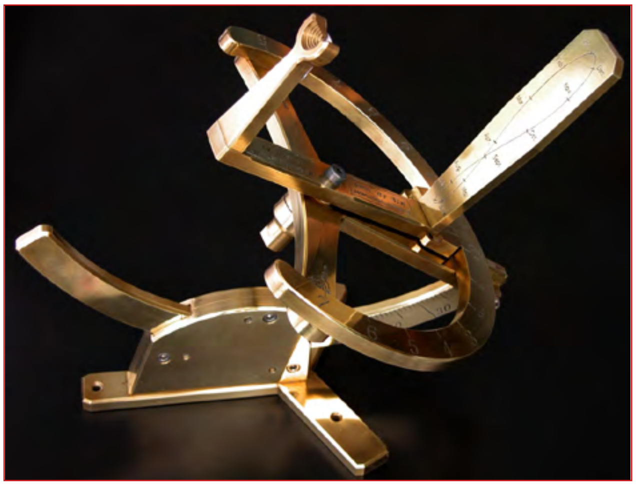
Pillinton Gibbs' mechanism. The precision-made green cam is attached to the circular bronze date-adjusmetment (see previous illustation).



Noon Mark by Anthony Sprent, University of Tasmania

The Equatorial version of Pilkington Gibbs.
This was installed in the British military barracks of the Khyber Rifles on the Pakinstan/Afghasistan border in 1923.
It is still looked after, with pride, by the Pakistan Rangers

Date Adjustment of Anthony Sprent Noon Mark



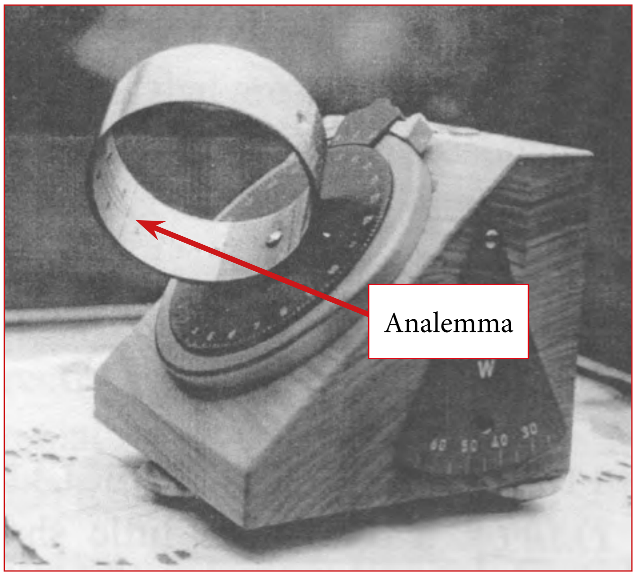
J.D Gard - Aten Heliochronometer - ca 2006

These dials used to be commercially available in bronze or aircraft-grade aluminium

return to "Table of Contents"

John Gunning - 2002 - Gunning Helichronometer, Belvoir Castle, Leicestershire UK.

photo © J Hannan-Briggs



Schilt Heliochronometer

NASS Jun 14 21(2):12 & Sci 5

Tempus Mundi, concept by Christian Wernli.

Available from www.helios-sonnenuhren.de/de/tempus-mundi

Prof Martin Jenkins - Cooke Heliochronometer - 2012. Made for his wife's birthday!

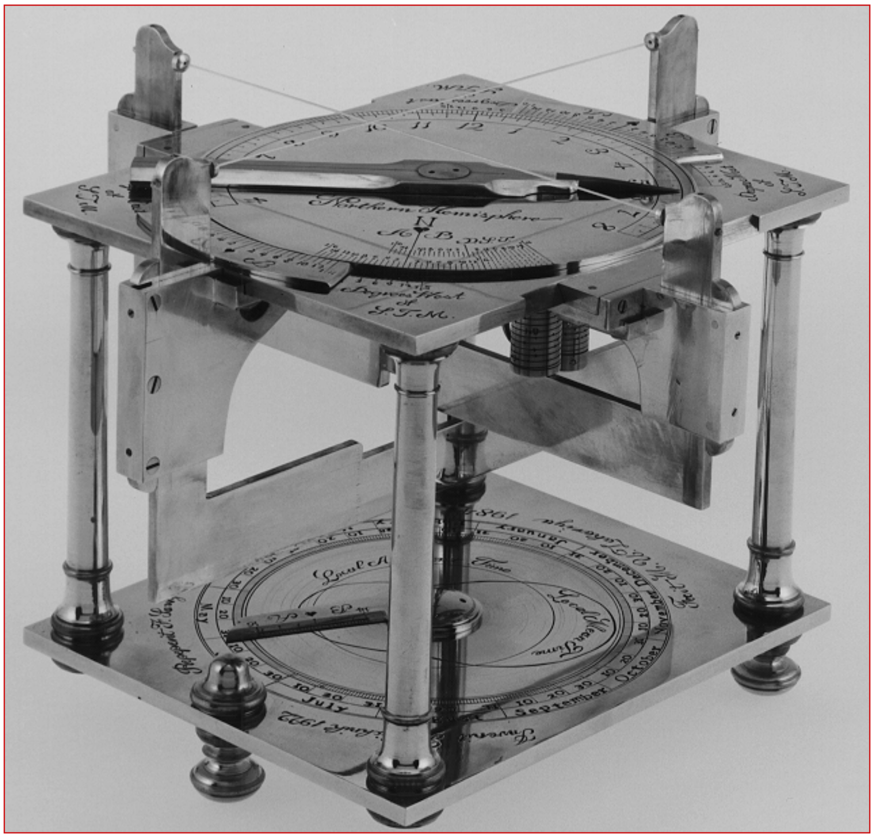
return to "Table of Contents"

Bifilar Dials and Other Esoterics

THE BIFILAR

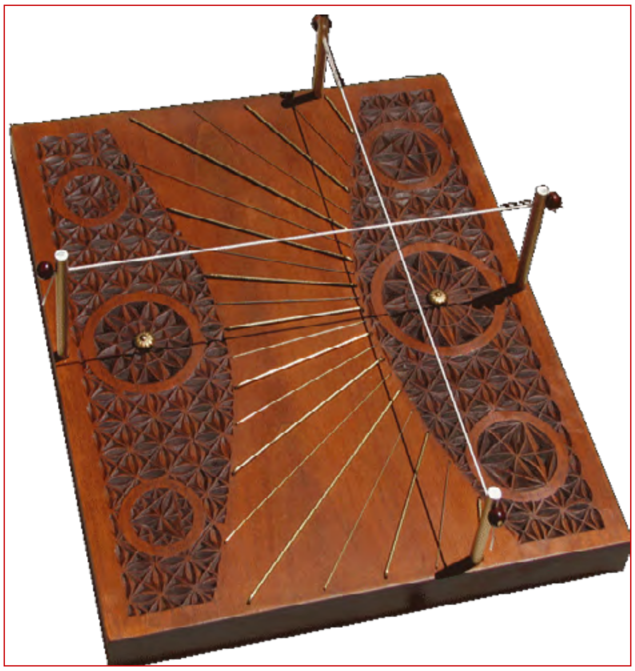
The Bifilar sundial was invented by the German mathematician Hugo Michnik in 1922. It has two non-touching threads parallel to the dial face. Usually the two threads are perpendicular to one another. The intersection of the two threads' shadows gives the local apparent time.

When the threads have the correctly calculated separation, the hour-lines on the horizontal surface are wquiangular, i.e. the hour-lines for successive hours are 15 degrees apart.



Equation Corrected Bifilar dial by Fred Sawyer & M.U. Zakariya -

'Bifilar Gnomics' zzBSS Feb 93 93(1):36-44 & BSS Feb 95 95(1):18-27 & Sci 1
'Bifilar Origins' NASS Sep 08 15(3):20-26 & Sci 4



Chip-Carved Bifilar Dial - Roberto Rattotti

www.rknet.it

Bifilar Dial - Sergio Garcia Doret - 2004

http://www.relogiodesol.com

Other Esoteric Designs

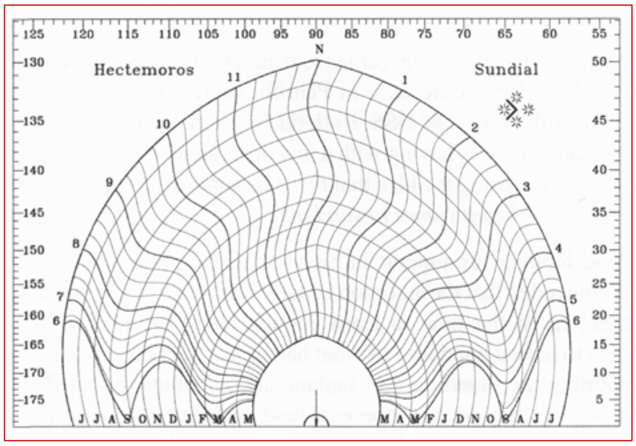
Ptolematic Coordinate Sundials

This strange series of dials utilise the pair of earth-bound great circles used by Ptolemy, rather than the more familiar celestial circles.

i) the hectemoros circle which passes though the sun itself and the east/west points on the horizon;

ii) the horarius circle which passes though then sun itself and the north/south points on the horizons.

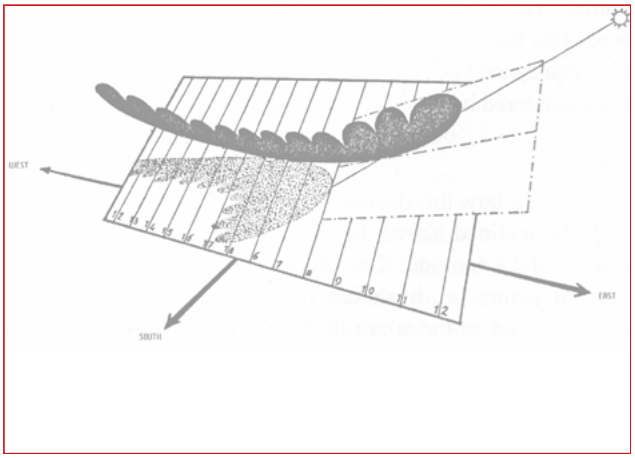
From these, Fred Sawyer developed a latitude independent mean time dial.



Sawyer's Latitude Independent Hectemoros Mean Time Dial

Ptolemaic Coordinate Sundials NASS Sep 98 5(3);17-24 & Sci 2

The Cycloid Dial



de Vries' Cycloid Polar Dial with Equispaced Hour Lines.

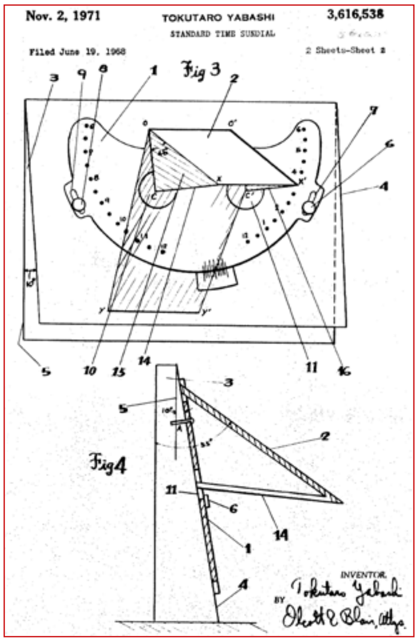
The shadow of gnomon's lower edge tells the time - 8:00 in this example - where the cycloid's shadow just touches an hour line. The upper edge is decorative.

Thijs de Vries : De Zonnewyzerkring, Jul 1980 80(6) and Fred Sawyer : The Cycloid Polar Sundial, NASS Dec 1998 5(4)

Yabashi Standard Time Sundial

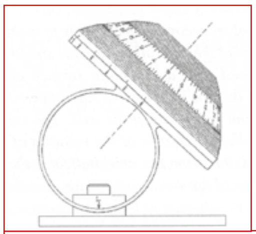
This interesting wide gnomon sundial artificially creates equiangular hour lines that are correct on the hour about the so-called Yabashi Point. Fred Sawyer has shown that it is not correct during the intervening minutes. Thus it is hardly a mean time dial. The US patent 3,616,538 was lodged in 1971.

However, perhaps recognising that it was not a 'proper' sundial, it was relodged in 1977 as patent 4,050,161. But it was now claimed as 'True North Meridian Indicator', for which it could be used for - on the hour!



Tokutaro Yabashi Standard Time Dial.

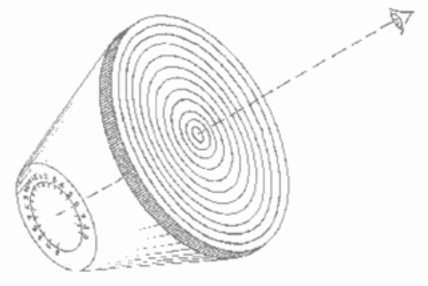
NASS Dec 00 7(4):27-29 & Sci 3

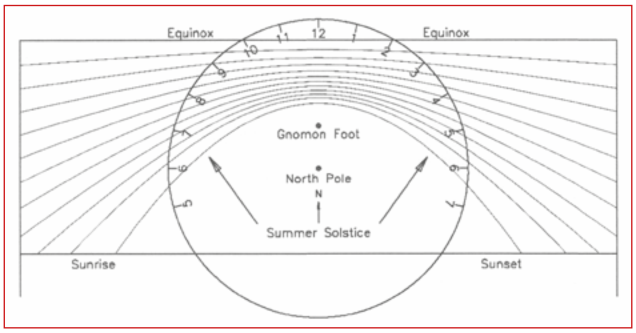


Gundlach's Shadowless Sundial

These use concentric reflective rings or could use a fresnel lens as in a CD. These were never brought into production

NASS Sep 99 6(3):23-26 & Sci 3



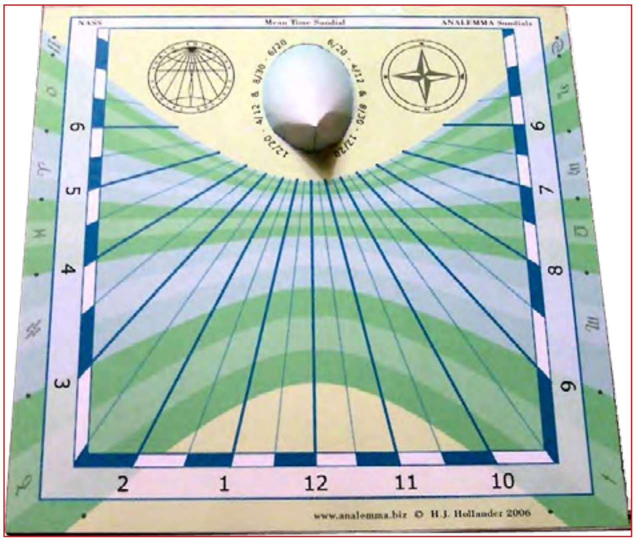


A Bipolar Azimuthal Equant Sundial

NASS Jun 05 12(20:13-16 & Sci 4

return to "Table of Contents"

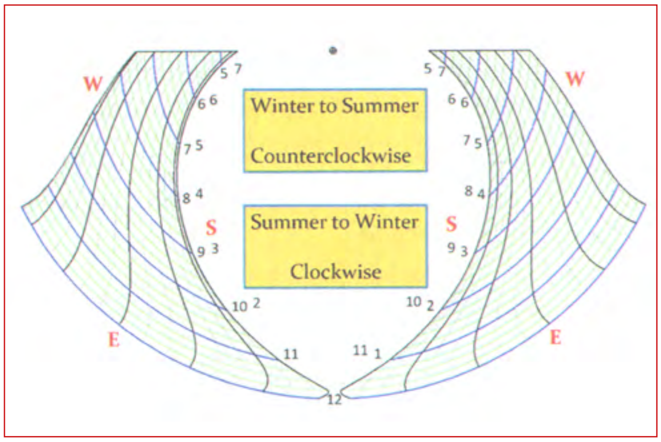
Polar Cone Dials



Mean Time Sundials with polar cone and vertical cone gnomons - Hendrix Hollander - www.analemma.nl.

NASS Compendium v13(3) Sept 2006 - 3 articles :
Mean Time Sundial With A Cone Gnomon : Hollander
Bi-Gnomon Sundials : Hollander
Equations For Hollander's Mean Time Sundial : Sawyer
This design won the 2006 Sawyer Dialing Prize

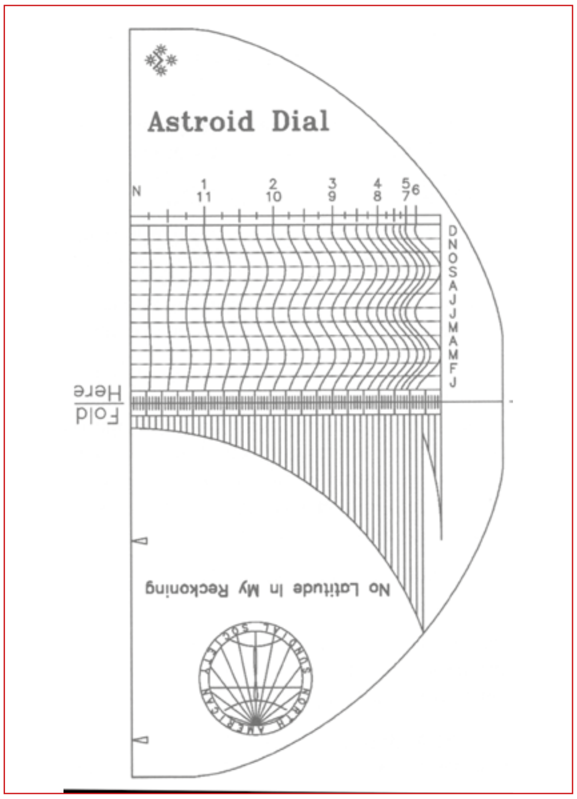
Solar Decliners



Sawyer's Mean Time Solar Decliners

NASS Mar 20 27(1):6-15 & Sci 5

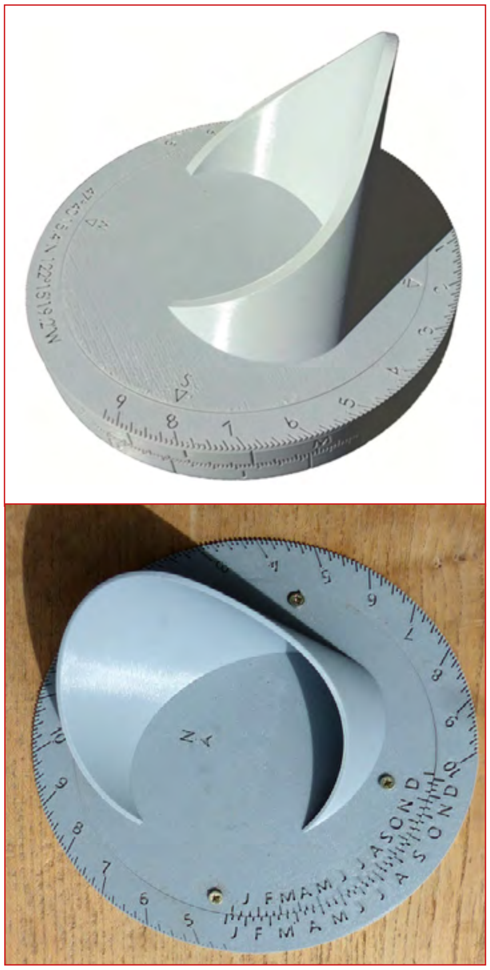
Astroids



A Portable Astroid Dial

NASS Dec 96 3(4) 18-25 & Sci 2

Polar Envelopes



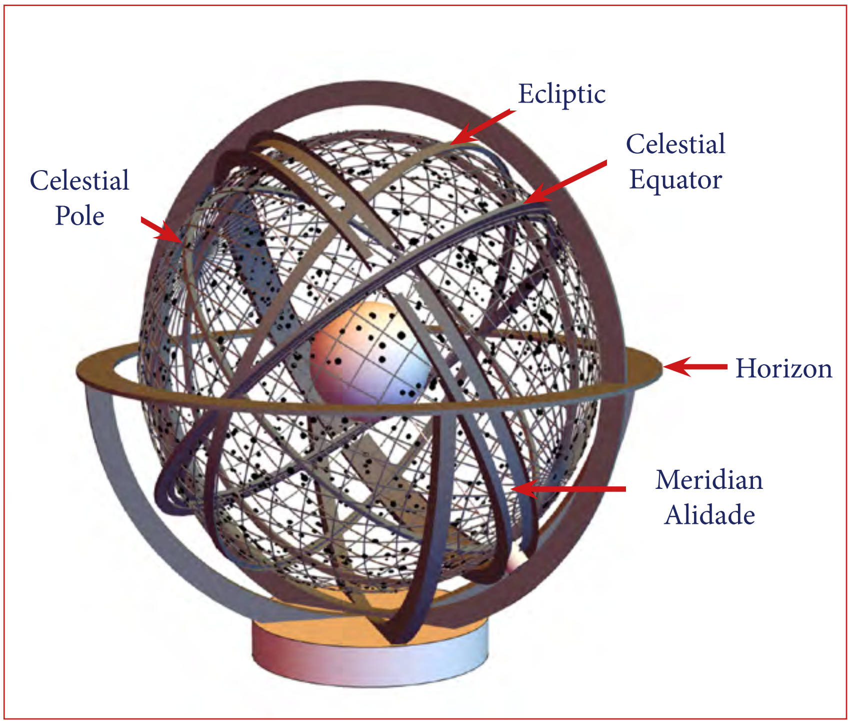
Two of Sawyer-Lelievre Polar Envelope Gnomon Dials
a family of dials, such as this, may be 3-D Printed

NASS Compendium v26(4) Dec 2019

Top : Latitude 47° North
photo Art Kaufman
Bottom : Latitude 47° North
photo Mike Shaw

return to "Table of Contents"

This beautiful device, set in the grounds of Linz University, represents the universe as seen by Joannes Kepler, who lived in Linz from 1612-1626, where he also finished the Rudolphine Tables.. The celestial globe is marked with the 1440 stars of 6 different magnitudes and 48 Ptolemaic constellations – as recorded in the his Tables.

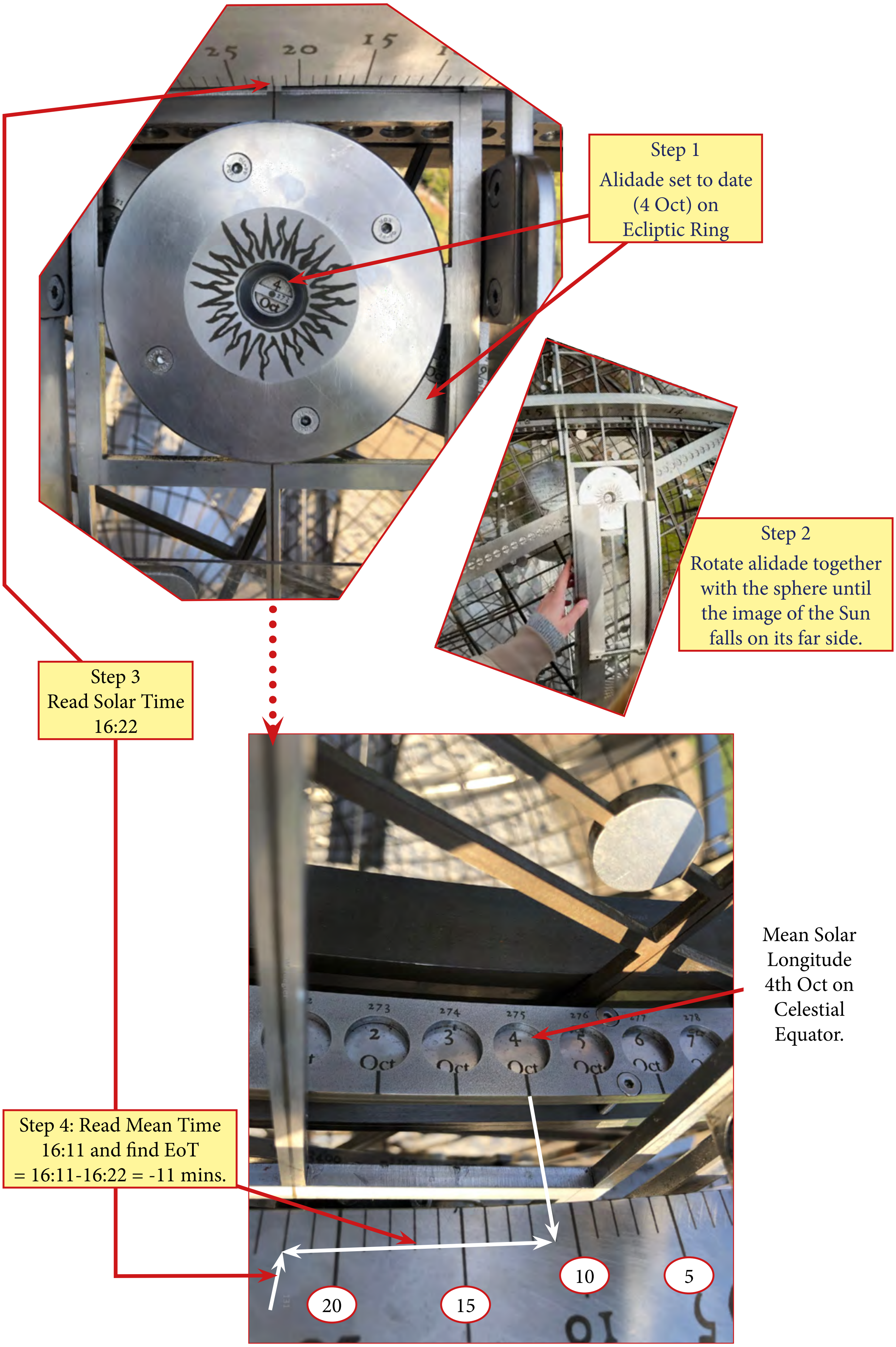


The Ecliptic is marked with the Solar Longitude according to Kepler's Laws, while the Equator is marked with the Solar Mean Longitude.

To read the time, the split Meridian Alidade is set the correct date, the whole sphere and alidade are rotated to align with the Sun. This is found when the image of the split falls on the opposite side of the alidade. Both Solar and Mean Time can then be read, from which the EoT can be directly derived.

The design is such that, in the longer term or as a teaching aid, the scale on Ecliptic can be moved to adjust for precession and the Ecliptic itself can be adjusted to account for changing obliquity. If both are adjusted, the dial should be accurate for < 1 minute for the next 8000 years.

return to "Table of Contents"



Step 1
Alidade set to date
(4 Oct) on
Ecliptic Ring

Step 2
Rotate alidade together
with the sphere until
the image of the Sun
falls on its far side.

Step 3
Read Solar Time
16:22

Step 4: Read Mean Time
16:11 and find EoT
= 16:11-16:22 = -11 mins.

Mean Solar
Longitude
4th Oct on
Celestial
Equator.

Setting and Reading Kepler's Cosmos

Jackoby's Spherical Noon Mark at Columbia University

Fred Sawyer - NASS 12(4) Dec 2005

Fred Sawyer - BSS Peterborough Conference 2024

The Noon Sphere in 1914

The Noon Sphere today with the cracked sphere gone. note the two symetrical noon plates.

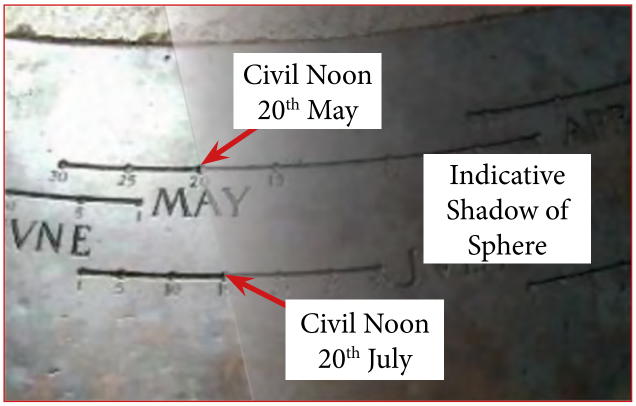
The Noon Mark sundial in Columbia University was installed in 1914. It consisted of a 7 foot, 15 ton granite ball sitting on a plinth with two bronze plates with noon marks in ***standard*** time. Previously two balls had cracked while being turned.

The dial was designed by Prof. Harold Jacoby to tell the time at noon, so that students could get to lectures on time. It carried the inscription 'Await the Hour – It will come'

Await the Hour – It will come

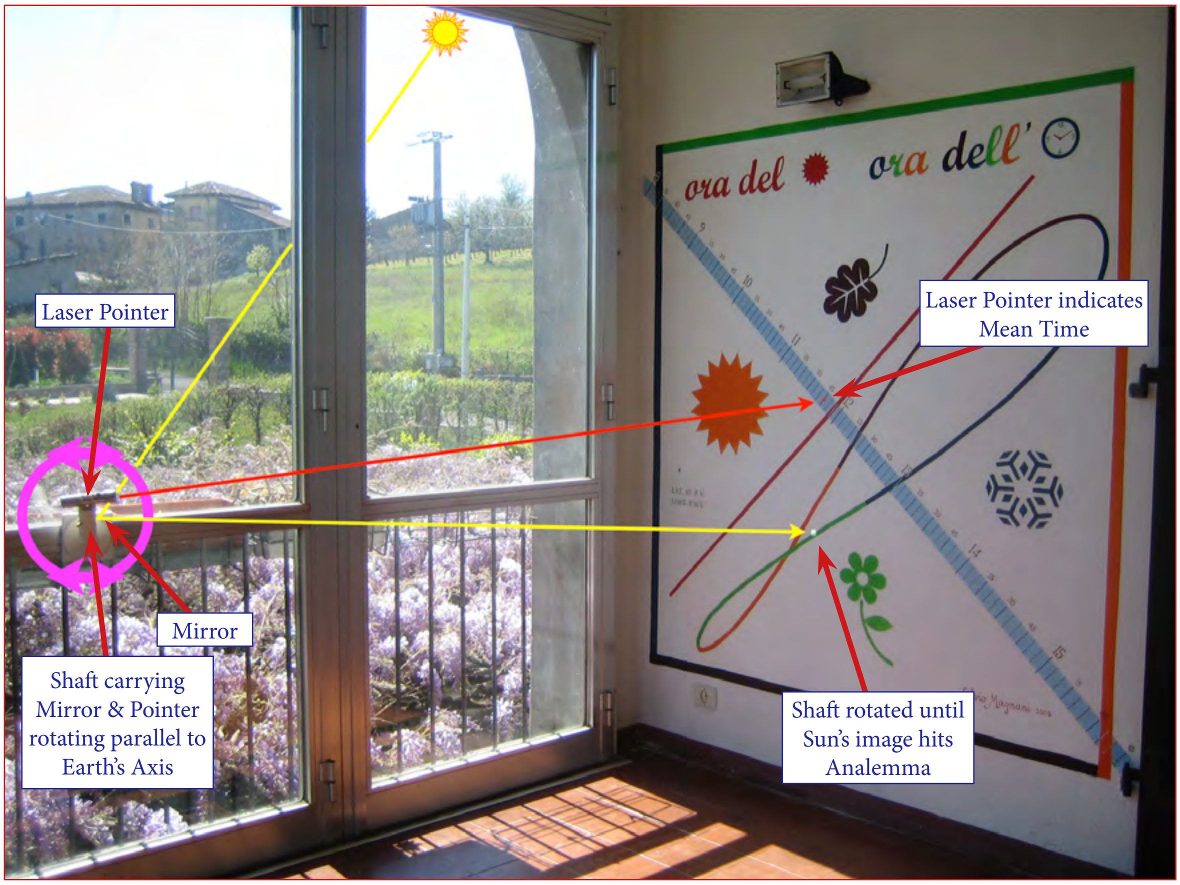
In 1947, the sphere cracked and was removed, leaving only the plinth, the noon marks and the inscription. It is still called 'The Sundial' and is still a central meeting point for students

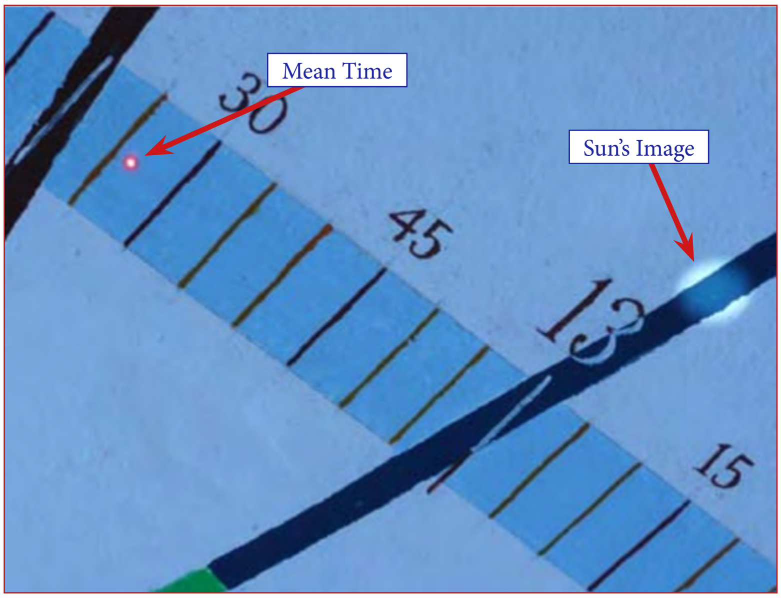
Part of one of the dial plates



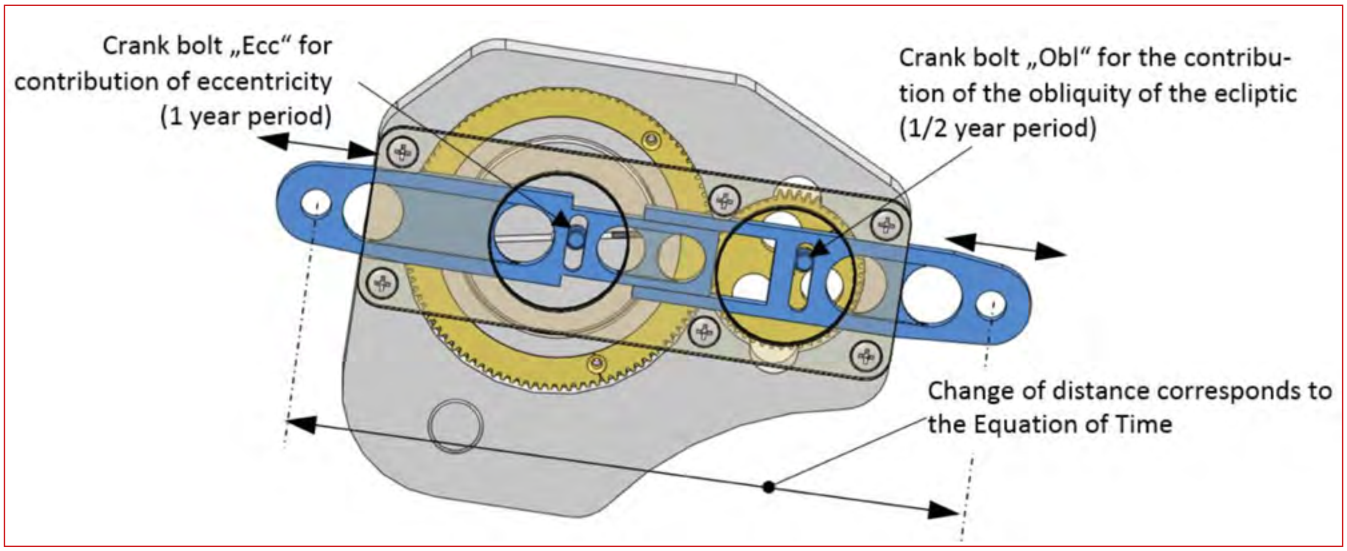
If the shadow of the sphere was as shown, it would have been standard time Noon on those two dates

return to "Table of Contents"


Laser Pointer
Laser Pointer indicates Mean Time
ora del
ora dell'
Mirror
Shaft carrying Mirror & Pointer rotating parallel to Earth's Axis
Shaft rotated until Sun's image hits Analemma


Mean Time
Sun's Image
30
45
13
15

The Heliochronometer of Schwarzenau, Austria



The Equation Mechanism

See also 'Descovitch Method' on page 113.

return to "Table of Contents"

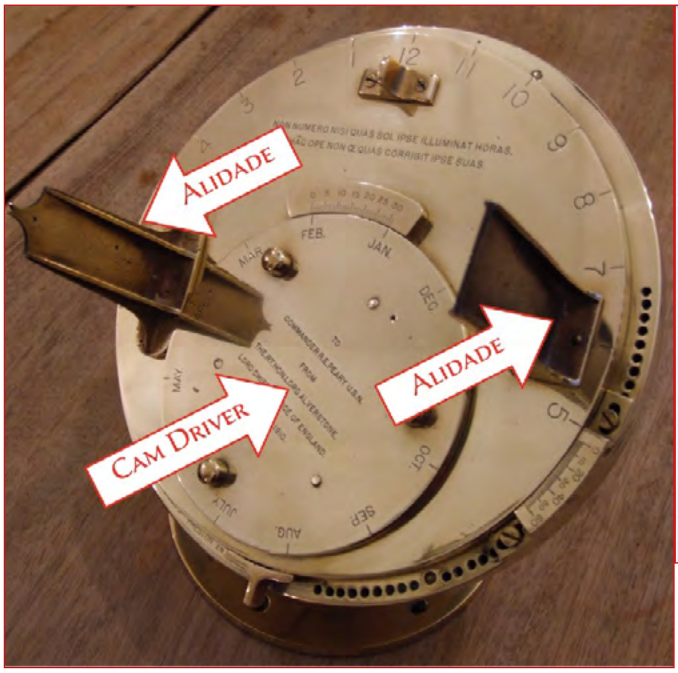
Mechanically Corrected Sundials

Profiles

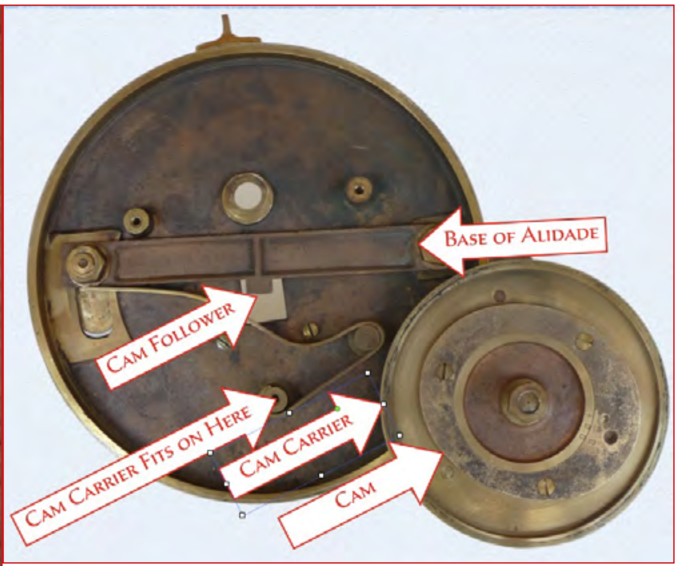
In this context, a profile is simply a graph of the equation of time wrapped around a cylinder. In Bill May's example below, the graph is a grove.

Bill May - 2000 - EoT profile directly drives the equatorial ring of his sundial via a pin which follows the grove cut in the cylinder. The cylinder is marked with the date. The red arrow head shows the month.

Single Cams



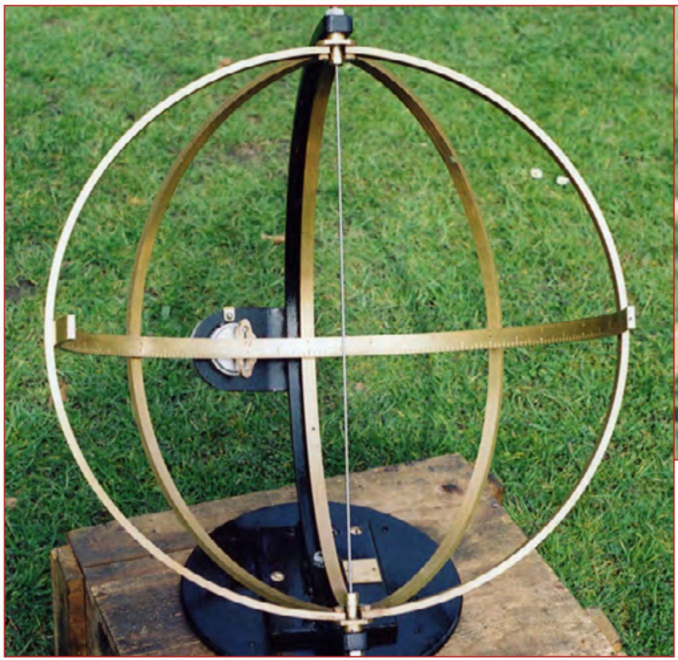
Pilkington-Gibbs Heliochronometer - ca 1910 - (this gold-plated dial, still in mint condition) was given to Commander Robert Peary, who first reached the North Pole by the Lord Chief Justice of England, (who had previously adjudicated in the international dispute over Arctic fishing rights and thus knew Peary)



the mechanical working of a Pilkington Gibbs - showing the cam and its follower, which is attached to the alidade .

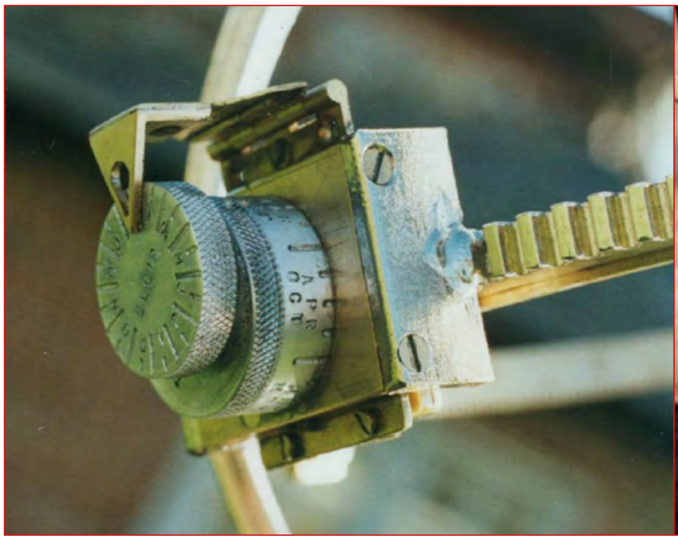
TUSI MECHANISM

A Tusi mechanism, see 'Generating a Sine Curve Mechanically' on page 109. can be found in Bill May's work. Here one cylinder sits inside another. This produces an effect similar to that of two gears (see Gears page). The inner (circular) cam carries an eccentric pin that connects to the equatorial ring of a dial. The inner cam is carried eccentrically inside the outer ring. The inner cam is turned wide in the year (Obliquity effect) ; the outer ring once (Eccentricity effect)



Bill May - ca 2000 - Equatorial Dial.

Inner cam carrying an eccentric pin driving the Hour ring.



Outer ring and inner cam, showing month graduations - version 1.

Outer ring and inner cam, showing month graduations - versions 2.

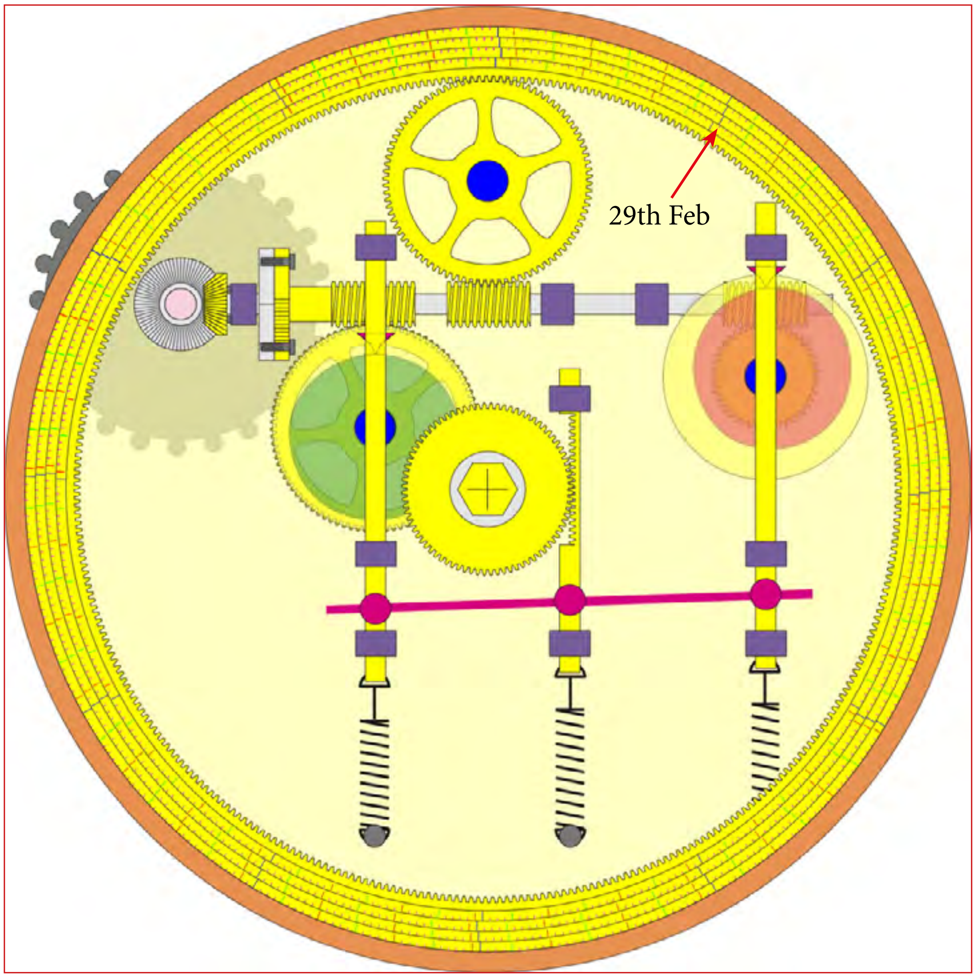
return to "Table of Contents"

DUAL CAMS

The easiest way to produce the EoT mechanically is with 2 (or more) cams, the output of which are added together. One cam turning once a year simulates the Eccentricity Effect. The other turning twice a year simulates the Obliquity Effect. See below.

For sophisticated mechanism (clocks or dials), designed to work in the longer term, this has the added advantage that the Eccentricity cam can be driven once per Anomalistic year (the time from one mean perihelion to the next = 365.259 636 days). While the Obliquity cam can be driven twice per Tropical year (the time from one mean vernal equinox to the next = 365.242 189 days) .

The output from each cam is then added with a 'wiffletree' or other mechanism, 'Mechanically Generating the Equation of Time' on page 109.



The inner working of the author's Dual cam design (never realized).
The Grey wheel is turned to set ...
(i) the outer leap-cycle date bezel.
(ii) the red Ellipticity-effect cam,
(iii) the green Obliquity effect cam.

The output from the two cams is added by the red whiffle tree. This drives the central yellow gear would be attached to an equiangular dial pointer.

return to "Table of Contents"

Part 3 - Equation Corrected Clocks

Equation Clocks - Background

This part of the book predominantly covers the period when Solar Time gave way to Mean Time. See 'The Golden Age of the Equation of Time' on page 4. Initially, in this period, the Equation was required to set the new-fangled clocks to provide traditional solar timekeeping. Thus the growth of Equation clocks.

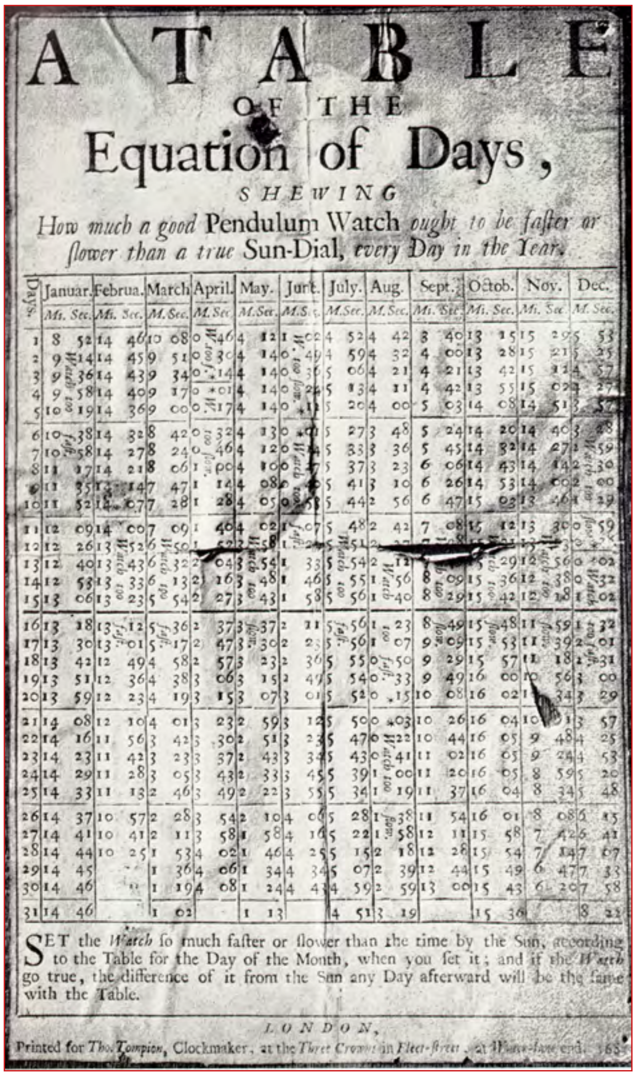
As time passed, clocks became more ubiquitous: church and town clocks tolled the hours. Thus, the role of the Equation was relegated to the use of sundials to the setting household clocks.

The remaining usefulness of the Equation was in marine navigation. See 'Marine Navigation' on page 80.

Equation Clocks - Forerunners

Printed Tables

Before mechanical methods became available, a cheap alternative was to provide a paper table to be stuck or mounted on the case of the clock. Or, in the case of pocket watches, paper roundels to be cut out and fitted inside the back case of the watch.



A TABLE OF THE Equation of Days,

SHEWING

How much a good Pendulum Watch *ought to be faster or ſlower than a true* Sun-Dial, *every Day in the Year.*

Days.	Januar.	Februa.	March	April.	May.	June.	July.	Aug.	Sept.	Octob.	Nov.	Dec.
	Ms. Sec.	Ms. Sec.	M. Sec.	M. Sec.	M. Sec.	M. Sec.	M. Sec.	M. Sec.	Mi. Sec.	Mi. Sec.	Mi. Sec.	M. Sec.
1	8 52	14 46	10 08	0 46	4 12	1 02	4 52	4 42	3 40	13 15	15 29	5 53
2	9 14	14 45	9 51	0 30	4 14	0 49	4 59	4 32	4 00	13 28	15 21	5 25
3	9 36	14 43	9 34	0 14	4 14	0 36	5 06	4 21	4 21	13 42	15 11	4 57
4	9 58	14 40	9 17	0 01	4 14	0 24	5 13	4 11	4 42	13 55	15 02	4 27
5	10 19	14 36	9 00	0 17	4 14	0 11	5 20	4 00	5 03	14 08	14 51	3 57
6	10 38	14 32	8 42	0 32	4 13	0 01	5 27	3 48	5 24	14 20	14 40	3 28
7	10 58	14 27	8 24	0 46	4 12	0 14	5 33	3 36	5 45	14 32	14 27	2 59
8	11 17	14 21	8 06	1 00	4 10	0 27	5 37	3 23	6 06	14 43	14 14	2 30
9	11 35	14 14	7 47	1 14	4 08	0 40	5 41	3 10	6 26	14 53	14 00	2 00
10	11 52	14 07	7 28	1 28	4 05	0 53	5 44	2 56	6 47	15 03	13 46	1 29
11	12 09	14 00	7 09	1 40	4 02	1 07	5 48	2 42	7 08	15 12	13 30	0 59
12	12 26	13 52	6 50	1 52	3 58	1 [illegible]	5 51	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	12 40	13 43	6 32	2 04	3 54	1 33	5 54	2 12	[illegible]	15 29	12 56	0 02
14	12 53	13 33	6 13	2 16	3 48	1 46	5 55	1 56	8 09	15 36	12 38	0 32
15	13 06	13 23	5 54	2 27	3 43	1 58	5 56	1 40	8 29	15 42	12 18	1 02
16	13 18	13 12	5 36	2 37	3 37	2 11	5 56	1 23	8 49	15 48	11 59	1 32
17	13 30	13 01	5 17	2 47	3 30	2 23	5 56	1 07	9 09	15 53	11 39	2 01
18	13 42	12 49	4 58	2 57	3 23	2 36	5 55	0 50	9 29	15 57	11 18	2 31
19	13 51	12 36	4 38	3 06	3 15	2 49	5 54	0 33	9 49	16 00	10 56	3 00
20	13 59	12 23	4 19	3 15	3 07	3 01	5 52	0 15	10 08	16 02	[illegible] 34	3 29
21	14 08	12 10	4 01	3 23	2 59	3 12	5 50	0 03	10 26	16 04	10 [illegible]	3 57
22	14 16	11 56	3 42	3 30	2 51	3 23	5 47	0 22	10 44	16 05	9 48	4 25
23	14 23	11 42	3 23	3 37	2 43	3 34	5 43	0 41	11 02	16 05	9 24	4 53
24	14 29	11 28	3 05	3 43	2 33	3 45	5 39	1 00	11 20	16 05	8 59	5 20
25	14 33	11 13	2 46	3 49	2 22	3 55	5 34	1 19	11 37	16 04	8 34	5 48
26	14 37	10 57	2 28	3 54	2 10	4 06	5 28	1 38	11 54	16 01	8 08	6 15
27	14 41	10 41	2 11	3 58	1 58	4 16	5 22	1 58	12 11	15 58	7 42	6 41
28	14 44	10 25	1 53	4 02	1 46	4 25	5 15	2 18	12 28	15 54	7 14	7 07
29	14 45		1 36	4 06	1 34	4 34	5 07	2 39	12 44	15 49	6 47	7 33
30	14 46		1 19	4 08	1 24	4 44	4 59	2 59	13 00	15 43	6 20	7 58
31	14 46		1 02		1 13		4 51	3 19		15 36		8 22

SET the *Watch* ſo much faſter or ſlower than the time by the Sun, according to the Table for the Day of the Month, when you ſet it; and if the *Watch* go true, the difference of it from the Sun any Day afterward will be the ſame with the Table.

LONDON,

Printed for *Tho. Tompion*, Clockmaker, at the *Three Crowns* in *Fleet-ſtreet*, at *Water-lane* end. 1683

The clockmaker, Thomas Tompion's Equation table of 1683. Some 12 years later, the first Astronomer Royal, John Flamsteed, wrote to Sir Isaac Newton "Tompion's a true table of equations, but made for a particular year perhaps, fits not the present" !

One of two of John Harrison's personal regulators, with Equation Table in his own hand, printed on pendulum door. This clock was set, not by the sun, but by the stars viewed between Harrisson's door post and his neighbour's chimney.

Watch Papers. In the 18th C, many pocket watches would be supplied with such cut-out papers that could be folded to fit inside the back of the watch

AN UNKNOWN MECHANISM

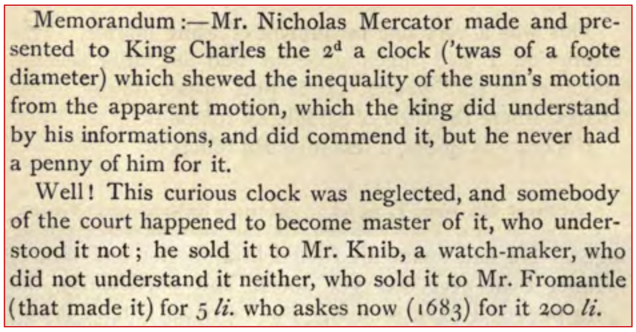
One of the earliest recorded Equation Clocks was designed by the mathematician, Nicholas Mercator (not Gerardus Mercator who invented the map projection), and made by Ahasuerus Fromanteel (whose son, John, bought pendulum technology from Amsterdam to London)

> "Next day, to the Royal Society, where one Mercator, an excellent Mathematician, produced his rare clock and new motions to perform the equations..."

wrote the diarist John Evelyn on 18 August 1666.

This may or may not be the clock described below...

NICHOLAS MERCATOR'S CLOCK



Memorandum :—Mr. Nicholas Mercator made and presented to King Charles the 2^{d} a clock ('twas of a foote diameter) which shewed the inequality of the sunn's motion from the apparent motion, which the king did understand by his informations, and did commend it, but he never had a penny of him for it.

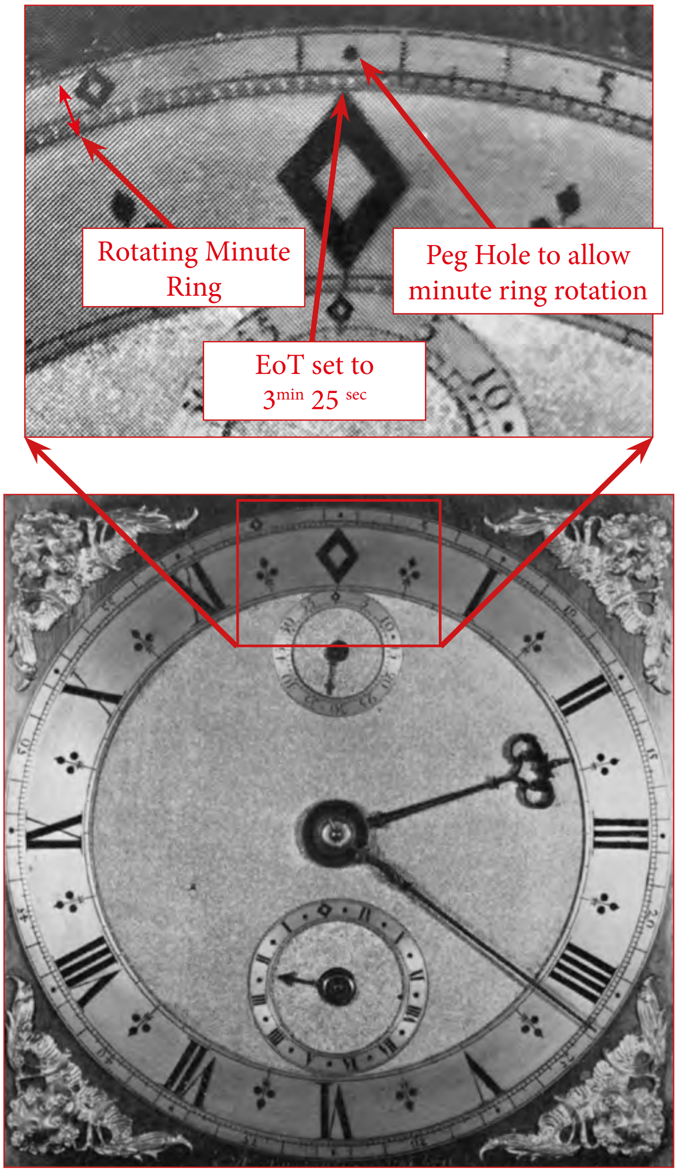
Well! This curious clock was neglected, and somebody of the court happened to become master of it, who understood it not; he sold it to Mr. Knib, a watch-maker, who did not understand it neither, who sold it to Mr. Fromantle (that made it) for 5 *li.* who askes now (1683) for it 200 *li.*

CLOCKS WITH A MANUALLY ADJUSTED HOUR RING

The first clearly known step in mechanically adjusting a clock was from Ahasuerus Fromanteel (1607–1693), whose son brought Huygens' pendulum technology from Amsterdam to London about a year after Huygens published his invention. Thus, he also understood the circular error in pendulum, which eventually led the adoption of the long seconds pendulum only swinging through a few degrees and the Anchor escapement.

In the clock face, shown the outer minutes ring (divided into 10-secs divisions) is manually adjustable for the EoT (using a printed table) by insertion of a pin into small peg holes. The smaller seconds dial is similarly adjustable. There was a friction fit for the moveable rings.

In this illustration, the seconds ring adjustment at 32^{secs} does not exactly match the minute ring adjustment at 3^{mins} 25^{secs}.



Fromanteel's manually adjusted minute ring ca 1680

Ref: "Some Outstanding Clocks over Seven Hundred Years" by H. Alan Lloyd, Leonard Hill Books, 1958.

return to "Table of Contents"

John Smith’s Tables

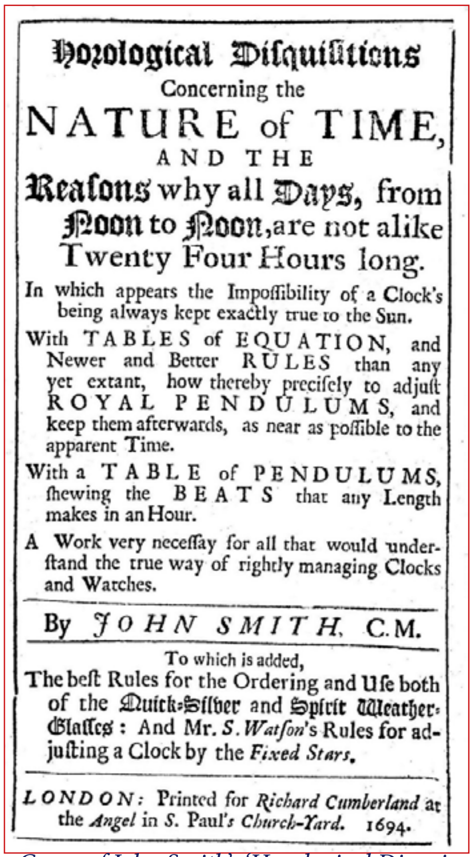
the author is grateful to Prof Frederick W. Sawyer III for the information in this section.

The Tables are designed to keep Clocks as close as possible to solar time

John Smith C.M. (1650 - 1730) was a colourful character. He was a clockmaker, hence the C.M. after his name. He wrote books. One was on the painting of sundials. Another extolled the heretical Socinian Controversy (by order of Parliament, these books was banned, ordered to be burnt and Smith forced to recant). Another on “The Curiosities of Common Water or the Advantages therefor in Preventing and Curing many Distempers”. This book went to 10 editions and was published in translation in both Germany & France!

His last books, however related Horological matters - especially to the lack of acceptance of mean time. He published a number of works on the Equation of Time. The last of these, in 1694, elucidated the 10 Rectifying Days of the year on which one would need to adjust one’s pendulum clock so that the clock would always indicate the time within 3½ minutes of Solar time. The title page of his book contains ....

> “With TABLES of EQUATION, and Newer and Better RULES than any yet extant, how thereby precisely to adjust ROYAL PENDULUMS, and keep them afterwards, as near as possible to the apparent (= Solar or God’s) Time.



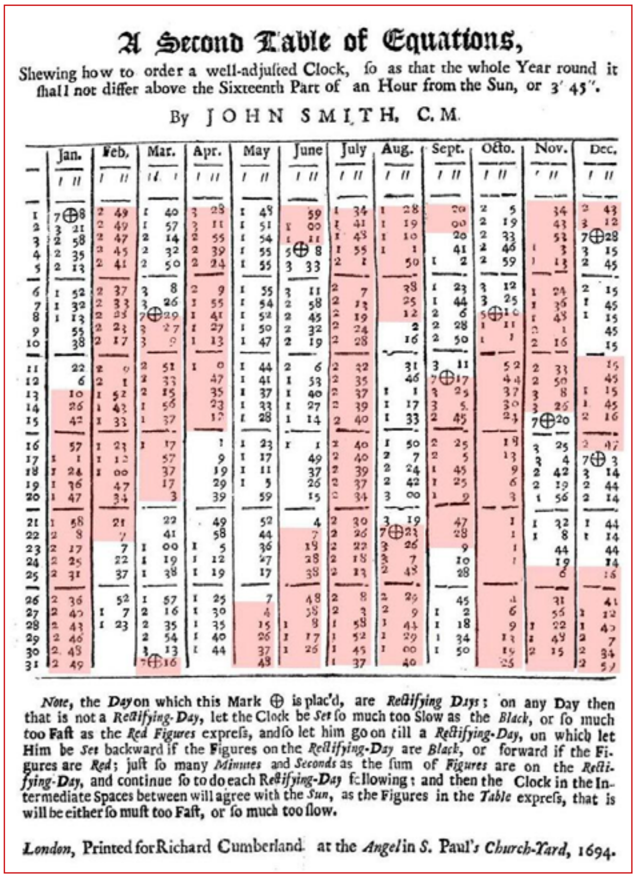
Horological Diſquiſitions
Concerning the
NATURE of TIME,
AND THE
Reaſons why all Days, from Noon to Noon, are not alike Twenty Four Hours long.
In which appears the Impoſſibility of a Clock's being always kept exactly true to the Sun.
With TABLES of EQUATION, and Newer and Better RULES than any yet extant, how thereby preciſely to adjuſt ROYAL PENDULUMS, and keep them afterwards, as near as poſſible to the apparent Time.
With a TABLE of PENDULUMS, ſhewing the BEATS that any Length makes in an Hour.
A Work very neceſſay for all that would underſtand the true way of rightly managing Clocks and Watches.

By JOHN SMITH, C.M.

To which is added,
The beſt Rules for the Ordering and Uſe both of the Quick-Silver and Spirit Weather-Glaſſes: And Mr. *S. Watſon*'s Rules for adjuſting a Clock by the *Fixed Stars*.

LONDON: Printed for *Richard Cumberland* at the *Angel* in *S.* Paul's *Church-Yard*. 1694.

Cover of John Smith’s ‘Horological Disquisitions concerning the Nature of Time’



A Second Table of Equations,

Shewing how to order a well-adjuſted Clock, ſo as that the whole Year round it ſhall not differ above the Sixteenth Part of an Hour from the Sun, or 3′ 45″.

By JOHN SMITH, C.M.

	Jan. ′ ″	Feb. ′ ″	Mar. ′ ″	Apr. ′ ″	May ′ ″	June ′ ″	July ′ ″	Aug. ′ ″	Sept. ′ ″	Octo. ′ ″	Nov. ′ ″	Dec. ′ ″
1	7 ⊕ 8	2 49	1 40	3 28	1 48	59	1 34	1 28	20	2 5	34	2 43
2	3 21	2 49	1 57	3 11	1 51	1 00	1 41	1 19	00	2 19	43	3 12
3	2 58	2 47	2 14	2 55	1 54	1 11	1 48	1 10	20	2 33	53	7 ⊕ 28
4	2 35	2 45	2 32	2 39	1 55	5 ⊕ 8	1 55	1 1	41	2 46	1 3	3 15
5	2 13	2 41	2 50	2 24	1 55	3 33	2 1	50	1 2	2 59	1 13	2 45
6	1 52	2 37	3 8	2 9	1 55	3 11	2 7	38	1 23	3 12	1 24	2 15
7	1 32	2 33	3 26	1 55	1 54	2 58	2 13	25	1 44	3 25	1 36	1 45
8	1 13	2 28	7 ⊕ 29	1 41	1 52	2 45	2 19	12	2 6	5 ⊕ 10	1 48	1 15
9	55	2 23	3 27	1 27	1 50	2 32	2 24	2	2 28	1 11	2 1	45
10	38	2 17	3 9	1 13	1 47	2 19	2 28	16	2 50	1 1	2 16	15
11	22	2 9	2 51	1 0	1 44	2 6	2 32	31	3 11	52	2 33	15
12	6	2 1	2 33	47	1 41	1 53	2 35	46	7 ⊕ 17	44	2 50	45
13	10	1 52	2 15	35	1 37	1 40	2 37	1 1	3 25	37	3 8	1 15
14	26	1 43	1 56	23	1 33	1 27	2 39	1 17	3 5	30	3 26	1 45
15	42	1 33	1 37	12	1 28	1 14	2 40	1 33	2 45	23	7 ⊕ 20	2 16
16	57	1 23	1 17	1	1 23	1 1	2 40	1 50	2 25	18	3 25	2 47
17	1 1	1 12	57	9	1 17	49	2 40	2 7	2 5	13	3 4	7 ⊕ 3
18	1 24	1 00	37	19	1 11	37	2 39	2 24	1 45	9	2 42	3 14
19	1 36	47	17	29	1 5	26	2 37	2 42	1 25	6	2 19	2 44
20	1 47	34	3	39	59	15	2 34	3 00	1 9	3	1 56	2 14
21	1 58	21	22	49	52	4	2 30	3 19	47	1	1 32	1 44
22	2 8	7	41	58	44	7	2 26	7 ⊕ 23	28	1	1 8	1 14
23	2 17	7	1 00	1 5	36	18	2 22	3 26	9	1	44	44
24	2 25	22	1 19	1 12	27	28	2 18	3 7	10	1	19	14
25	2 31	37	1 38	1 19	17	38	2 13	2 48	28		6	16
26	2 36	52	1 57	1 25	7	48	2 8	2 29	45	4	31	41
27	2 40	1 7	2 16	1 30	4	58	2 3	2 9	1 2	6	55	1 12
28	2 43	1 23	2 35	1 35	15	1 8	1 58	1 48	1 18	9	1 22	1 42
29	2 46		2 54	1 40	26	1 17	1 52	1 29	1 34	13	1 49	2 7
30	2 48		3 13	1 44	37	1 26	1 45	1 00	1 50	19	2 15	2 34
31	2 49		7 ⊕ 16		48		1 37	40		25		2 57

Note, the *Day* on which this Mark ⊕ is plac'd, are *Rectifying Days*; on any Day then that is not a *Rectifying-Day*, let the Clock be *Set* ſo much too Slow as the *Black*, or ſo much too Faſt as the *Red Figures* expreſs, and ſo let him go on till a *Rectifying-Day*, on which let Him be *Set* backward if the Figures on the *Rectifying-Day* are *Black*, or forward if the Figures are *Red*; juſt ſo many *Minutes* and *Seconds* as the ſum of *Figures* are on the *Rectifying-Day*, and continue ſo to do each *Rectifying-Day* following; and then the Clock in the Intermediate Spaces between will agree with the *Sun*, as the Figures in the *Table* expreſs, that is will be either ſo much too Faſt, or ſo much too ſlow.

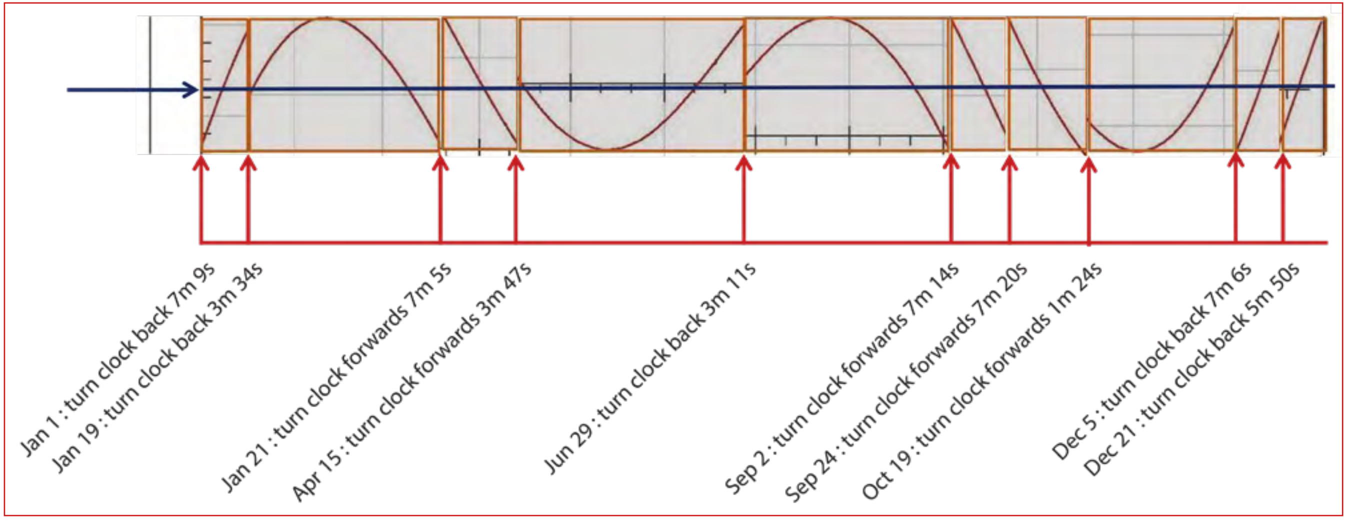
London, Printed for Richard Cumberland at the *Angel* in *S.* Paul's *Church-Yard*, 1694.

John Smith CM’s table of Rectifying Days. The figures in the pink areas were originally printed in red

Note, the Day on which is the Mark is plac’d are ***Rectifying Days***; on any Day then that is not a ***Rectifying Day***, let the Clock be Set so much Slow as the ***Black***, or so much to Fast as the ***Red*** Figures express, and so let him go on till a ***Rectifying Day*** on which let him be Set backward if the Figures on the ***Rectifing-Day*** are ***Black***, or forward if the figures are ***Red***, just so many ***Minutes*** and ***Seconds*** as the sum of ***Figures*** are on the ***Rectifing Day***; just so many ***Minutes*** and ***Seconds*** as the sum of ***Figures*** are on the ***Recifying-Day*** and continue so to do each ***Rectifying-Day*** following; and then the Clock in the Intermediate Spaces between will agree with the ***Sun***, as the Figures in the ***Table*** express, that is will be either so much too Fast, or so much too slow.

The Instructions are somewhat unclear to the modern reader. However, they have been deciphered, bought up to date and improved Prof Frederick W. Sawyer III. Following his rules, one’s clock will be within a few seconds of God’s time

- On 1st January, one must set one’s clock to match Apparent (=Solar) Time. Set the clock back by 7m 9s
- On 19th Jan, set the clock back a further 3m 34s
- On 21st Jan, set the clock forward 7m 5s
- etc. etc. See overleaf



Sawyer's improvement on John Smith's Rectifying Days

VARIABLE PENDULUM LENGTH

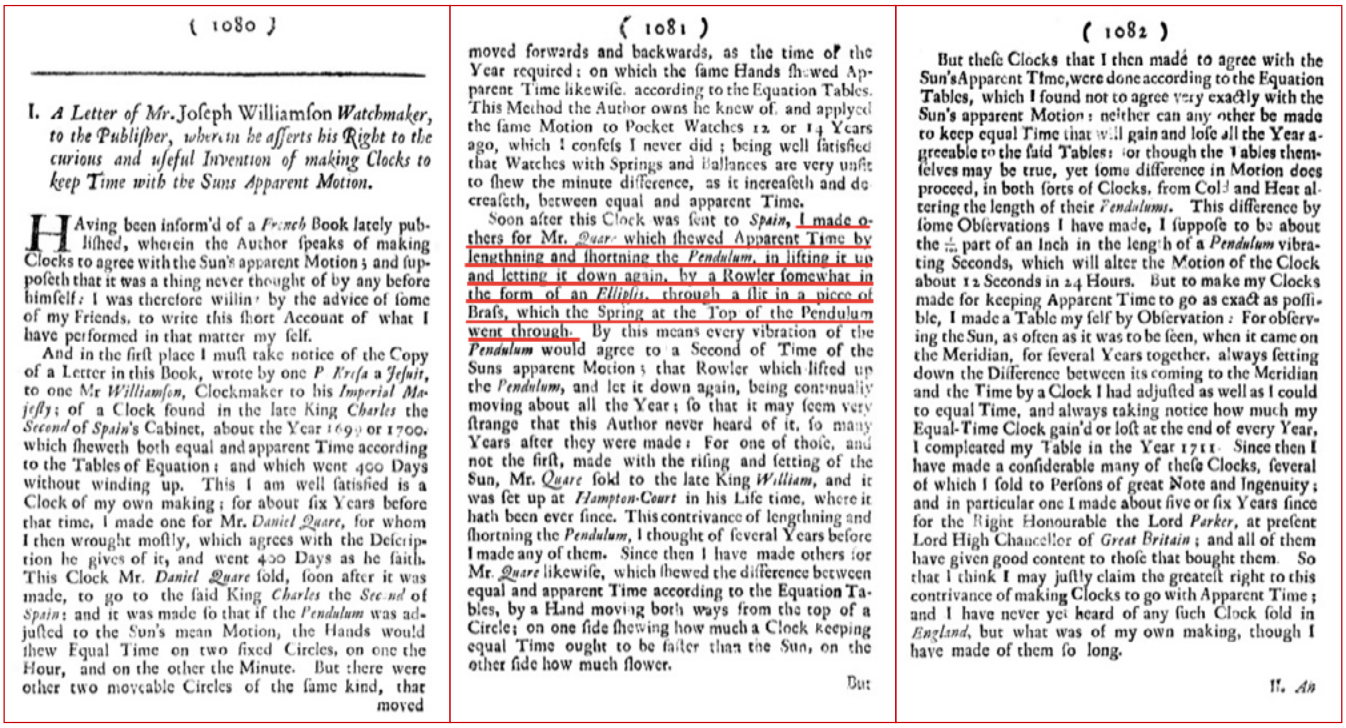
Joseph Williamson - who worked for the famous clockmaker Daniel Quare, in the letter that follows suggests that he was father of all Equation mechanisms. The letter was written to the Royal Society in order protect his name, He does however claim ...

> "I have made others for Mr Quar(e), which showed Apparent Time by lengthening and shortening the Pendulum in lifting it up and setting it down again, by a Rowler somewhat in the form of an Ellipsis, through a slit in piece of brass, which the spring at the top of the pendulum went through".

Manual adjustment of a pendulum by moving the suspension spring through a slit was a common method used later to adjust clocks for temperature variation or wear & tear.

The following is a quotation on Variable Pendulum Clocks from the Encyclopédie edited by Denis Diderot and Jean le Rond d'Alembert in the 18th century.

> Father Alexandre's year wheel carries an ellipse on which rests a lever that carries the pendulum, suspended by a spring that fits closely through a slit in a cock, like conventional seconds pendulums. The spring can rise and fall in the slit; the cock is the center of oscillation, and it is mounted on the movement plate. To produce the apparent variations of the sun, Father Alexandre makes the pendulum longer and shorter by means of the ellipse; its diameters are determined in relation to the lengthening or shortening the pendulum requires in order to gain or lose a given amount in 24 hours. He gives extensive details on the subject on p .147 of his book.



(1080)

I. *A Letter of Mr.* Joſeph Williamſon *Watchmaker, to the Publiſher, wherein he aſſerts his Right to the curious and uſeful Invention of making Clocks to keep Time with the Suns Apparent Motion.*

HAving been inform'd of a *French* Book lately publiſhed, wherein the Author ſpeaks of making Clocks to agree with the Sun's apparent Motion; and ſuppoſeth that it was a thing never thought of by any before himſelf: I was therefore willing by the advice of ſome of my Friends, to write this ſhort Account of what I have performed in that matter my ſelf.

And in the firſt place I muſt take notice of the Copy of a Letter in this Book, wrote by one *P. Krefa* a *Jeſuit*, to one Mr *Williamſon*, Clockmaker to his *Imperial Majeſty*; of a Clock found in the late King *Charles* the *Second* of *Spain*'s Cabinet, about the Year 1699 or 1700. which ſheweth both equal and apparent Time according to the Tables of Equation; and which went 400 Days without winding up. This I am well ſatisfied is a Clock of my own making; for about ſix Years before that time, I made one for Mr. *Daniel Quare*, for whom I then wrought moſtly, which agrees with the Deſcription he gives of it, and went 400 Days as he ſaith. This Clock Mr. *Daniel Quare* ſold, ſoon after it was made, to go to the ſaid King *Charles* the *Second* of *Spain*: and it was made ſo that if the *Pendulum* was adjuſted to the Sun's mean Motion, the Hands would ſhew Equal Time on two fixed Circles, on one the Hour, and on the other the Minute. But there were other two moveable Circles of the ſame kind, that

moved

(1081)

moved forwards and backwards, as the time of the Year required: on which the ſame Hands ſhewed Apparent Time likewiſe, according to the Equation Tables. This Method the Author owns he knew of, and applyed the ſame Motion to Pocket Watches 12 or 14 Years ago, which I confeſs I never did; being well ſatisfied that Watches with Springs and Ballances are very unfit to ſhew the minute difference, as it increaſeth and decreaſeth, between equal and apparent Time.

Soon after this Clock was ſent to *Spain*, I made others for Mr. *Quare* which ſhewed Apparent Time by lengthning and ſhortning the *Pendulum*, in lifting it up and letting it down again, by a Rowler ſomewhat in the form of an *Ellipſis*, through a ſlit in a piece of Braſs, which the Spring at the Top of the Pendulum went through. By this means every vibration of the *Pendulum* would agree to a Second of Time of the Suns apparent Motion; that Rowler which lifted up the *Pendulum*, and let it down again, being continually moving about all the Year; ſo that it may ſeem very ſtrange that this Author never heard of it, ſo many Years after they were made: For one of thoſe, and not the firſt, made with the riſing and ſetting of the Sun, Mr. *Quare* ſold to the late King *William*, and it was ſet up at *Hampton-Court* in his Life time, where it hath been ever ſince. This contrivance of lengthning and ſhortning the *Pendulum*, I thought of ſeveral Years before I made any of them. Since then I have made others for Mr. *Quare* likewiſe, which ſhewed the difference between equal and apparent Time according to the Equation Tables, by a Hand moving both ways from the top of a Circle; on one ſide ſhewing how much a Clock keeping equal Time ought to be faſter than the Sun, on the other ſide how much ſlower.

But

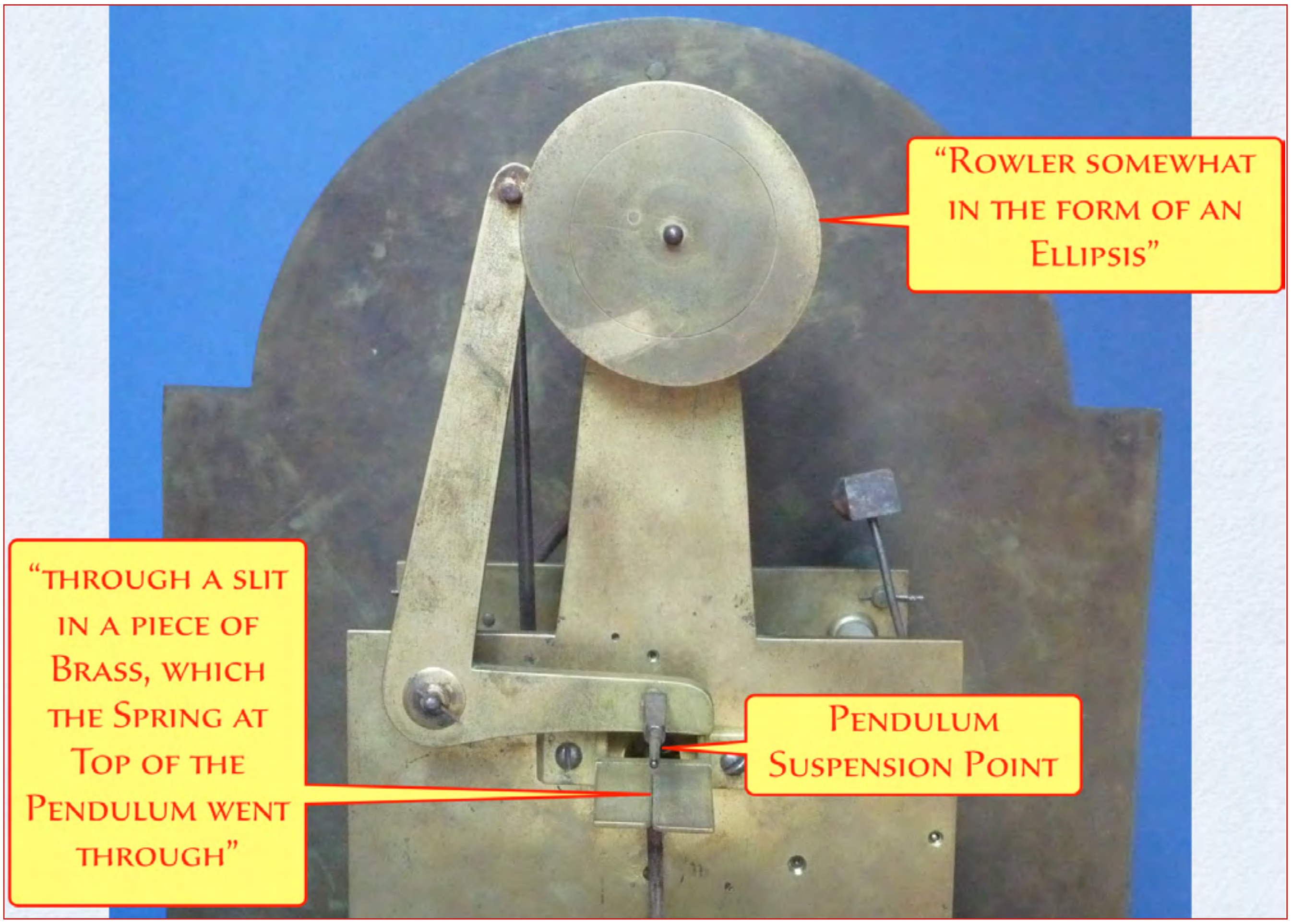
(1082)

But theſe Clocks that I then madé to agree with the Sun's Apparent Time, were done according to the Equation Tables, which I found not to agree very exactly with the Sun's apparent Motion: neither can any other be made to keep equal Time that will gain and loſe all the Year agreeable to the ſaid Tables: for though the Tables themſelves may be true, yet ſome difference in Motion does proceed, in both ſorts of Clocks, from Cold and Heat altering the length of their *Pendulums*. This difference by ſome Obſervations I have made, I ſuppoſe to be about the $\frac{1}{100}$ part of an Inch in the length of a *Pendulum* vibrating Seconds, which will alter the Motion of the Clock about 12 Seconds in 24 Hours. But to make my Clocks made for keeping Apparent Time to go as exact as poſſible, I made a Table my ſelf by Obſervation: For obſerving the Sun, as often as it was to be ſeen, when it came on the Meridian, for ſeveral Years together, always ſetting down the Difference between its coming to the Meridian and the Time by a Clock I had adjuſted as well as I could to equal Time, and always taking notice how much my Equal-Time Clock gain'd or loſt at the end of every Year, I compleated my Table in the Year 1711. Since then I have made a conſiderable many of theſe Clocks, ſeveral of which I ſold to Perſons of great Note and Ingenuity; and in particular one I made about five or ſix Years ſince for the Right Honourable the Lord *Parker*, at preſent Lord High Chancellor of *Great Britain*; and all of them have given good content to thoſe that bought them. So that I think I may juſtly claim the greateſt right to this contrivance of making Clocks to go with Apparent Time; and I have never yet heard of any ſuch Clock ſold in *England*, but what was of my own making, though I have made of them ſo long.

II. *An*

Joseph Williamson's letter to the Royal Society asserting his claim to have invented variable length Pendulum Clock

return to "Table of Contents"



Willamson's Variable Length Pendulum Mechanism

Photo by Andrew James, used with permission

His theory certainly has the merit of simplicity, but to endorse it we must ignore its inherent practical disadvantages. A single error brings the whole structure down; the slightest error in the curve will produce a noticeable variation in the hands. Assume that inaccuracy in the ellipse makes the pendulum too short by 1/12 ligne; it will gain 12 seconds in 24 hours, etc., every beat during that time will be shorter than necessary, and this error multiplied by the numbers will give 12 seconds for just 1 point, and the same for every day. Moreover, this method is not practical with today's heavy pendulums, whose virtues have been well demonstrated in our time by M. de Rivaz.

Finally, I do not see the advantage of a pendulum that divides time into unequal parts only. However, it was appropriate to mention this design, for the understanding it gives of the whole matter of equation; in addition, I am convinced that knowledge of all kinds of mechanisms is a great help in obtaining certain results in other constructions, even though they have no obvious connection with what inspired the initial idea. Nothing must be overlooked regarding the mechanical arts, but we must always assume understanding in the person who makes a new application to other objects.

translation from Encyclopedia of Diderot & d'Alembert
https://quod.lib.umich.edu/d/did/index.html

return to "Table of Contents"

Equation Clocks - Cams

The Invention of the Kidney Cam

The true mechanical equation clock was made possible by the kidney cam. This cam's shape encoded the Equation of Time and, if rotated once a year, could power a clock dial or hand to show solar time.

Whether the kidney-cam was invented by Christian Huygens or Robert Hook is open to debate. Certainly Huygens, whose brother, Constantijn, Dutch ambassador in London and was in touch with London's clockmakers, claimed the distinction. On the other hand, Robert Hook also claims to have been the inventor. He was an experimental genius second to none, with a huge interest in horology and Thomas Tompion was his friend and disciple. It was the manufacture of a perfect quadrant that Tompion made to Hook's design in 1674, that propelled the former into the London's intellectual & social limelight.

Cam Clocks with Rotating Minute Ring

There is argument as to who pioneered these clocks, whether Thomas Tompion for the King Charles II of England or Joseph Williamson/Danial Quare for King Charles II of Spain. The latter clock was lost, probably in a fire in the Alcazar in Madrid in 1734.

Illustrated here is that of Tompion. Clocks of this quality run for a whole year on a single winding.

Robert Hooke - 1635-1703. There is no certain portrait of Hooke. This 'Portrait of a Mathematician' by Mary Beale ca: 1680 may or may not be Hooke. It does not fit contemporary descriptions of him as "in person, but despicable" : "melancholy, mistrustful, and jealous" : "cantankerous, envious, vengeful": "somewhat crooked, pale faced"

Christina Huygens - 1629-1696

Thomas Tompion - 1639-1713

Illustrations are from
Royal Collection, www.rct.uk/collection.
R.W. Symonds, 'Thomas Tompion, His Life & Work'
Batsford Press 1991
H. Alan Lloyd, 'Some Outstanding Clocks over 700 Years'
Leonard Hill (Books) 1958

The equation mechanism of the Tompion clock had a kidney cam attached behind the Date dial. At the other end of the cam follower, via a pivot, is a gear segment which rotates the apparent minute ring. This can be seen overleaf in the 1703 Tompion and Banger clock made for King George III.

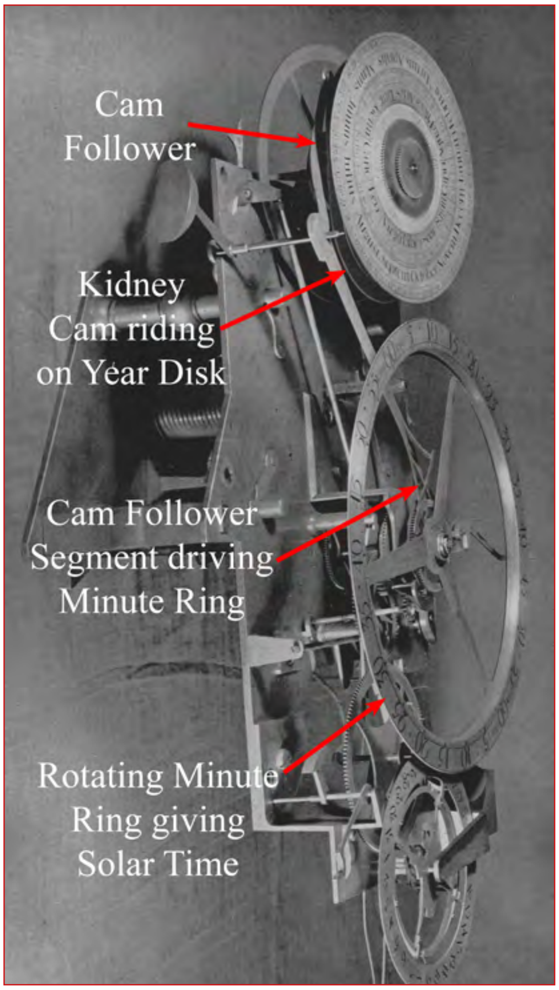
The latter clock bears the inscription under the dial "Tho Tompion London Invent" - which is the basis for the assumption that Hooke/Tompion invented the use of the kidney cam. Edward Banger worked for/with Tompion and married the latter's niece

Tompion - 1695 - Morning Room, Clarence House - Note at Noon on the inner Mean Time Ring, the outer Solar Time Ring reads 54 minutes

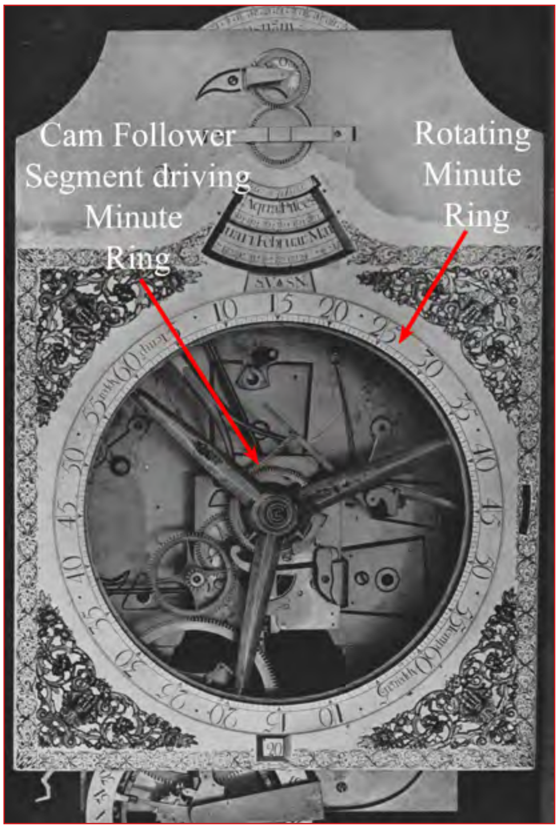
return to "Table of Contents"

Tompion & Banger - 1703 - Royal Collection. Showing the static Mean Time dial and the rotating Solar Time ring. Note the date is 1st January Julian calendar, being 11th Jan Gregorian, and the difference between the two scales is some 6 1/2 mins - which is about the right value for the Equation on that date.

Detail of Tompion & Banger - 1703. Note the Date scale reads 2nd April Julian (SV) and 13th April Gregorian (SN). On this day, the clock gives an EoT of -1 min 20 secs. Tompion's own table (see page 91) for this date in 1683 gives an EoT of 2 mins 4 secs



Tompion & Banger - 1703 - Showing workings Cam & Follower



Tompion & Banger - 1703 - Showing Cam Follower Driving Minute Ring

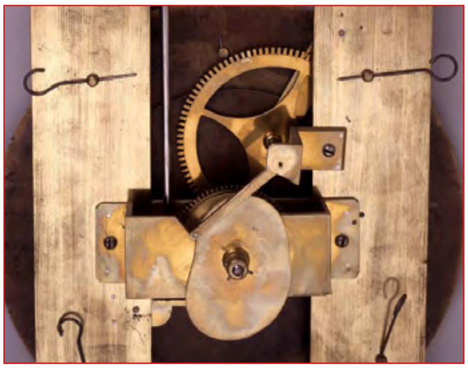
return to "Table of Contents"

CAM CLOCKS WITH A SEPARATE FACE INDICATING THE EOT AS CLOCK FASTER/SLOWER

Joseph Williamson working for Danial Quare made a number of these type of clocks. Quare made the main clock, Williamson made the equation mechanism that was a separate device. The most magnificent is held in the British Library - shown below.

Danial Quare/Joseph Williamson - 1710 - British Museum.

The secondary Equation & Calendar Dial. The Long hand indicates the Date- the Sun indicates the Equation of Time, as Clock Fast or Clock Slow

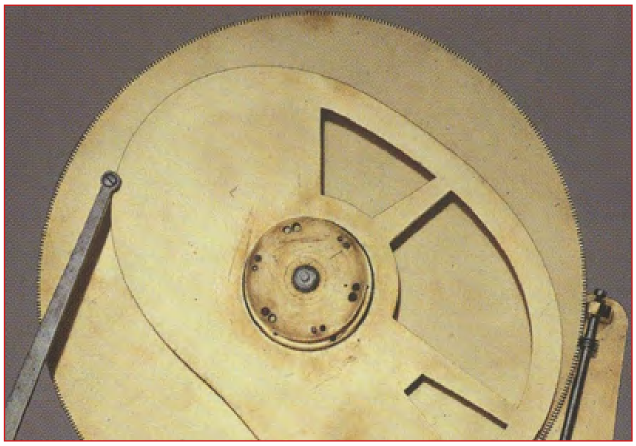


The vertical shaft comes from above driven by the main clock mechanism giving the Date directly and the Equation of Time via the kidney cam.

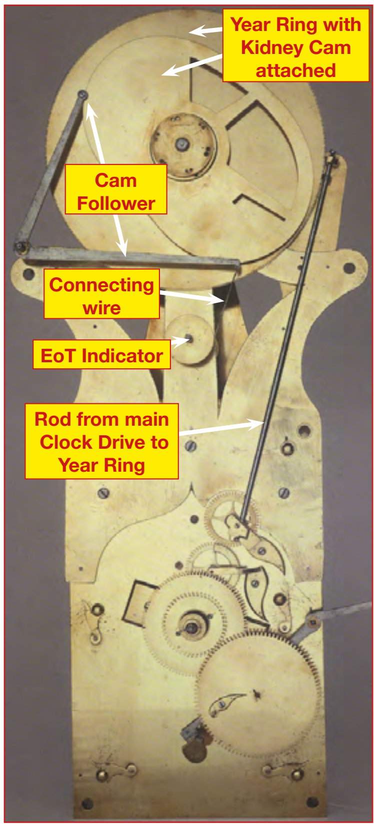
Another example is Tompion's clock and sundial made for the city of Bath, in recognition of him being made a freeman of the city. It is of interest since it is only known clock and sundial pair

Tompion - 1707 - Pump Room, City of Bath

Tompion's Sundial on a window sill, close to the clock. The sundial and the Equation of Time indication were used to set the clock



Equation Mechanism from Front

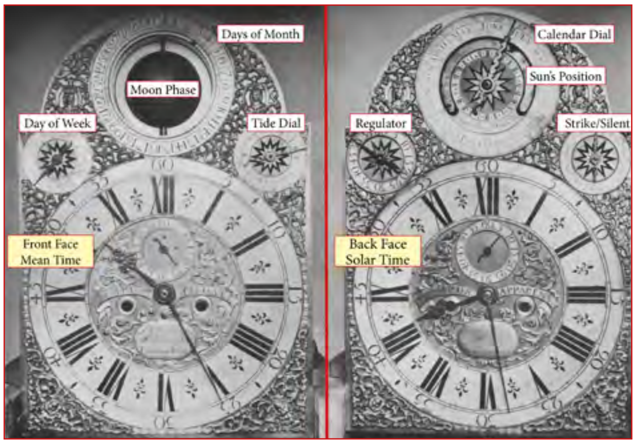


Equation Mechanism from Rear

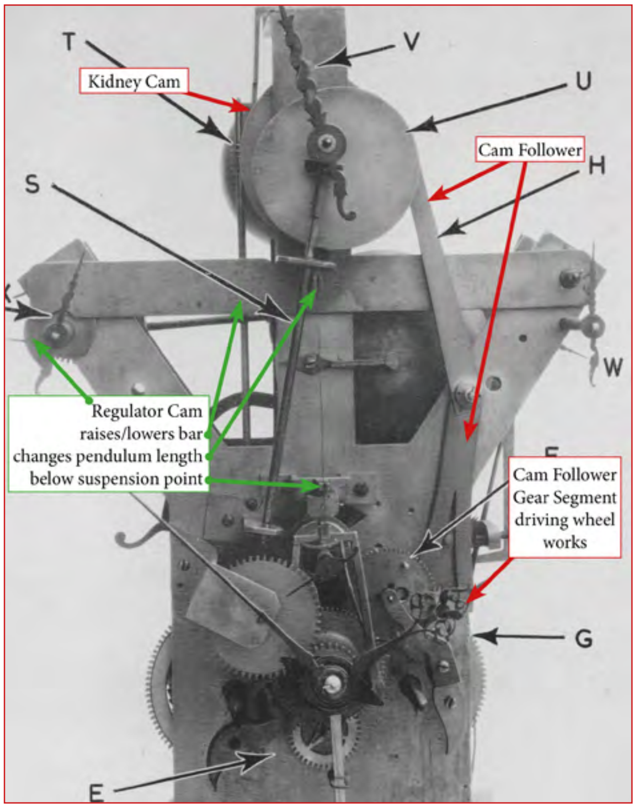
return to "Table of Contents"

CAM CLOCKS ADDING THE EQUATION TO MEAN TIME TO GIVE MEAN AND APPARENT TIME

The next major improvement came, once more, from Joseph Williamson in 1720, when he succeeded in mechanically adding the Equation of Time to pendulum derived Local Mean Time to produce a clock that indicated Solar Time. This marks the invention of differential gearing to mechanically sum two rotary motions together. ‘The Differential Adder’ on page 110. The clock had two faces back-to-back - one indicating mean time, the other solar time. The clock therefore was designed not to be placed against a wall.

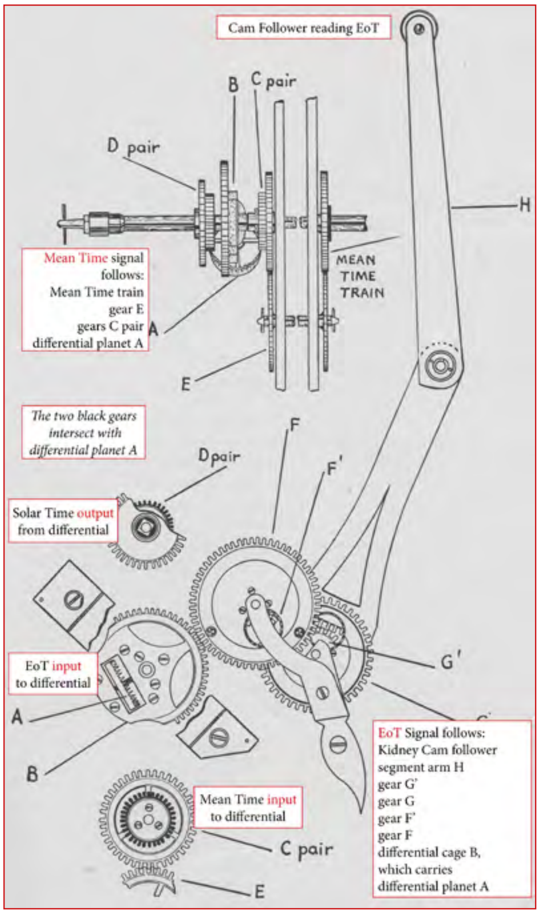


Danial Quare/Joseph Williamson - 1710.
Front & Back Faces



Regulator Cam
raises/lowers bar
changes pendulum length
below suspension point

Danial Quare/Joseph Williamson - 1710
Rear Solar Time Face, showing
a) in Red the Kidney Cam, Follower & Segment Arm
b) in Green, the pendulum length adjustment mechanism.

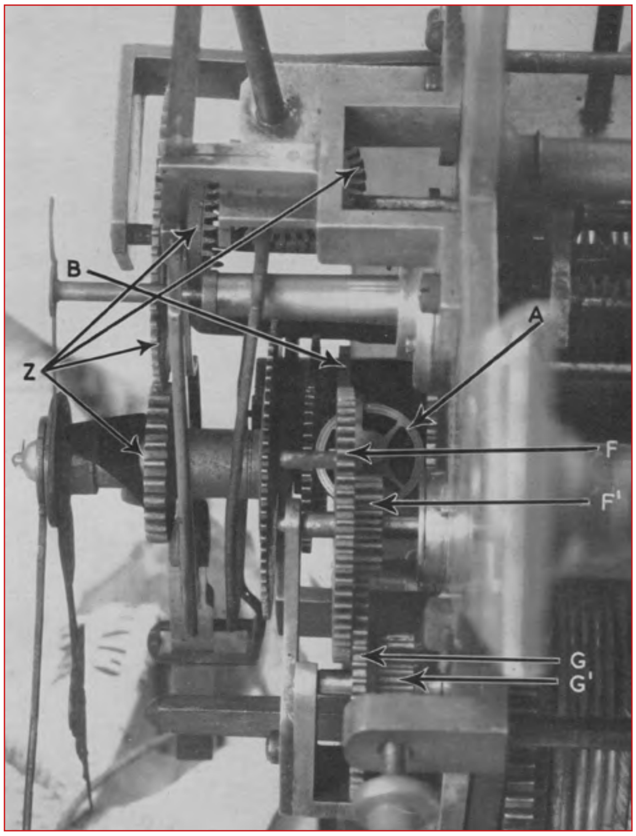


Mean Time signal
follows:
Mean Time train
gear E
gears C pair
differential planet A

The two black gears
intersect with
differential planet A

EoT Signal follows:
Kidney Cam follower
segment arm H
gear G'
gear G
gear F'
gear F
differential cage B,
which carries
differential planet A

Details of the Differential Adder.



Danial Quare/Joseph Williamson - 1710
Details of the Differential Adder. The gear names as the same as the diagram on the left.

Two other examples form the British Royal Collection - showing mean & solar time. These were complex astronomical clocks - specifically made to the specifications of King George III, around the time of the 1770's transits of Venus. The King was avidly interested in astronomy and time-keeping.

See www.rct.uk/collection

Eardley Norton - 1765 for King George III - Royal Library Windsor. Front Face of Astronomical Clock. Note the 24 hour dial with golden solar hand, black mean time hand and position of the sun in the sky in the background.

Christopher Pinchbeck - 1768

Note again the golden solar hand, black mean time hand. This photo was taken at 10:45 am, when the background golden sun, corresponds with the hour hand.

THE LAST SIGNIFICANT IMPROVEMENT

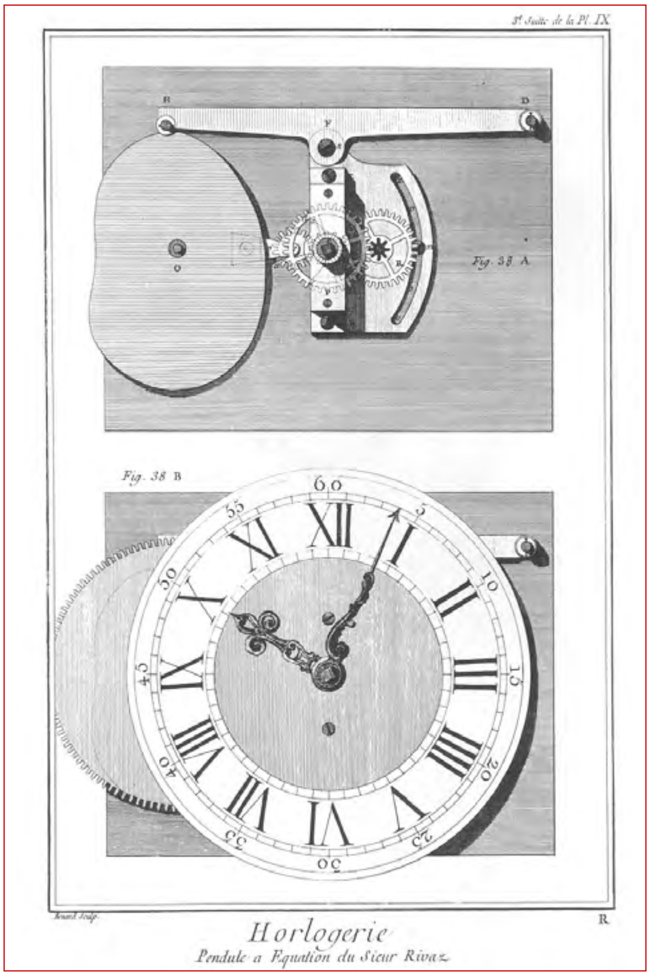
Abraham Louis Breguet - 1747-1823

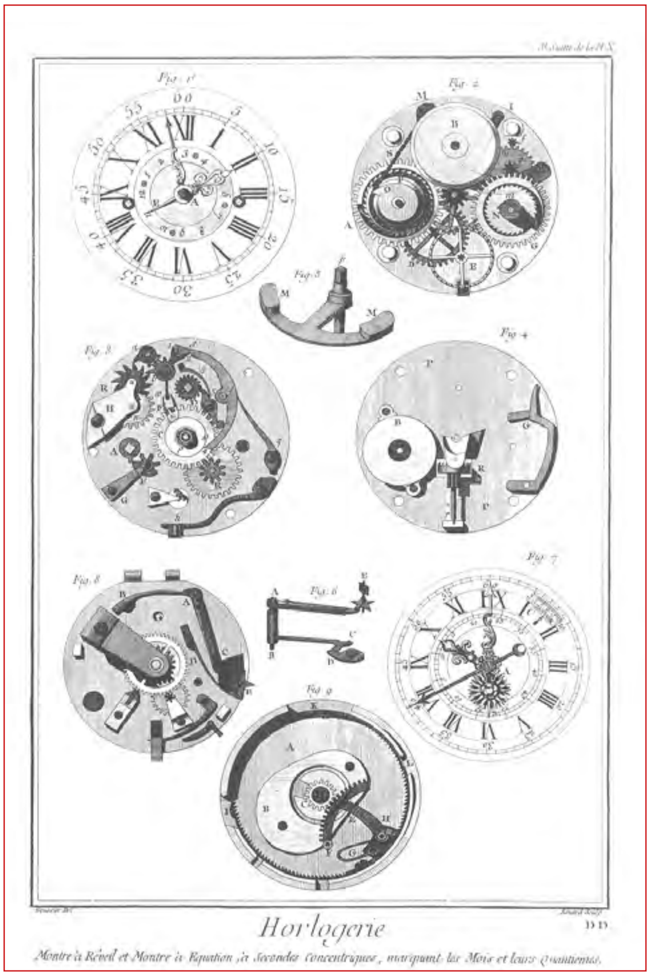
Further refinements were made in a watch by the master watchmaker, Abraham-Louis Breguet in 1793. These involved hourly sampling of the Kidney Cam to avoid continuous drag between the cam & the mechanism as a whole. See his exquisite Marie-Antoinette watch below and a recent re-creation of the mechanism by the Breguet company.

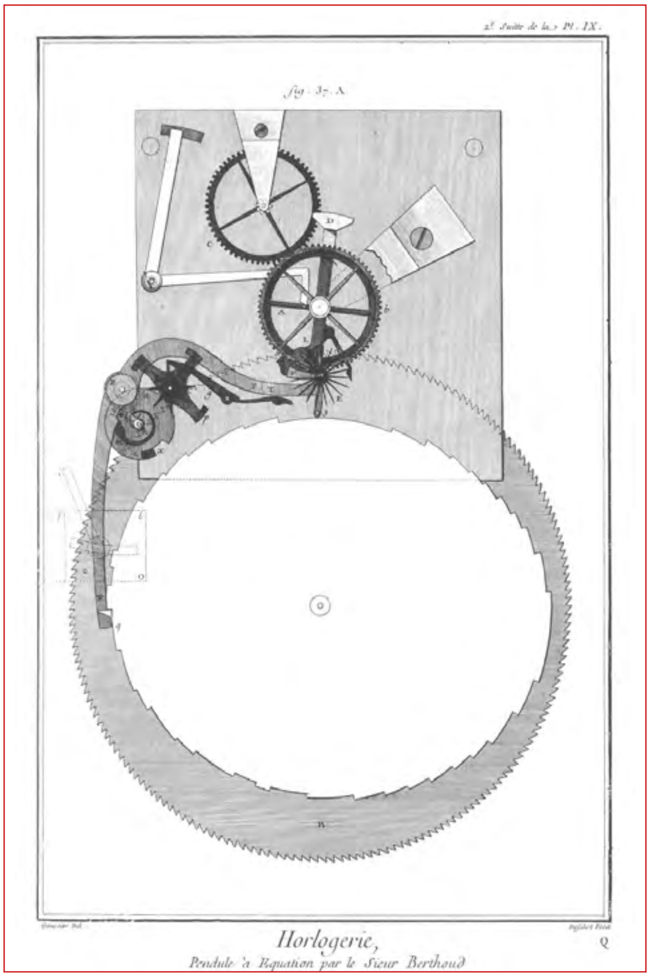
Breguet No. 160 "The Grand Complication," commonly known as the Marie-Antoinette. The cam is visible, but not the intermittent sampler.

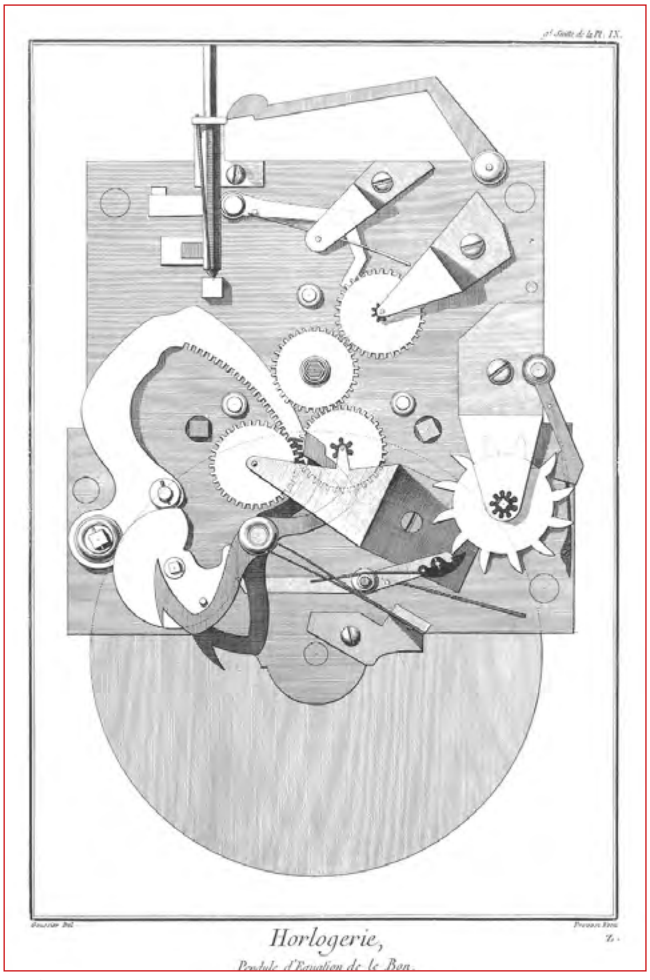
The scythe-shaped cam follower arm is engaged to the kidney cam by the spring at the bottom of the image. This spring is normally disengaged from the follower arm by the small crescent shape in the centre. This rotates to intermittently, releasing the spring and allowing an EoT reading to be taken

www.breguet.com.



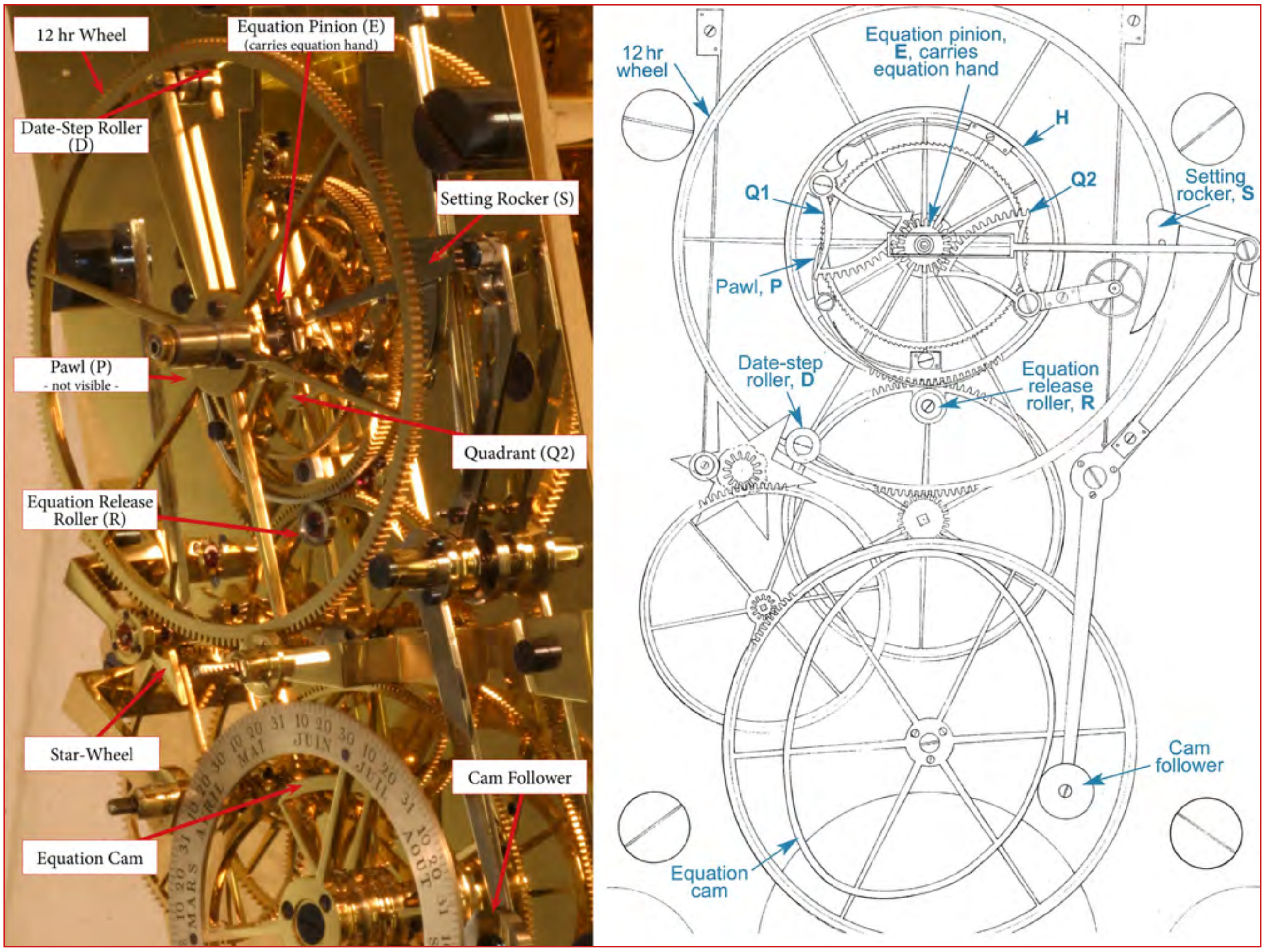






Four of the Plates from Encyclopédie of Diderot and d'Alembertin the section on Horlogerie - all showing the use of a cam

return to "Table of Contents"



Buchanan's Deryck Noakes' Clock

Buchanan's Deryck Noakes' Clock

See www.buchananclocks.com

"Conversion from solar mean time to true solar time is via a conventional kidney cam. As in Breguet's clock the correction is sampled once an hour and the offset left locked, with no force or drag on the kidney cam, until the next sample is taken.

The mechanism, below, is based on a pair of quadrants, Q1, Q2, mounted within the hour wheel, H, which offset the equation pinion, E, on which is mounted the equation hand, in solid gold. The offset this achieves is frozen by a long-tailed pawl, P, which locks in the finely toothed rim of the equation wheel and is realised when the tail of the pawl arm contacts the release roller, R, on the adjacent intermediate wheel, to which it is geared 1:1 wheel. When the pawl is released, the spring on the anti-backlash quadrant, Q1, tries to rotate the equation pinion anti-clockwise but is controlled by its twin, Q2, and its contact with rocker, S, pivoted onto the equation cam. As the equation release roller is small, and as it and the tail of the pawl are both mounted in the periphery of adjacent wheels, contact friction is minimal and contact time is short, probably about a minute.

This arrangement has the advantage that the relationship between the two can be controlled with precision by moving one wheel relative to the other a tooth at a time.

The date-step roller, D, on the 12-hour wheel indexes a star-wheel twice per day, which through a train of (6/15 x(100/12) x 219 ensures that kidney cam rotates once per year"

Quoted from Horological Journal April 2006, with permission

THE FRICTIONLESS LAST IMPROVEMENT

The only significant improvement over Breuget's cam sampling method can be found in one of Greubel Forsey's Tourbillon Watches. See following page for illustration.

In this case, the traditional kidney shaped cam is replaced by the strange semi-transparent shaped object with coloured edges. This is an integral part of month indicating ring. There is no cam follower, so technically it is not a cam. But with no follower, there no frictional drag involved with the reading of the Equation of Time. When the edge of the indicator is red, the value read is positive on the scale, when blue, it is negative.

return to "Table of Contents"

The only weakness in this otherwise elegant and interesting mechanism, is that the EoT indication is on the back side of the watch!

Greubel Forsey
'QP à Equation'

costing
ca ¾ million $US

Greubel Forsey Equation Cam.
If the colour of the edge of the cam is Red,
the scale should read as +ve :
if Blue -ve

EQUATION WATCHES

The Equation of Time lives on as a 'complication' in modern ultra hight end watches. They are beautiful and wonders of design, mechanical genius and horological excellence. But, in the greater scheme of things, of little importance - not one contains the longitude correction that might make the readings of practical use.......! In particular, because none of the four illustrated gives the correct value of EoT. This is not really surprising given the tiny scale of the cams used.

Panerai L'Astronomo Luminor 1950 Tourbillon.

EoT : 4 mins incorrect for indicated date

Audemars Piguet Jules Audemars Equation of Time.
EoT : 4 mins incorrect for indicated date

Breguet Marine Equation Marchante
EoT by 'Sun' hand : almost correct for indicated date

Jaquet Droz La Chaux-de-Fonds Equation of Time
EoT : 1 min incorrect for indicated date

return to "Table of Contents"

Equation Clocks - Gears

With the invention of the differential gear adder, output from a kidney cam could be added to mean time to give a clock dial directly reading both mean and solar time. But a number of other mechanisms have been incorporated into astronomical clocks

RITTENHOUSE'S CLOCK AT DREXEL UNIVERSITY

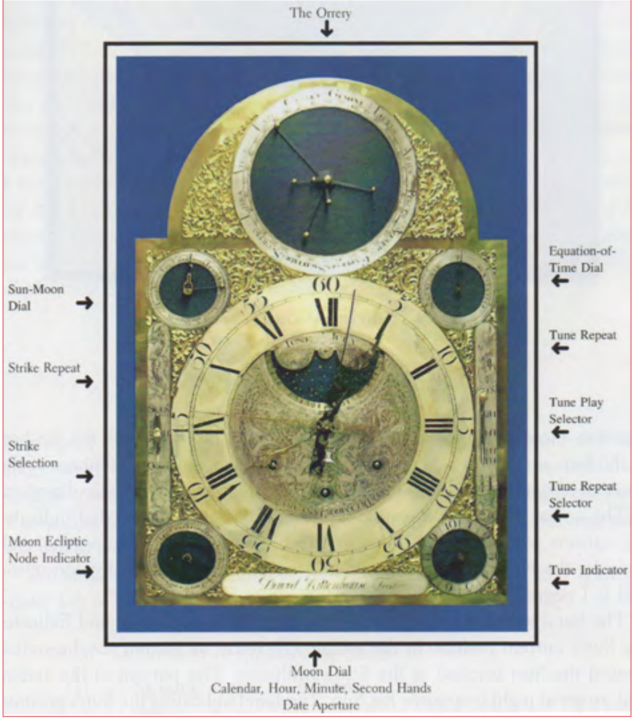
Probably the first significant early gear driven Equation clock was made in 1773 in Pennsylvania by the polymath David Rittenhouse. His astronomical musical clock is a masterpiece.

The three images below are from "The Most Important Clock in America" by Ronald R. Hoppes : The American Philosophical Society : 2009

David Rittenhouse - 1732 – 1796 - Politician, Surveyor, Inventor, Mathematician Astronomer & Clockmaker

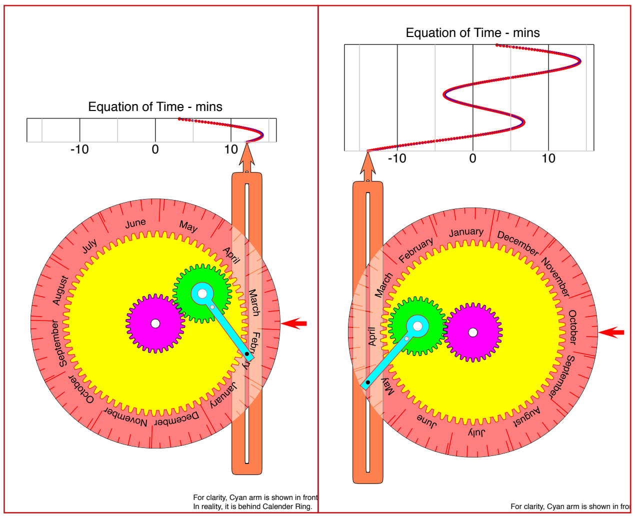
The Rittenhouse Clock in Drexel University

The Equation Face



Face of the Clock

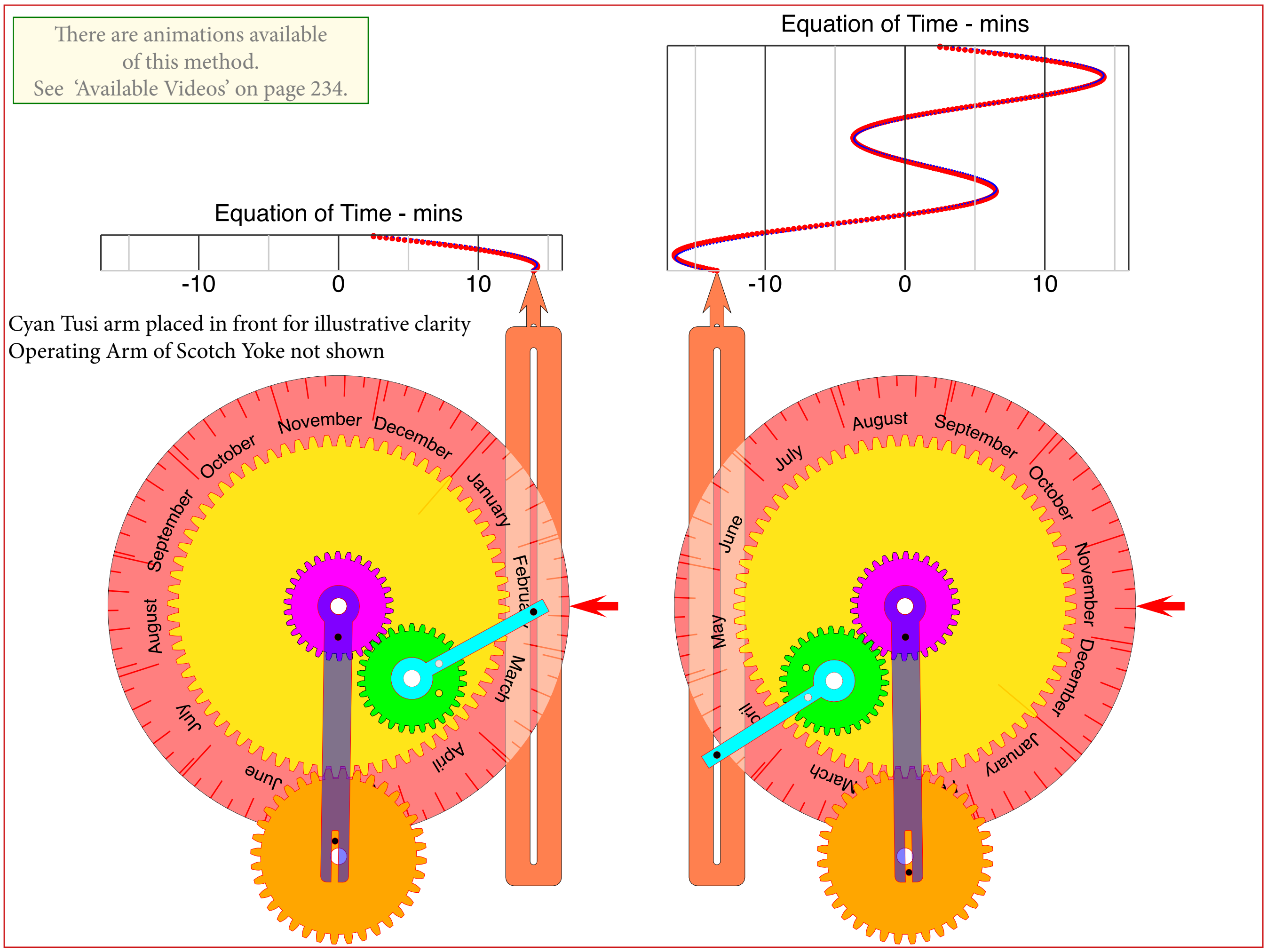
Rittenhouse was aware that the generation of the Equation of Time using two gears would, at best, have an accuracy of around 60 seconds of the true value, using a Tusi mechanism - with a single fixed gear with planet gear of the same size rotating about it - as shown below. Here, the distance between the stationary magenta gear and the rotating green is proportional to the Elliptical component of the EoT, while the length of the cyan arm is proportional to the Obliquity component. The relative positions of the green gear and cyan arm on 1st January reflects the different phases of these two signals : one originating at Perihelion (ca. 3 January) the other at the Equinox (ca, 20 March).



Two movements in a standard 2-Tusi mechanism - Rittenhouse's modification shown overleaf.

return to "Table of Contents"

There are animations available
of this method.
See 'Available Videos' on page 234.



Cyan Tusi arm placed in front for illustrative clarity
Operating Arm of Scotch Yoke not shown

Rittenhouse Tusi Modification

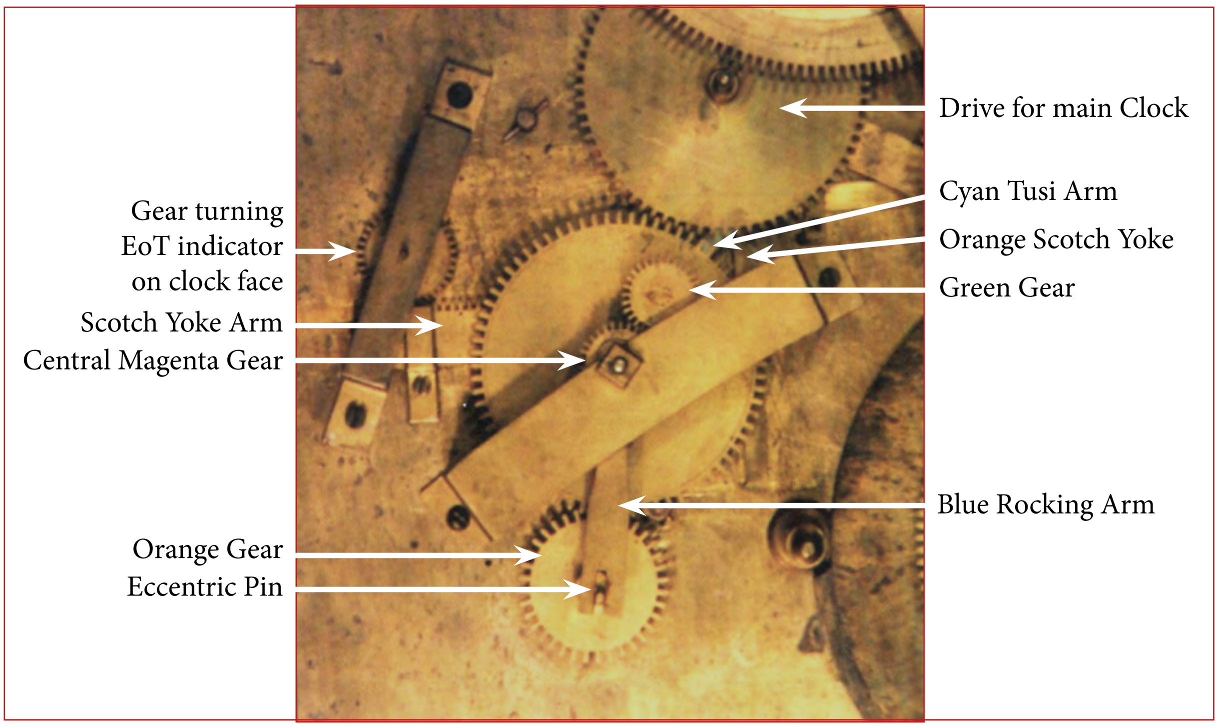
Seeking further refinement, he modified the design as shown overleaf.

Rittenhouse's modification involved the addition of the blue arm attached to the purple central cog. This rocked backwards and forwards by the small pin In the lower orange gear

The mechanism solves the Equation thus:

EoT=7.665 sin(A) + 9.665 sin(B) - 0.31 sin(C)

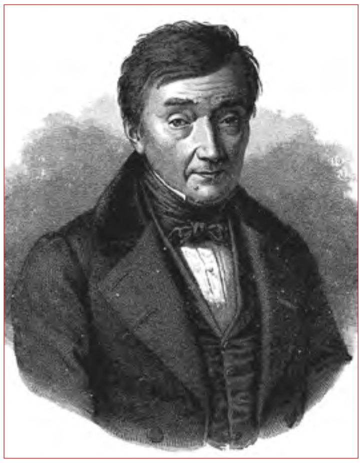
where A = 360°/365.25 × N



The Back of the Rittenhouse Clock

return to "Table of Contents"

$B = 360°/365.25 \times (N - 81)$



Jean-Baptiste Schwilgué
1776-1856

$C = 360°/365.25 \times (N - 173)$

THE SCHWILGUÉ STRASBOURG CLOCK

The Strasbourg Cathedral Clock was built on 1353. It was restored in 1574. By 1788, it stopped completely : worn out and no longer astronomically correct. It 1842, the clock mechanics were completely rebuilt by Jean-Baptiste Schwilgué, who had admired the 'dead' clock as a boy.

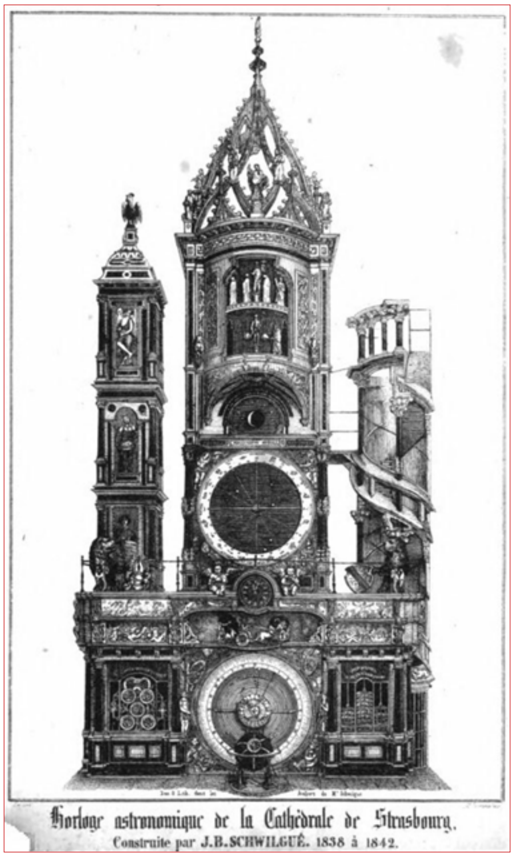
The Equation Mechanism works through both gears and profiles (the latter being the vertical equivalent to a cam).

The astronomically technical advance in this mechanism is the introduction of both the anomalistic year (365.259636 days between successive mean perihelions) and the tropical year (265.242189 days between successive mean vernal equinoxs). Since these are different, perihelion is moving towards the vernal equinox, so over the long term, the Equation will vary.

Although two profiles are used for the two prime harmonics of the EoT, the third harmonic is introduced by incorporation of a helical gear - which adds extra vertical height to the obliquity profile.

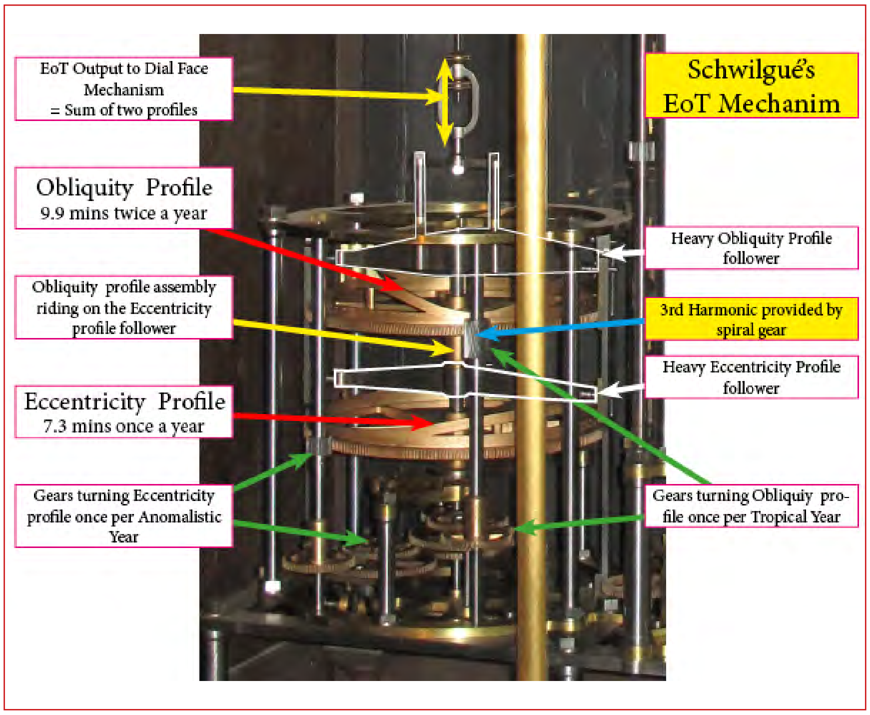
Personal Communication with Andrew James

The Clock's Solar & Lunar Dial. Also showing the time of sunrise and sunset



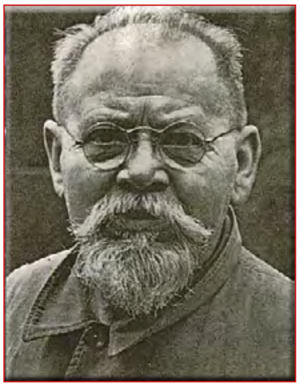
Strasbourg Clock - The Equation mechanism is at the bottom right corner. Illustration from Schwilgué's own documentation

The Clock today



Schwilgué's EoT Mechanism in Detail

The Solar (left) and Lunar (centre & right) Computer



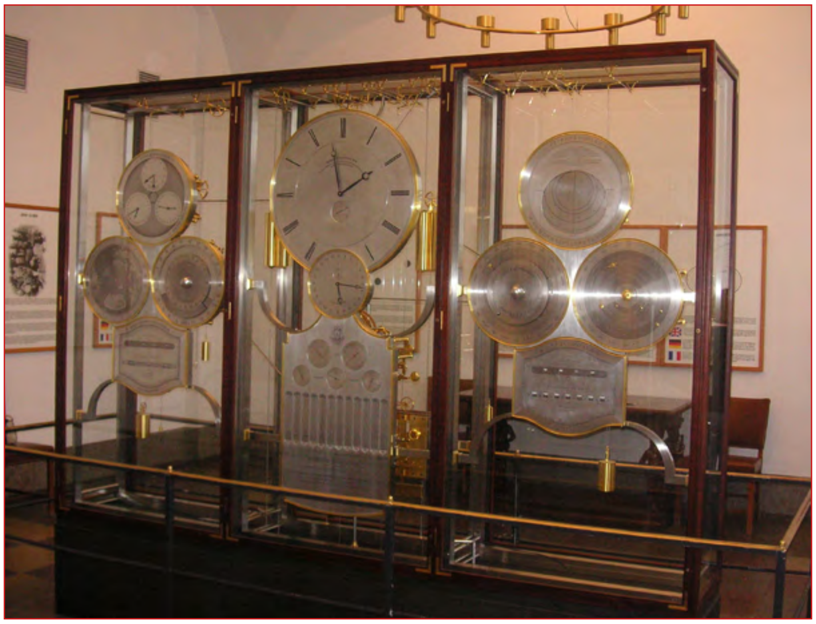
Ref. The black & white illustrations, following, were taken from Otto Mortensen's book "Jens Olsen's Clock" - 1957 - Copenhagen Technological Institute. No longer in print.

Jens Olsen 1872-1945.

Almost certainly, the most significant Equation modelling has been accomplished by the famous Jens Olsen clock that can be seen in the town hall in Copenhagen. The clock - comprising more than 15,000 parts, was completed after the death of Jens Olsen in 1955 by Otto Mortensen, who was Olsen's friend and collaborator.

Interested in clocks since a child, Olsen had been engrossed in Schwilgué's clock in Strasbourg. He became a clockmaker. When he was about fifty, he completed his calculations for the world clock he had envisioned as a boy. The ideas were them approved by a senior Danish astronomer. However, it took another twenty years to acquire the funds necessary to build the clock.

In 1943, when he was 71 years old, and Denmark was under German occupation, the Technological Institute of Copenhagen placed a workshop and staff at Olsen's disposal and work on the clock began in earnest. In a show of national pride, the workmwas kept secret form the occupying forces After his death in 1945, his colleague, Otto Mortensen, took over the project and, after its successful completion in 1955, it was placed in the town hall.

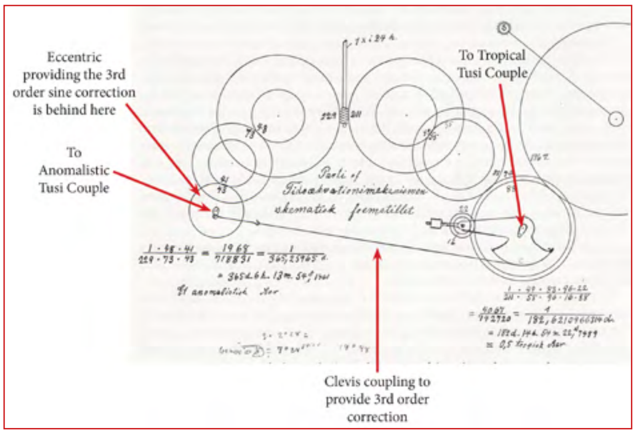


The Front of the Jens Olsen Clock. The top left face shows the Equation of Time, Local time and Solar Time.

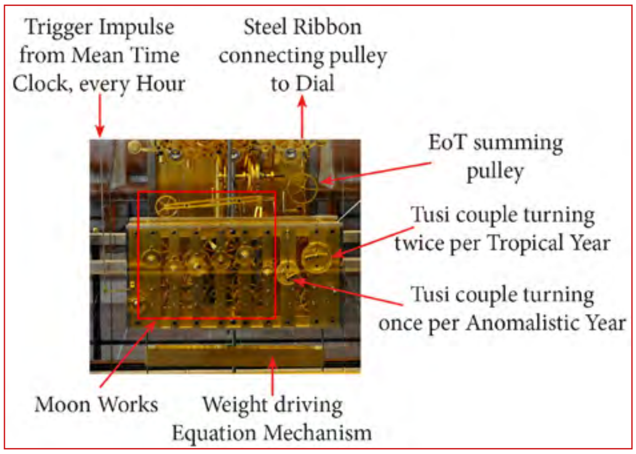
Detail of Equation Clock Face. TOP - (A) Equation of Time & (B) EoT longitude corrected (these hands are fixed 9 mins 47 second apart ; the longitude difference between CET's 15° East and Copenhagen. LEFT Local Mean Time. RIGHT True Solar Time. The clock was not running when this photo was taken (see the vast difference between local and true solar time)

return to "Table of Contents"

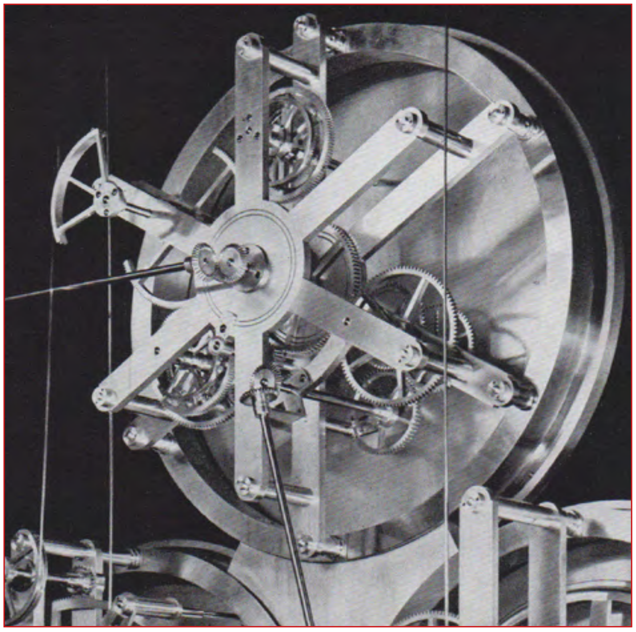
The Rear of the Jens Olsen Clock. The rectangular section (bottom centre) generated the 5 sine components of the lunar motion and, at the right , the 2 main & 1 small sine components of the Equation of Time. The connection between the signal generation and the Dial (now top right) is via a steel ribbon.



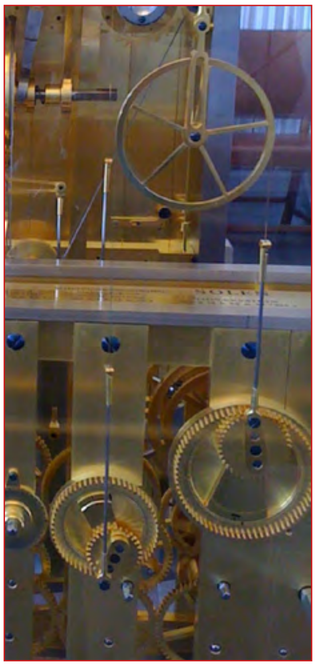
Jan Olsen's own diagram showing the generation of Tropical and Anomalistic year rotations



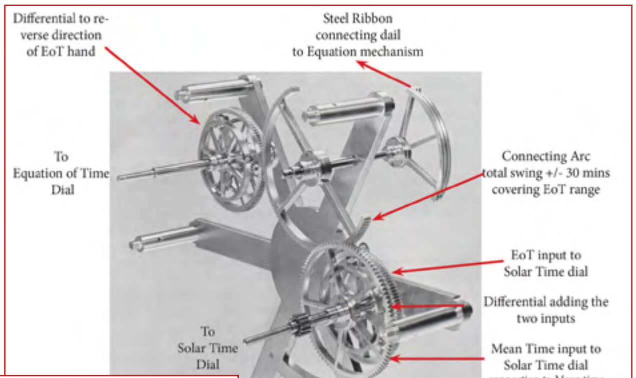
Moon & Equation Works



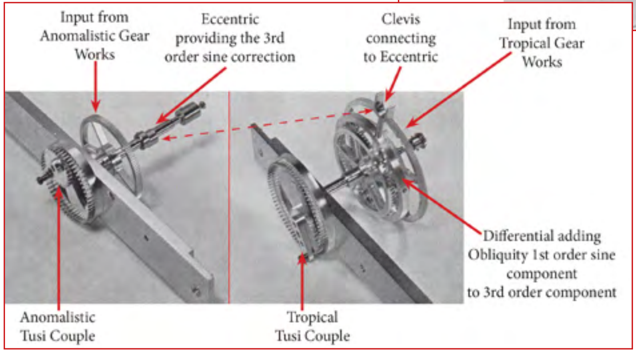
Detail of Rear of Equation Face : Whole works - top left is the quarter-round sector with steel ribbon from Equation mechanism and to left centre, the connection rod to main pendulum mean time mechanism



The 2 Tusi Couples



Further Detail



Detail of the 2 Tusi Couples and the 3rd order correction

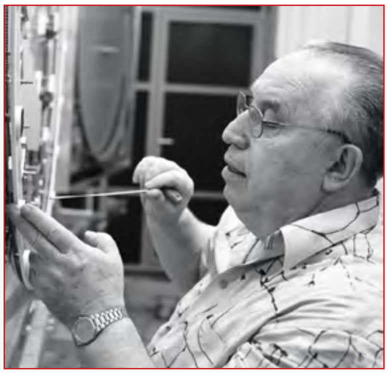
return to "Table of Contents"

SCHEURENBRAND'S FESTO HARMONICES MUNDI

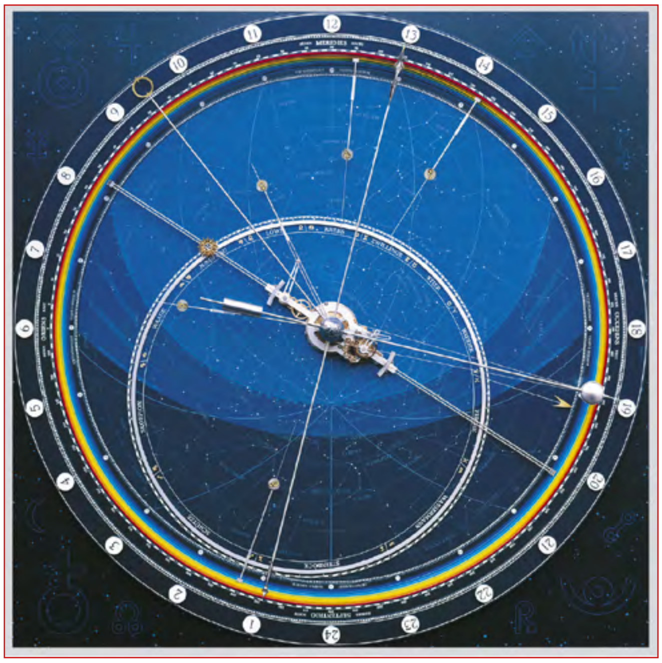
Ref: Brochure from Festo Group publications

Prof Dr Hans Scheurenbrand worked as Director of Research and Development at the Festo Group - a leader in automation technology. This remarkable modern clock, exquisitely engineered, is in the Festo headquarters in Esslingen, Germany. It comprises a calendar clock, an astrolabe in the form of an astronomical clock, an artistically designed glockenspiel featuring 76 bells and 40 tuned bars, covering 3 octaves.

The Equation of Time mechanism is a double sine mechanism based on circular cams added with differentials.



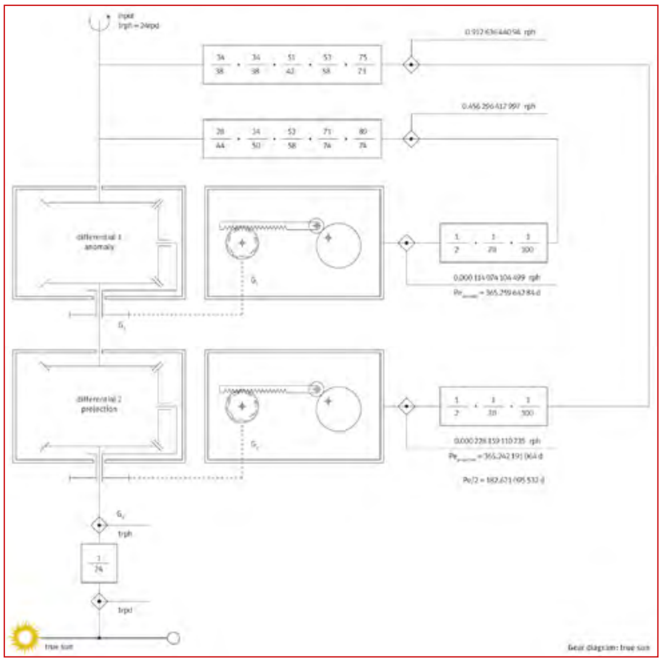
Prof. Dr.-Ing. Hans Scheurenbrand



Festo Harmonices Mundi Astronomical Face

Auxiliary Dials showing summer & winter Civil Time (left) and Solar Time & the Equation of Time (right) with gearing for the Ellipticity effect driven by the anomalistic year gearing (bottom left) and the gearing for the Obliquity effect driven by the tropical year gearing (bottom right)

Face detail, showing Solar Hand and EoT Indicator

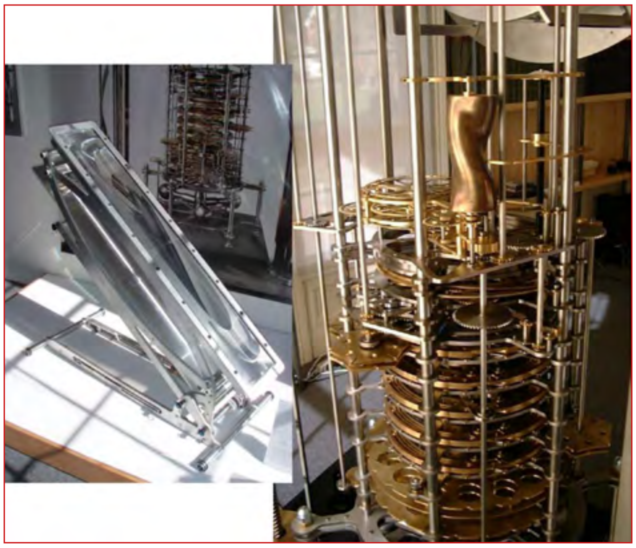


Festo Harmonices Mundi EoT Gearing for anomalistic and tropical years and the summation with differentials to produce the Equation

COMING FULL CIRCLE : THE CLOCK OF THE LONG NOW : GEARS & CAM

The Sun once more corrects the Clock

https://longnow.org/ &
The Clock of The Long Now: Time and Responsibility
by Stewart Brand: 2000: ISBN 978-0465007806



Prototype 1 - Solar Noon Correction Lens and the 10,000 years EoT Cam

Prototype 1 - 10,000 year EoT Cam Replica

Prototype 2 - showing the scale of part of the mechanism

The Clock of the Long Now is an extraordinary project to build a clock which - with appropriate maintenance - will run for 10,000 years. A prototype (some 3 metres tall) has been completed and has toured the science museums of the world. A second prototype, being built inside a mountain in Texas, is made of ceramics and stainless steel. The final clock will be built in Mount Washington in Nevada.

The clock runs with a torsion pendulum, but needs to be corrected every now and then. This will be done by south-facing quartz lens sensing solar noon. The focused sun will heat a wire that will trip a mechanism to read the EoT cam over it's 10,000 year life

Schematic of Prototype 2

Prototype 2 - The Equation Cam

return to "Table of Contents"

Colophon

Background to the book

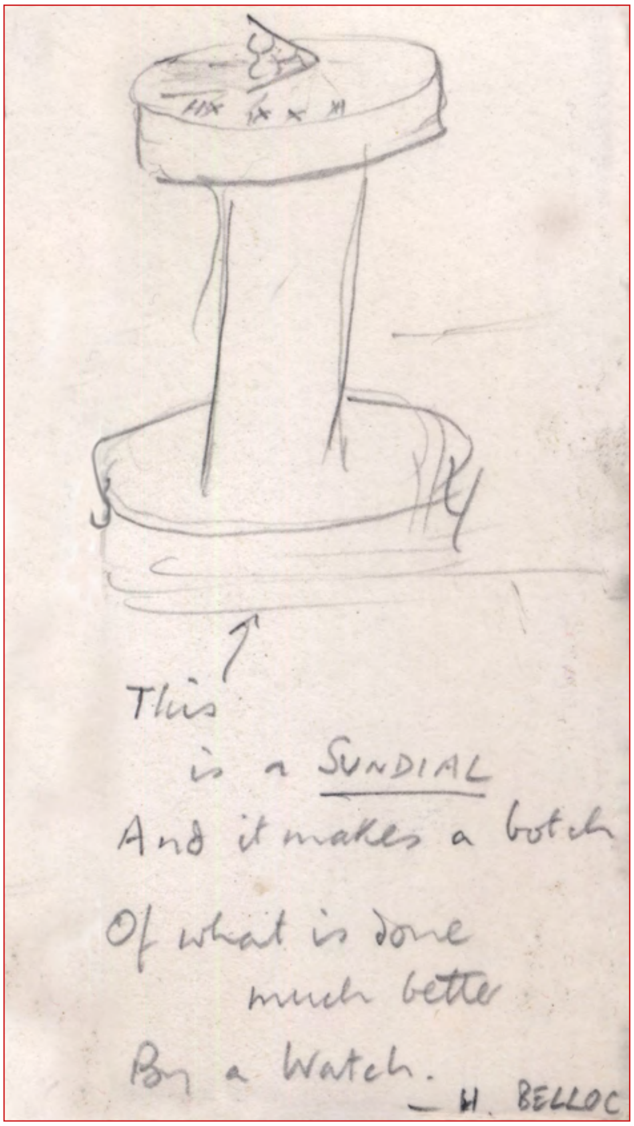
George Mallory, is famously quoted as having replied to the question, “Why did you want to climb Mount Everest?” with the retort “Because it’s there”.

So, when my friends ask why I write a book about a subject that is so esoteric and of such irrelevant in these times, I reply in a similar vein.

My interest in the subject started when I was about 6 years old. I discovered that the sundial in our Rectory garden seldom showed the same time as my father’s Longine watch.

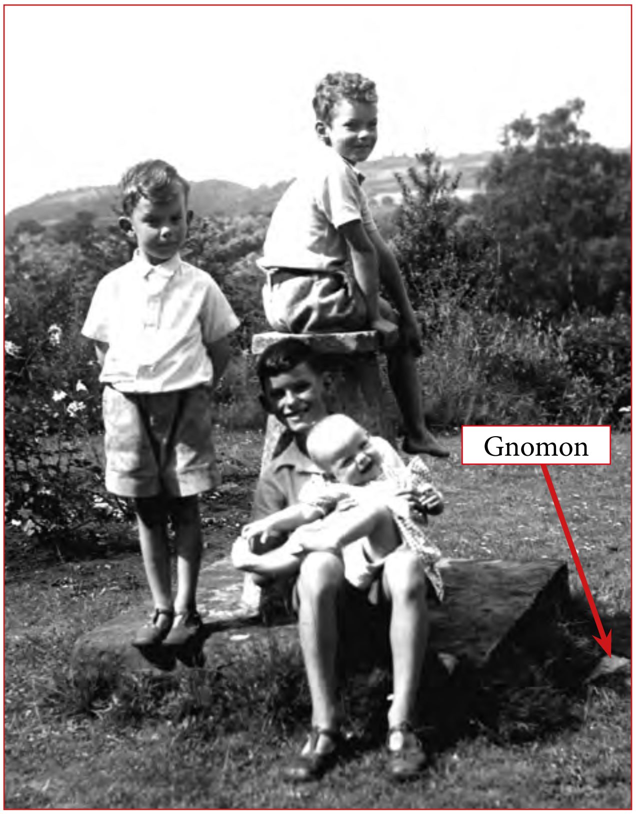
However, the chiming of the church clock always agreed with his watch. My father explained that the gnomon on the sundial was meant to be fixed firmly to the dial.

He did not mention that he checked his watch to the 9 o’clock pips on the BBC news and then he set the church clock when he read Martins next morning...



This
is a SUNDIAL
And it makes a botch
Of what is done
much better
By a Watch.
— H. BELLOC

My father drew me this little picture.
The text is a cherihew by Hilaire Belloc.



The Rectory Sundial in 1950 - gnomon somewhere on the ground.The author on the left - with his brothers and sister.

The next step in my engagement with the Equation came when I was working in the offshore oilfields in Persia. I lived in an Italian oil camp on a bleak, sandy, mist enclosed island some miles from the coast. Wonderful food and wine in abundance. But no feminine presence and the evening entertainment was poker, blue movies or drunkenness. So I studied solar dynamics and became acquainted with the equation of time through my uncle. (For his explanation see ‘A Minimalistic Approach’ on page 25.).

When I retired in 2002, more time was spent on solar astronomy, sundials, the history of timekeeping and the equation of time in particular. I spent 12 years lecturing internationally on these subjects.

Finally in 2019, my son, Edward, asked me where the output of all my studies was leading. First a website was generated - and then this book was the result.

This book is set in Adobe’s Minion Pro font and prepared using Adobe InDesign on an Apple Mac computer. The 3D images were created with the free Pov-Ray software. All other images were prepared with the free PlotDevice software.

Contact

If I am still alive and compos mentis, I can be reached at Kevin@Karney.com, and welcome correspondence.

return to "Table of Contents"

return to "Table of Contents"

APPENDICES

return to "Table of Contents"

Basic Positional Solar Astronomy : Part 1. Essential Parameters and the Equation of Time

The history of astronomy and timekeeping goes back many millennia. The terms used reflect this long history - and can be confusing to the non-astronomer. The author certainly became en-mired in this confusion – and this paper largely reflects how he sorted it out in his own mind. This paper hopes to chart some clarification.

The essential solar parameters needed by the gnomonist are:

- Right Ascension and Declination of the Sun – both Mean & True,
- Equation of Time, $^{\text{See Note 1}}$
- Altitude and Azimuth of the Sun,
- Time of Sunrise, Sunset and Solar Noon.

Part 1 of the Series will define the basic astronomical terms that are needed, how Coordinated Universal Time & Greenwich Mean Sidereal Time are calculated and charts the route needed to calculate the Equation of Time.

Part 2 will detail a method that can be used to calculate the Right Ascension and Declination of the Sun and the other parameters above. The classical astronomical method based on Kepler's single body approach will be used. This approach is satisfying since, with only a few basic astronomical parameters, one may derive the parameters listed above with *far* greater accuracy than is generally required for the most sophisticated sundial design.

Part 3 presents a little Fourier theory and some simple formulae - derived by Fourier analysis - that allow rapid and accurate calculation of the Equation of Time, Declination and Right Ascension, for those who do not want to bother with the complete calculations

HISTORY

We will skip lightly over those thousands of years, when Unequal or Seasonal hours were in use. When Scientific or Common hours were introduced by the Arabs in late mediaeval times, time was told by the Solar Time, now called Local Apparent Time. Noon was when the sun was at its zenith. The vast majority of sundials still tell Common hours.

However, around the Enlightenment, with ever increasing international maritime trade, the navigators' need for accurate longitude determination spurred the need for clocks that ticked uniformly with the rotation of the Earth around the Equator. Such clocks tell Mean Time. However, the Sun moves around the Ecliptic at 23° to the equator and its elliptical orbit means that it does not appear to move uniformly. So there is an *imaginary* Sun - the Mean Sun,- moving uniformly around the Equator which takes the place of the real Sun and tells such time. One cannot see an imaginary sun. But, since the Stars do appear to move uniformly around the Equator, they are used to measure Sidereal Time. This, in turn, with a suitable conversion, is used to determine accurate Mean Time.

The discrepancy between Mean and Solar Time is called the Equation of Time. Ancient Greek astronomers understood this discrepancy and, around 150 AD, Claudius Ptolemy gave a succinct description of the geometries that give rise to this non-uniformity and methods with which to calculate it. It was not until the time of Kepler in 1621 that the Earth's elliptical orbit was fully understood and some years later, Newton showed that Kepler's theories could be explained by his Laws of Gravity.

Until the arrival of the telegraph - there was little option but to set one's clock by a sundial, albeit corrected, if needed, for the Equation of Time. It was not until the late 19th century, the introduction of the telegraph and the demands of the railway companies allowed cross-country dissemination of accurate mean time, determined by astronomers. Thus, bit-by-bit, Local Solar Time was gradually displaced by Local Mean Time and thereafter by National Mean Time. GMT was introduced in 1880 in the UK. The changes wrought by the subsequent conversion of GMT to Universal Coordinated Time (UTC) and the introduction of Atomic Time are of irrelevant magnitude to the gnomonist.

BACKGROUND AND APPROACH TAKEN

The elliptical nature of the solar orbit gives rise to one difference between Solar Time and Mean Time - which is approximately sinusoidal with a yearly period, phased with perihelion in January (when the Sun is closest to the Earth) and with magnitude of some 7.4 minutes. Calculating this difference is a problem of dynamics.

The 23.4° obliquity between the Ecliptic and the Equator gives rise to a second difference - which is somewhat sinusoidal with a six-monthly period, phased with the Vernal Equinox in March and with magnitude of some 9.9 minutes. Calculating this difference is a problem of spherical trigonometry.

The fact that most of us do not live on our Time Zone meridian (plus the introduction of Summer or Daylight Saving time) provides the third difference between Solar Time and that told by our watches. This correction involves a simple arithmetic calculation.

The calculation of the Sun's Altitude and Azimuth for any time/date and location is once again a problem of spherical trigonometry.

return to "Table of Contents"

The traditional geocentric view is used - the Sun travelling around the Earth. While one 'knows' that the Earth revolves around the Sun, it is common to refer to the converse. It is only a matter of one's frame of reference. It makes no calculational difference when considering just the Sun & Earth. The Earth's longitude with respect to the Sun is just 180° difference from the Sun's longitude with respect to the Earth. On the other hand, a heliocentric view makes it much easier to explain the movement of the Planets in relation to the Earth.

Since this paper is meant to present the basics, it makes certain simplifications to definitions and equations consistent with the provision of results at levels of accuracy that are *more than sufficient* for the needs of the gnomonist. Pedants should read the notes at the end where I have tried to be more precise.

The figures are correctly calculated for a given place, viz Athens - Time Zone 2 and for a given date/time - 2nd February 2013 at 11:30 a.m. local civil time. See Note 3

THE CELESTIAL SPHERE

It has been practice throughout the ages to place the Earth at the centre of the Celestial Sphere. Fig. 1 shows the Celestial Sphere viewed from the medieval Empyrean - the place outside the Stars - where God is.

> *The Celestial Sphere is an imaginary sphere of arbitrarily large radius, concentric with the Earth and rotating upon the same axis. All objects in the sky can be thought of as projected upon the celestial sphere. The celestial equator and the celestial poles are the outward projections of the Earth's equator and poles.*

The Ecliptic at 23.4° from the Celestial equator is the path around which the Sun appears to move.

An essential point on the Celestial Sphere is one of the two intersections of the Celestial Equator and the Ecliptic. The point chosen is the point when the Sun crosses the celestial equator during the northern hemisphere spring and is called the Vernal Equinox. Somewhat confusingly, it is also called the First Point of Aries. These terms are used more-or-less interchangeably. Strictly speaking, the First Point of Aries is a direction in the sky, while the Vernal Equinox is a moment of time. The First Point of Aries is the prime origin for all measurements made along the Celestial Equator and the Ecliptic. Confusingly, the First Point of Aries is no longer in the astronomical Constellation of Aries. It was - in classical Greek times - but as a result of Precession see Note 4, it is now in the Constellation of Pisces. See Note 5

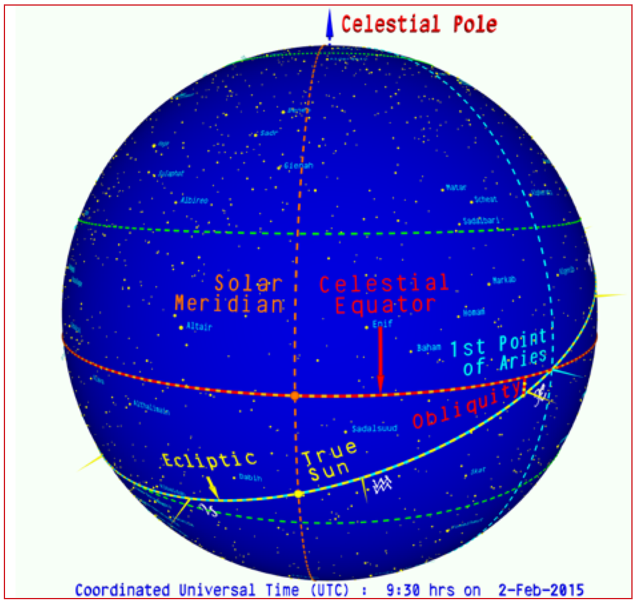


Fig. 1. The Celestial Sphere

ASTRONOMICAL NOMENCLATURE & DEFINITIONS

Since the Stars appear to rotate around the Earth with exemplary uniformity. See Note 2 24 hours of time equates to 360° of rotation. Hours and Degrees can be used interchangeably with a conversion factor of 15.

Traditionally, some parameters (e.g. Right Ascension) are quoted in hrs/mins/secs) and some parameters (e.g. Hour Angles) are quoted from -180° to +180°.

In all the figures and calculations below, parameters are in Degrees +ve West to East. This ensures a consistent arithmetic and the avoidance of sign errors. This is the international convention, though not always used in gnomonics, e.g. in the BSS Sundial Glossary Ref. 1. Otherwise, Glossary symbols are used throughout. A summary of the abbreviations and their translation is given in Table 1 towards the end of text. Definitions below are given on indented paragraphs.

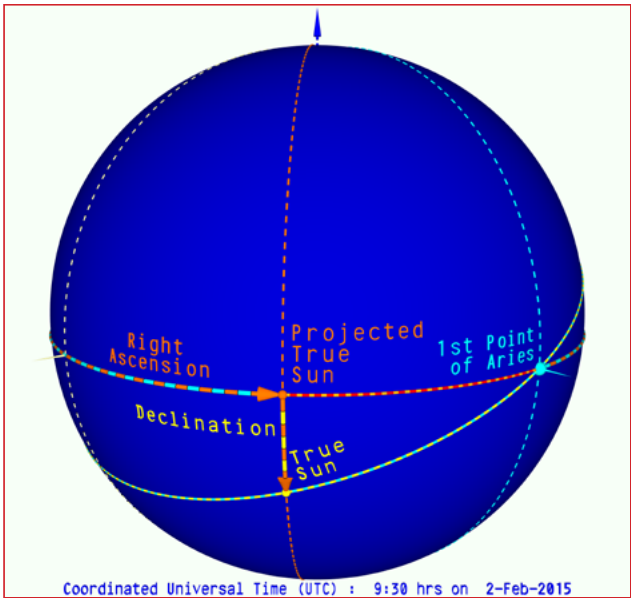


Fig. 2. Declination & Right Ascension

return to "Table of Contents"

ZENITH & MERIDIAN

The Zenith is the point on the Celestial Sphere directly above the observer. (The opposite point on the Sphere is the Nadir).

A meridian is a great circle on the celestial sphere that passes through the North & South Celestial Poles and either through a point on the Celestial Sphere or through the Zenith of an observer on the Earth's surface.

Meridians are analogous to line of longitude on the Earth's surface. Angles between meridians (as angles between lines of longitude) are measured around the Celestial Equator.

RIGHT ASCENSION & DECLINATION

We are concerned with the position of the Sun on the Celestial Sphere. This is measured by Right Ascension & Declination. See Fig. 2. These are equivalent to our terrestrial Longitude & Latitude, except that...

- Declination uses the Celestial equator, running from +90° to -90° - positive towards the north, negative towards the South.
- Right Ascension is measured along the celestial equator and the 1st Point of Aries as origin. It is measured anti-clockwise - when viewed from the North Celestial Pole. This is the direction in which the Earth rotates and in which the Sun appears to move. Traditionally, RA is quoted in Hours/Minutes/Seconds, running from 0 to 24 hrs. But Degrees are generally used in this paper.

The Sun moves around the Ecliptic at very approximately 365/360° per day, so its RA and Decl are continuously changing. In Part 2 of this series, we will see how solar dynamics can used to calculate the Sun's RA & Declination for any given time and date.

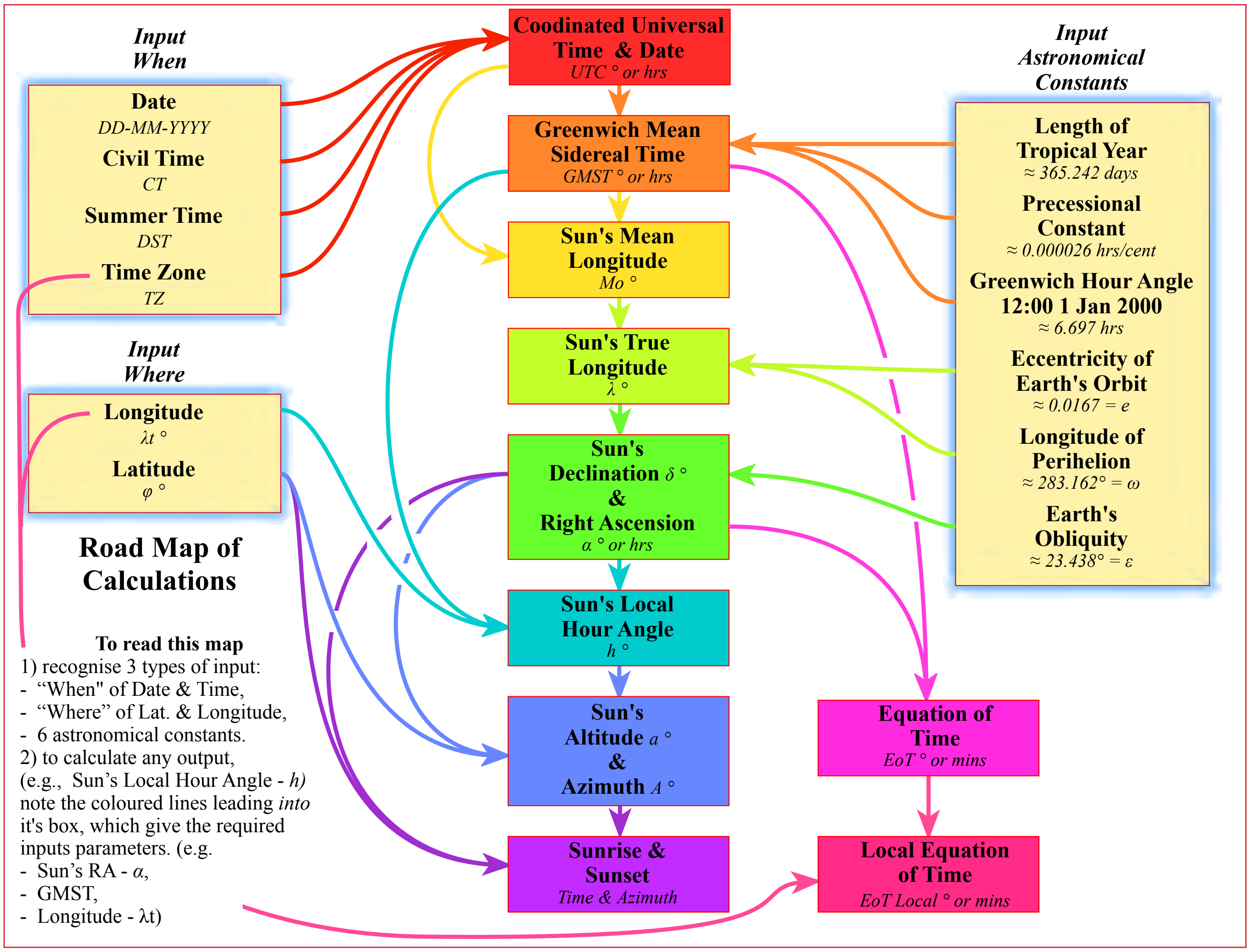
In passing, we should note that...

- the planets (from Greek πλανήτης αστήρ "wandering star") move near to the Ecliptic in somewhat erratic manner (from a geocentric point of view) so their RA & Decl are also continuously changing.
- the RA and Decl of any star is effectively constant. [See Note 6]
- RA and Decl have nothing to do with the daily spinning of the Earth about its axis.

WHAT ARE WE TRYING TO CALCULATE...

Fig. 3 charts the path along which calculations are made. The start is made by provision of three classes of input…

- the "When", the local time and date;
- the "Where", the terrestrial Latitude and Longitude of the Observer (or the Sundial);



To read this map

1) recognise 3 types of input:
- "When" of Date & Time,
- "Where" of Lat. & Longitude,
- 6 astronomical constants.

2) to calculate any output, (e.g., Sun's Local Hour Angle - *h)* note the coloured lines leading *into* it's box, which give the required inputs parameters. (e.g.
- Sun's RA - *α*,
- GMST,
- Longitude - λt)

Fig. 3. The Calculation's Road Map

return to "Table of Contents"

- the 6 astronomical constants required – 3 of which are not quite constant.

In this part of the series,

- the simple connection between local Civil Time and date – which we hear on the radio and read from our watches – and Coordinated Universal Time – UTC – is established.
- the more complicated connection between Greenwich Mean Sidereal Time - GMST - and UTC is made
- the connection between UTC and Sun's Mean Longitude is made
- the formulae to establish the Equation of Time and the Longitude Correction is introduced.

Coordinated Universal & Standard Time

Some gnomonists eschew civil time and rely entirely on 'true' or Solar time – it is noon when the Sun is South. The author respects this view. But he personally feels it is of paramount importance that the gnomonist should be capable to explain to our young why the sundial reads a different time to that on their watch or mobile phone. Hence the apparently perverse starting point of Civil – rather than Solar - Time.

It was the advent of the railways that forced society to adopt mean time so that the same time was used everywhere in a country (or in large portions of a country – as in Russia or the USA). The global starting point was Greenwich Mean Time – GMT. This has morphed, with minor changes, in Coordinated Universal Time – UTC.

> *Coordinated Universal Time (UTC) is 12 + the hour angle at Greenwich of the Mean Sun. The hour angle being converted from degrees to hours at 360°/day.*

Although the 'tick' of UTC now relies on atomic clocks, its formal definition is in terms of the Mean Sun. The Mean Sun – which is an imaginary body…

> *The Mean Sun is an abstract fiducial point at nearly the same Hour Angle as the Sun, but located on the mean celestial equator of date and characterized by a uniform sidereal motion along the equator at a rate virtually equal to the mean rate of annual motion of the Sun along the ecliptic.*

As an example, when the Mean Sun's meridian has moved west by 15° (or 1 hour) from the Greenwich meridian, UTC = 12 + 1 = $13{:}00^{\text{hrs}}$, which is what one would expect. The term 'fiducial' is the technical term for a point that is a fixed and trusted basis for comparison. In simple terms...

- the *mean* sun is an *imaginary* body that *uniformly* moves around the *Equator,* once in one tropical year.

on the other hand...

- the *true* Sun, moves *non-uniformly* around the *Ecliptic*, once in one tropical year. The true sun is thus 'out-of-angle' with the axis which creates our day/night.

In passing, we should note that...

- the 'Tropical' Year is the time taken for the sun *(on average)* to pass through the 1st Point of Aries - 365.242 191 days. Note that our leap year system gives a 'Calendrical' Year of (365.25 x 400 - 3) / 400 = 365.242 500 days, which closely matches the length of the Tropical Year, ensuring that the Calendar does not drift away from the Seasons.
- Atomic Time is kept in sync with the solar definition of UTC by the occasional insertion of Leap Seconds, which compensate for the gradual slowing of the Earth's rotation.

UTC is a surrogate for Solar time in providing a universal and uniform time scale. The Mean Sun's position has zero declination and its Right Ascension increases uniformly from 0° at the Vernal equinox to 360° at the next Vernal equinox.

In the 1880s, Greenwich Mean Time was established as legal time across the UK. Other countries offset their own mean time by integral number hours (or half hours) before or after Greenwich - thus introducing the Time Zones. So Standard Time was created. Greenwich Mean time morphed with minor changes into Coordinated Universal Time (now UTC).

> *Standard Time – ST - is Mean Time on the Time Zone meridian of that area. Time Zone meridians are (usually) in 15° Longitude increments away from the Greenwich meridian.*

Standard Time may be further moderated by the introduction of Summer or Daylight Saving to give Civil Time - CT. In winter, Civil Time is the same as Standard Time. Civil Time is the legal binding time in a given Time Zone.

$$UTC^{hrs} = ST^{hrs} - Time\ Zone^{hrs\ (+ve\ East\ of\ Greenwich)} \text{ .... Equ 1.1}$$

$$UTC^{hrs} = CT^{hrs} - Time\ Zone^{hrs} - DST^{hrs} \text{ .... Equ 1.2}$$

Calculations of solar positions need both a time and a date, and it must be recognised that if the correction in Eqn. 1 lead to a different day in Greenwich than that of the observer, a correction is needed..

$$\begin{aligned} &\text{if } UTC^{hrs} > 24 \\ &\qquad UTC^{hrs} = UTC^{hrs} - 24 \ \&\ Date^{day} = Date^{day} + 1 \\ &\text{if } UTC^{hrs} < 0 \\ &\qquad UTC^{hrs} = UTC^{hrs} + 24 \ \&\ Date^{day} = Date^{day} - 1 \\ &\text{................Eqns 1.3} \end{aligned}$$

Finding Greenwich Mean Sidereal Time

Before atomic clocks, the problem with GMT was that it was based on an imaginary mean Sun. Thus it was not measurable, especially by navigators trying to calculate longitude. They require an entirely uniform, definable and measurable time scale that accords with the axis of spin of the Earth and which is independent of the vagaries of the Sun's apparent movement. This is provided by the stars - so-called Sidereal Time (from the Latin word 'sidus' meaning 'star').

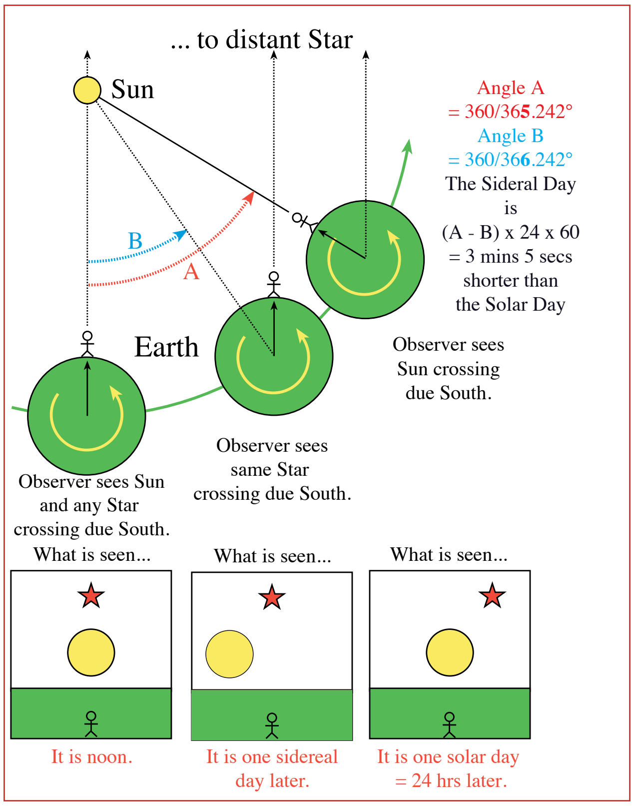


Fig. 4. Sidereal Time -v- Solar Time

On successive nights, it is easy to measure 'transits' of any star i.e. when it has its highest altitude in the sky. Thus the stars began to be used as time-keepers and so-called sidereal day was defined by successive transits of any star through an observer's meridian. The introduction of Sidereal time was the start of the gradual decline of Sundials as civilization's primary time keeper. Astronomers – rather than gnomonists – gradually became Masters of Time

The sidereal day is not the same as the solar day. Fig. 4 shows a solar day, *defined by the transit of the sun*, as compared with the sidereal the day, *defined by the transit of a star*. There are 366.242 transits of a given star in the same time as 365.242 transits of the sun. This is because the Sun itself has circled one revolution against the stars. The ratio 366.242/ 365.242 = 1.002738 will crop up again in our calculations.

Against this background,

> *Greenwich Mean Sidereal Time (GMST) is the angle along the celestial equator from the Mean Vernal Equinox (1st Point of Aries) to the Greenwich meridian.*

Both Sidereal Time and UTC record an evenly ticking cycle that completes each tropical year. Therefore, it is possible to define UTC explicitly in terms of Sidereal Time. This definition is 'owned' by the International Astronomical Union.

$$\begin{aligned} GMST^{hrs} = (&6.697\,374\,558^{hrs} \\ &+ 0.065\,709\,824\,419\,08 \times D_0^{days} \\ &+ 1.002\,737\,909\,35 \times UTC^{hrs} \\ &+ 0.000\,026 \times T^2) \bmod 24 \quad \text{.............. Eqn 1.4} \end{aligned}$$

D_0 is the number of days from 12:00 $^{\text{hrs}}$ on 1st January 2000 – the so-called Epoch$_{2000}$.- until the mid-night that starts the day in question. T is the number of Julian Centuries of 36,525 days from Epoch$_{2000}$ until the moment of time in question. The 'mod' function reduces the answer to fall between 0 and 24 hours. This is a slight simplification of the complete definition. For ultimate but unnecessary accuracy... $^{\text{See Note 10}}$.

The numbers in this definition are not arbitrary.

- 6.697 374 558 was the Greenwich hour angle of the Sun at Epoch$_{2000}$.
- 0.065 709 824 419 08
  = 24 $^{hrs/day}$ / 365.242 191$^{days/tropical\ year}$

  which ensures that, in one tropical year, GMST increases by 24 hours, corresponding to the extra sidereal day in the tropical year.

  D_0 is the number of days from Epoch$_{2000}$ to midnight of the day in question.
- 1.002 737 909 35
  =366.242 19$^{sidereal\ days/year}$/365.242 191$^{tropical\ days/year}$

  this converts from normal to sidereal hours.
- 0.000 026 x T^2 accounts for Precession. $^{\text{See Note 4}}$

  T is the number of Julian Centuries (of 36525 ^{days}) from the Epoch$_{2000}$.

  Note that three of the six input astronomical constants are involved in this definition.

Since our years and months are of variable length, any given date and time combination is not directly amenable to mathematical formulae, so a strictly linear time/date scale is used throughout the astronomical world. This is the Julian Date (JD).

The Julian Date is the number of decimal days that have elapsed since noon coordinated universal time (UTC), 1st January, 4713 BC. $^{\text{See Note 8.}}$ However for these calculations, times from Epoch$_{2000}$ (12:00 $^{\text{hrs UTC}}$ on 1st January 2000) are needed, which is the Julian Date reduced by 2451545.0

In passing, we may note that...

$$Date^{days}_{Epoch\ 2000} = JD^{days} - 245\ 154\ 5.0^{days} \quad \text{.................... Eqn 1.5}$$

Date/Time $_{\text{Greenwich}}$ is given by YYYY $^{\text{years}}$, MM $^{\text{months}}$, DD $^{\text{days}}$, HH $^{\text{hrs}}$, MM $^{\text{mins}}$ then to obtain the D_0 - *during this century* - apply the following formula:

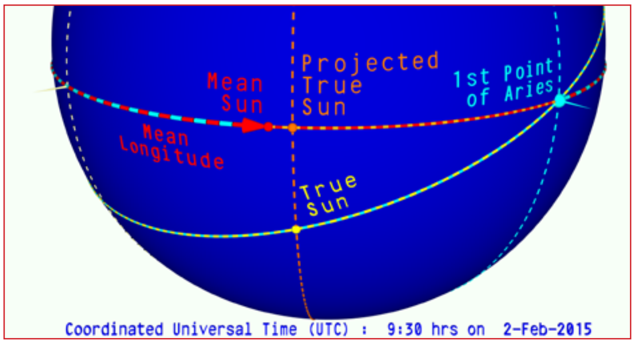


Fig. 5. The Mean Sun & Mean Longitude

$$
\begin{aligned}
bbb &= 367 \times YYYY - 730531.5 \\
ccc &= -\operatorname{int}\left(\left(7\times\operatorname{int}\left(YYYY+\left(MM+9\right)/12\right)\right)/\ 4\right) \\
ddd &= \operatorname{int}\left(275\times MM/9\right) + DD \\
D_{today} &= \left(HH + MM/60\right)\ /\ 24 \\
D_0 &= bbb + ccc + ddd^{\text{See Note } 9} \\
T &= (D_0 + D_{today})\ /\ 36525 \qquad \text{.................. Eqn. 1.6}
\end{aligned}
$$

Why these formulae work is a mystery to the author... The 'int' function removes the fractional part of the calculation just made. The 'mod' function reduces the result until it lies between 0 & 24.

FINDING THE SUN'S MEAN LONGITUDE

Referring once more to Fig. 3, the next thing to calculate is the Mean Sun's Longitude. This may also be referred to as the Mean Sun's Right Ascension. It is measured along the Celestial equator, from the 1st Point of Aires See Figs. 5 & 6.

In the latter, working from out to in, see the various arcs...

- the Sun's Mean Longitude - M_O - origin 1st Point of Aries
- GMST - origin 1st Point of Aries
- UTC - origin at the Nadir (the opposite point) from the Mean Sun. This reflects the definition of UTC (see above) - or more obviously the fact that our $0{:}00^{hrs}$ at midnight is 180° away from mean noon, the moment when the Mean Sun's Hour Angle is 0°
- complimentary arc 180 - UTC

From the figure, it is apparent that...

$$\mathrm{M}_0^{deg} = \mathrm{GMST}^{deg} - \mathrm{UTC}^{deg} + 180^{deg} \qquad \text{.................. Eqn. 1.7}$$

INTRODUCING THE SUN'S RIGHT ASCENSION AND THE EQUATION OF TIME

The Sun's Right Ascension was introduced above, see Fig. 2. Putting this together with the definition of Mean Longitude, we can find the Equation of Time. See Figs. 7 & 8. From the arcs in Fig. 7, it may be seen the Equation-of-Time

$$\mathrm{EoT}^{deg} = \mathrm{M}_0^{deg} - \alpha^{deg} \text{... Eqn. 1.8}$$

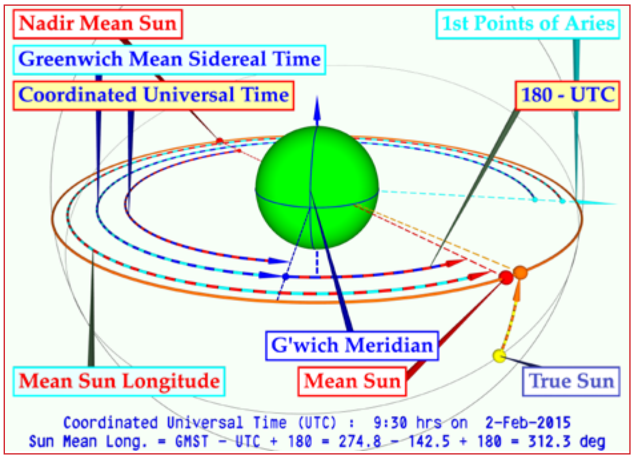


Fig. 6. GMST, UTC & Mean Longitude

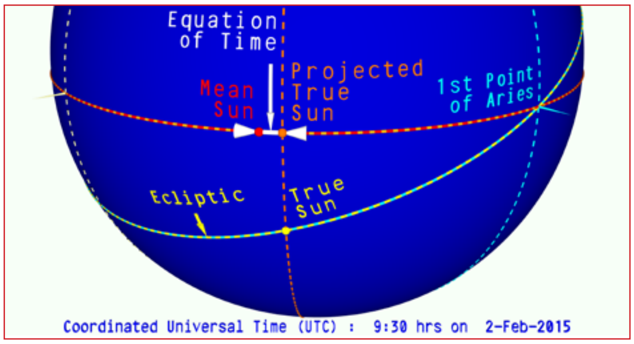


Fig. 7. The Equation-of-Time

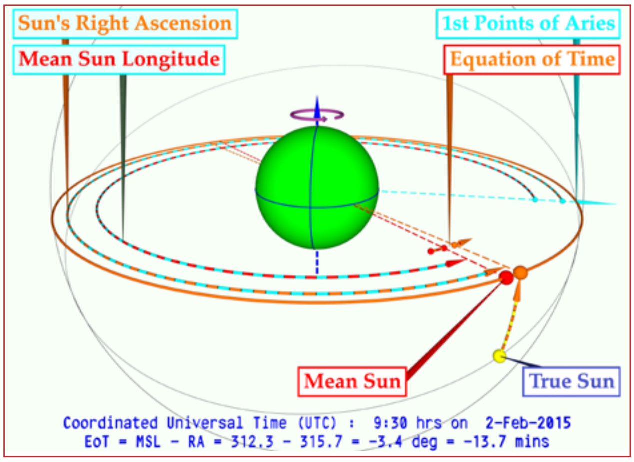


Fig. 8. The Equation of Time - see Eqn. 1.8

Combining Eqns. 1.7 & 1.8...

$$\mathrm{EoT}_{astronomical}^{deg} = \mathrm{GMST}^{deg} - \alpha^{deg} - \mathrm{UTC}^{deg} + 180^{deg} \text{... Eqn. 1.9}$$

All of these are explicitly known except for the Right Ascension of the Sun. This will be computed in Part 2 of this series. Those interested in gnomonics tend to use the inverse of this definition (i.e. the correction to be made to sundial time to get mean time) and want the results in minutes, thus...

$$EoT_{gnomonical}^{mins} = -\ 4 \text{ x } EoT_{astronomical}^{deg} \text{ ....Eqn. 1.10}$$

return to "Table of Contents"

In passing we may note that...

the formal definition from the all powerful Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris & Nautical Almanac Ref. 2, is:

> *.. As from 1965..... The equation of time will then be defined as the correction to be applied to 12h + Universal Time to obtain the Greenwich Hour Angle Sun,......... ; it is now so tabulated in the almanacs for navigators and surveyors...*

This implies...

$$12^{degs} + UTC^{degs} + EoT^{degs}_{astronomical} = GHA^{degs}_{Sun} \text{ or}$$

$$EoT^{degs}_{astronomical} = GHA^{degs}_{Sun} - UTC^{degs} - 12^{degs} \quad \text{.....Eqn. 1.11}$$

But, by definition...

Thus Equation 1.9 is the same as Eqn. 1.11

THE LONGITUDE CORRECTION

Solar noon at 1° west of a Time Zone meridian is 4 mins of time after Solar noon on the Time Zone meridian. Thus, if we wish to correct our sundials to provide what our watches read, we must apply an additional offset - the Longitude Correction ...

$$\sigma^{deg} = Time\ Zone^{hrs} \times 15^{deg/hr} - \lambda^{deg}_{t} \text{ ....Eqn. 1.13}$$

So we may conclude that - if we coin a new term...

$$EoT^{mins}_{Local} = EoT^{mins}_{Gnomonical} + \sigma^{mins} \text{ ....Eqn. 1.14}$$

For a standard sundial (i.e. one whose hour lines are not longitude corrected and whose noon line on the North/South meridian), it is suggested that any correction tables or graphs should indicate EoTLocal, with the additional comment that DST Hours should be added in the Summer.

SUMMING UP

The Table, below, sums up the various formulae, presented above. It can be seen that, for at any date/time/location, all the parameters can be deduced or calculated from one another - provided that the Right Ascension of the Sun can be found. These calculations, together the conversion to Azimuth and Altitude, Sunrise and Sunset. will be presented in Part 2 of this series.

Parameter	Symbol	Formula in degrees	Example	
Date		given	2nd Feb 2013	
Observer's Longitude, +ve east of Greenwich	LON or λ_t	given	23.717°	23° 43' 00"
Observer's Time Zone, +ve east of Greenwich	TZ	given	60°	2 hrs
Observer's Summer Time or Daylight Saving Hours	DST	given	0°	0 hrs
Observer's Civil Time	CT	given	172.500°	11:30 am
Observer's Standard Time	ST	CT - DST	172.500°	11:30 am
Coordinated Universal Time	UTC	ST - TZ	142.500°	9:30 am
Greenwich Mean Sidereal Time Calculated in terms of Date & UTC	GMST	(see Eqns. 1.1, 1.2, 1.3,1.4)	274.761°	$18^{hr}\ 19^{min}\ 02^{sec}$
Sun's Right Ascension Calculated in terms of Date & UTC	RA or α	(see Part 2)	315.673°	$21^{hr}\ 02^{min}\ 41^{sec}$
Equation of Time: Local Mean to Dial Time (Astronomical Convention) See Note below	$EoT_{Astronomical}$	GMST – α – UTC +180° = GMST – α – (CT - DST - TZ) + 180°	-3.413°	$-13^{min}\ 39^{sec}$
Equation of Time: Dial to Local Mean Time (Gnomonist's Convention)	$EoT_{Gnomonical}$	$- EoT_{Astronomical}$	3.413°	$13^{min}\ 42^{sec}$
Longitude Correction	σ	TZ - λ_t	6.283°	$25^{min}\ 08^{sec}$
Equation of Time: Dial to Standard Time	EoT_{Local}	$EoT_{Gnomonical} + \sigma$	9.696°	$38^{min}\ 47^{sec}$
Note: The Equation of Time calculated in this way may - depending on the time of day and year - give spurious looking results as a result of the cross-over from 24 hrs back to 0 hours. To correct, if $EoT^{mins} < -36$ then add 48, if $EoT^{mins} < -12$ then add 24.				

Table. Basic Calculations

return to "Table of Contents"

Notes

1. Various astronomical terms use the qualifier 'equation of…': the equation of time, the equation of centre, the equation of the equinoxes, the equation of origins, the equation of light. The term coming from Greek to Arabic to the mediaeval Latin 'equato' as in Equato Diem for EoT. In all cases, 'equation of…' means the difference between what is observed and the mean values of the phenomenon in question.
2. The Earth's rotation is not completely uniform. Not only does the position of the North and South Poles wander, but the rate of rotation is slowing in a somewhat random fashion by a number of seconds per decade. This is believed to be caused by tidal friction and crustal movements. This gives rise to the inclusion of 'leap seconds', mentioned in Note 7.
3. Two free software packages : 'Persistence of Vision', a precise 3-D simulation package & a precise 2-D NodeBox were used to prepare the graphics. The data required to draw the Stars in Fig. 1 was derived from the Right Ascension & Declinations of the 1000 brightest stars, readily found on the internet. All the figs used precisely drawn in accordance to the routines described in this document & Part 2 of the series.
4. Nothing on Earth or the Heavens is moving uniformly... In particular, the Earth's axis is slowing gyrating like an out-of balance spinning top. This effect - called Precession - has a long period of 25,600 years. It is caused by the torque induced by the Sun & Moon's gravitational pull on the equatorial bugle in the Earth's shape. Over time. Precession moves the position of the Vernal Equinox through the Sky. Most of the significant effect of precession, in these calculations is subsumed in the definitional formula for Mean Time. In addition to Precession - and primarily because of tidal forces between the earth and the moon - the axis of the earth is vibrating such there are complex minor variations in the position of the Vernal Equinox and the Obliquity of the axis. This is called Nutation. The effects are minor in the context of this paper. But precession and nutation lead to some potentially confusing nomenclature within astronomy. The terms **mean** equator, **mean** obliquity, **mean** equinox, **mean** sidereal time indicate that the effects of nutation are averaged out. (However, **mean time** has an entirely different context.) The term...**of date** indicates that precession has been considered, while...**of Epoch** refers to mean values on 1 January 2000, thus without precession. The term **apparent** indicates that all precessional, nutational and any other effects have been taken into account - i.e. it is what you will actually get on a given date/time.
5. The Reader should not confuse the astronomical *Constellation* of (e.g.) Pisces with the astrological *House* of Pisces. The two were the same in antiquity. The astrological Houses split the year into 12 equal portions starting at Aries on the Vernal Equinox. This is tropical astrology.

   However there is another branch - called Sidereal astrology, which does recognise the shift in constellations due to Precession.
6. In fact, since our galaxy is expanding, the stars do move relative to one another - their so-called 'proper motion' - but at usually imperceptible rates, unless they are close to the Sun. For example, the declination of our second closest star Alpha Centauri is changing at some 13 seconds of arc per year
7. The current basis for international timekeeping is Temps Atomic International (TAI). This is kept by an array of some 200 atomic clocks, kept in 30 countries around the world. These clocks 'tick' using the vibrations of the Cesium atom. The international standard second is the time taken for 9,192,631,770 cycles of radiation emitted during the transition between two hyperfine levels of the ground state of cesium 133 at 0° Kelvin. 24 x 60 x 60 x 365.242198781 of these original atomic seconds were matched to the length of the tropical year in 1900.

   The practically used time standard is Coordinated Universal Time (UTC) = TAI + a number of 'leap seconds', which are added to correct for the slight slowing of the Earth's rotation. This correction is made to maintain the historic and cultural/religious connection needed to align timekeeping with the 'tick' of the average solar day There have been 35 leap seconds added since 1971. As far as the gnomonist is concerned, UTC equates to the old Greenwich Mean Time - a term now abandoned.

   In order to sense when leap seconds are required and for other astronomical reasons, a further time scale confusingly called Universal Time (UT) is counted from 0 hours at midnight, with the unit of duration of the mean solar day. This is measured by observing the daily motion or various starts and extraterrestrial radio sources. The measured time is called UT0, which is then corrected to UT1, to account for the wobbling of the earth as a result of polar motion. The difference between UT1 (the 'astronomical' tick and UTC (the 'atomic' tick) is referred to as Delta T. Daily values of Delta T are published every week and forward forecast for 6 months. If Delta T exceeds 0.8 seconds, a further leap second will be introduced either on the following 30 June or 31 December.

   Moves - mostly from the computing industry - to abandon Leap seconds have led to an international symposium in 2012. Decisions have been deferred. China consider it important to maintain a link between civil and astronomical time due to Chinese tradition. This may be the clinching argument.

   The serious student of time or of planetary movement must also know all about Terrestrial Time (TT), Geocentric Coordinate Time (TCG), Barycentric Dynamical Time (TDB) and Barycentric Coordinate Time (TCB). These are generally concerned with the relativistic components of time keeping.

return to "Table of Contents"

8. The Julian date system was invented by Joseph Justus Scaliger (1540-1609), a French classical scholar, in 1582, when he invented the Julian period, named after his father, Julius Caesar Scaliger. This was a period of 7,980 = 28 x 19 x 15 years.

28 is the number of years in the Julian calendar that it takes for dates to fall again on the same days of the week, the so-called Solar cycle.

19 is the number of years in the Metonic cycle, devised by Meton of Athens in 432 BCE, although known in China as early as 2260 BCE. The basis of ancient Greek, Jewish, and other calendars, it shows the relationship between the lunar and solar year. In 19 years of exactly 365.25 days each (the Julian, or solar year), there are 235 lunar cycles, with seven of these years having a 13th, or embolistic, month. At the end of the cycle, the phases of the moon recur on a particular day in the solar year. The Metonic cycle was important because it established a lunar calendar having a definite rule for intercalary months, and didn't get out of phase with the cycle of tropical (seasonal) years.

15 is the number of years in the ancient Roman cycle of Indiction, a 15-year period used for taxation. It was used by Emperor Constantine beginning in 312 CE, and continued not only during the Middle Ages, but was used in the Holy Roman Empire until Napoleon abolished it in 1806.

Scaliger chose 12:00 UT, 1 January 4713 BCE as the day 0.0 of the Julian system, since it was the nearest past year when all three cycles - Solar, Metonic and Indiction - exactly coincided. The present Julian period will end at 12:00 UT, 31 December 3267. (Adapted from Ref. 7.)

9. The observant reader will note that the introduction of Julian Date is not strictly necessary. It has been included since it is a frequently used astronomical term. In this case, the numbers a,b,c, & d are all that are required - providing the days since 1st Jan 2000.

10. This equation is an approximation - but good to 0.1 secs over the current century, see Ref. 8. For the ultimate precision, see Ref. 9 and the IAU SOFA computational routines in Ref. 10.

11. For greater precision, one may follow the route taken by the US Naval Observatory's MICA Ref. 14 program uses the expression...

$$EoT^{hrs}_{Astronomical} = GMST^{hrs} + EoE^{hrs} - 12^{hrs} - UTC^{hrs} - RA^{hrs}_{Sun} \quad \text{............. Eqn. 1.15}$$

RA is Apparent Geocentric, True Equator and Equinox of Date. See Note 4 for meaning of *apparent* and *of Date*.

EoE is the Equation of Equinox, which is a small correction to account for nutation (typically of +/- a few seconds).

12. "Now let me see," the Golux said, "if you can touch the clocks and never start them, then you can start the clocks and never touch them. That's logic, as I know and use it..." James Thurber in *The 13 Clocks.*

References

There are a vast number of useful books on Astronomy....

1. J. Davis: *BSS Sundial Glossary:* 2nd Edn, The British Sundial Society (2004)
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5. G.J. Toomer: *Ptolemy's Almagest:* Princeton Press (1998).
6. R.M. Green: *Spherical Astronomy:* Cambridge University Press (1985)
7. Tom Alburger: *The Origin of Julian Days* : www.magma.ca/~scarlisl/DRACO/julian_d.html
8. *Approximate Sidereal Time* : US Naval Oceanography Portal: http://www.usno.navy.mil/USNO/astronomical-applications/astronomical-information-center/approx-sider-time
9. George H. Kaplan :*The IAU Resolutions on Astronomical Reference Systems, Time Scales & Earth Rotation Models :* US Naval Observatory, Circular 179
10. *Standards of Fundamental Astronomy* : IAU SOFA : http://www.iausofa.org

The following were used to calibrate and verify the calculations herein...

11. Peter Duffet-Smith: *Practical Astronomy with your Calculator:* Cambridge Univ. Press, Cambridge (1988).
12. Jean Meeus: *Astronomical Algorithms:* Willman-Bell, Richmond (1998).
13. *Horizons Software:* NASA/JPL: (2012) http://ssd.jpl.nasa.gov/?horizons

    This software uses JPL's DE405 routines which are the gold standard for Solar & Planetary ephemerides.
14. *Multiyear Interactive Computer Almanac - 1800 - 2050:* US Naval Observatory: (2012). This is a high precision astronomical program, that (e.g.) provides EoT to an accuracy of 0.1 second.

Adapted from NASS Compendium 25(3) September 2010 -publshed as 'Basic Asatronomy for Gnomonists'

Basic Positional Solar Astronomy - Part 2. Calculating the Sun's Right Ascension, Declination & EoT

Calculations Required

In Part 1 of this Series, we learnt how to calculate the Greenwich Mean Sidereal Time - GMST, together with the formulae needed to calculate the Equation-of-Time - EoT. In this part we will see how the Sun's actual position in the sky may be found, in terms of...

- the Ecliptic: the Sun's Longitude - λ
- the Equator: its Right Ascension - α or RA - and Declination - δ
- the Local Hour Angle - h
- the Horizon: its Altitude -a - and Azimuth - A
- the approximate times of Sunrise - h_{sr} and Sunset - h_{ss}

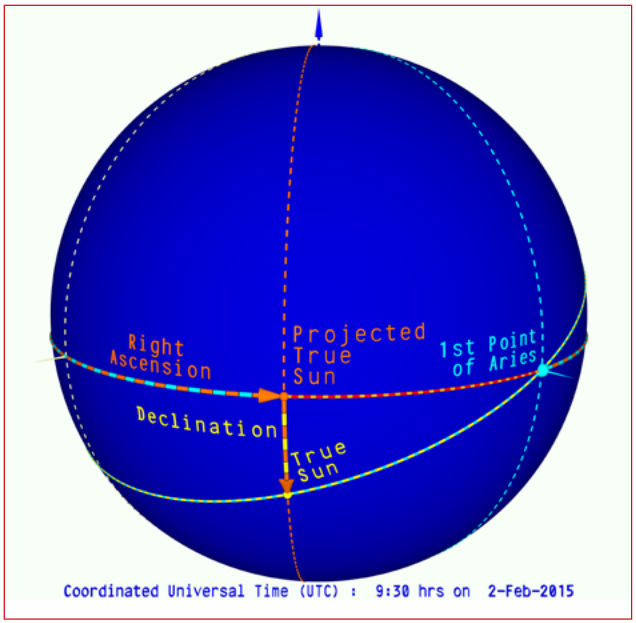

Fig. 1 : It is required to find the actual position of the Sun - in terms of Declination & Right Ascension. The True Sun projected onto the Celestial equator provides the Right Ascension.

Once the RA is found, the Equation of Time can be computed.

$$EoT^{deg}_{astronomical} = GMST^{deg} - \alpha^{deg} - UTC^{deg} + 180^{deg} \text{.......Eqn. 2.1}$$

Figs 1 to 3, repeated from Part 1, show illustrates the essential definitions and show graphically the Equation of Time.

There are two steps in calculating the Sun's Right Ascension & Declination, it is necessary to...

(i) find its position on the Ecliptic. This is the Solar Longitude - λ - which is measured around the Ecliptic, with 0° at the 1st Point of Aries. This is a dynamical problem.

(ii) convert the Solar Longitude (measured around the Ecliptic) to Declination - δ - and Right Ascension - α - (measured around the Equator, but also with 0° at the 1st Point of Aries.)

Figs 4 to 8 show these steps graphically.

Calculating the True Sun's Longitude

This calculation for any given instant relies on three facts...

1 the Longitude of Mean Perihelion [see Note 2] - ω - when the Earth is closest to the Sun, which corresponds to a date around 3rd January. This value is, once more, not exactly constant. Perihelion is moving towards the Vernal Equinox at the rate of 0.17° per century. For convenience, we will use... [See Note 1]

$$\omega^{deg} = 248.545\,360 + 0.017\,196 \times YYYY \text{ .... Eqn. 2.2}$$

where YYYY is the year

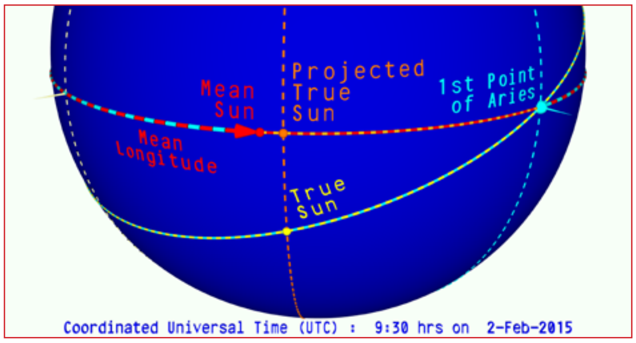

Fig. 2 : Since out civil time-keeping system is tied to the diurnal rotation of the Earth, we have chosen the position of a 'ficticious' Mean Sun on the Celestial Equator as our primary civil time keeping system. We can calculate its position - the Mean Longitude, since it is connected to GMST (see Part 1). The Mean sun rotates around the Celestial Equator once per year.

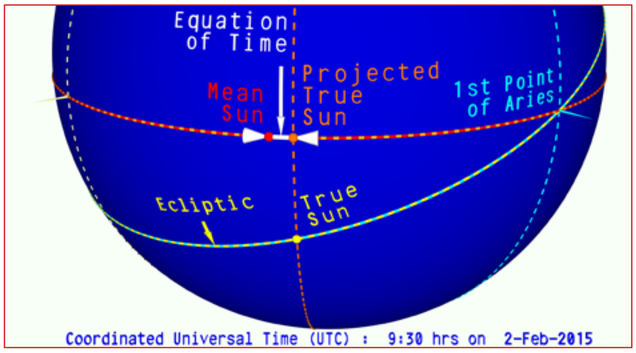

Fig. 3 : The difference between Mean Longitude and Right Ascension is the Equation-of-Time.

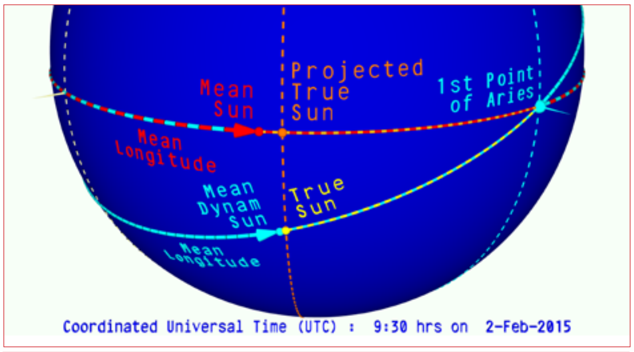

Fig. 4 : It is necessary to invoke the Dynamical Mean Sun, another fictious Sun: this time on the Ecliptic. It is. It rotates uniformly around the ecliptic, once per year (as does the Mean Sun). Thus, its position is also defined by the Mean Longitude - but measured along the Ecliptic.

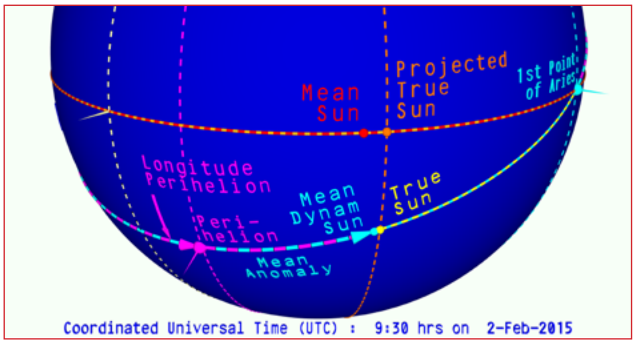

Fig. 5 : The dynamics of the elliptical movement of the True Sun is tied to Perihelion - when the sun is closest to the Earth. The Longitude of Perihelion (origin 1st Point of Aries) is an astronomically known fact. The Mean Longitude is equal to Longitude of Perhelion + the Mean Anomaly.

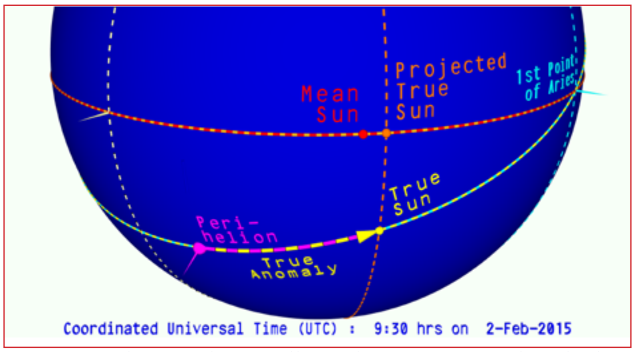

Fig. 6 : Keplarian physics allows the True Anomaly, which is the position of the True Sun with respect to Perihelion, to be calculated in terms of the Mean Anomaly.

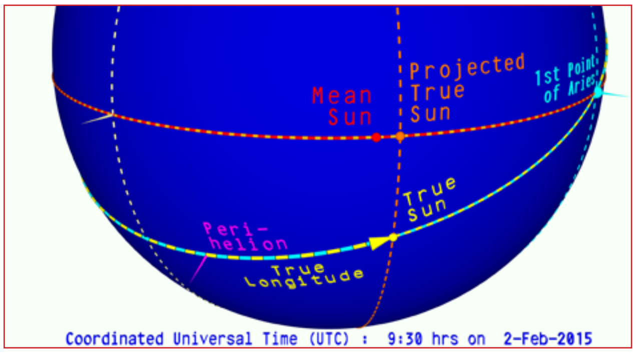

Fig. 7 : Adding the True Anomaly to the Longitude of Perihelion yields the True Longitude of the Sun.

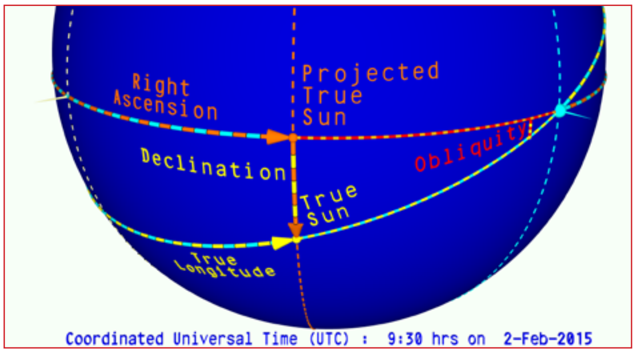

Fig. 8 : Spherical Trigonometry, involving the True Longitude and the Obliquity, yields both Right Ascension & Declination

2 the Sun's apparent orbit is an ellipse - Kepler's First Law - with eccentricity - e - of 0.016 713. This value is not actually constant, but varying marginally... See Note 1

$$e = 0.017585 - 0.438 \times \left(\mathrm{YYYY} / 1\,000\,000\right)$$ .... Eqn. 2.3

3 the apparent movement of the Sun obeys Kepler's Third Law - that a line joining the Earth to the Sun will sweep out equal areas in equal times.

This calculation requires the introduction of some new concepts and some very old mediaeval terms. Whereas we have used the 1st Point of Aries as our prime celestial origin, for elliptical orbits, we use instead the direction of Perihelion, when the Sun is closest to the Earth. Refer to Fig. 9, which is in the plane of the Ecliptic, unlike those illustrations in Part 1 of the series, which are in the plane of the celestial equator. For illustrative clarity, this shows an elliptical orbit of eccentricity of 0.4. The true value is a minute 0.0175, which if used in the diagram would make the elliptical path visually indistinguishable from a circle

Note the following...

- the *Earth*, at the centre of the illustration
- the *True Sun*, travelling on an ellipse, with the Earth at one of the ellipse's foci. Its position in relation to Perihelion - when the sun is closest to the earth - is called the *True Anomaly* - λ
- the imaginary *Mean Dynamical Sun* on the Celestial Ecliptic, a circle centred on the Earth. This body uniformly travels around the Ecliptic once in a tropical year. It is coincident with the Mean (equatorial) Sun at the 1st Point of Aries. It is thus the exact equivalent to the Mean Sun (on the Equator). Importantly, referenced to the 1st Point of Aries, at any moment in the year, its longitude on the Ecliptic is identical to the longitude of the Mean Sun on the Equator. Hence it can be calculated in terms of GMST. Its position in relation to perihelion is called the *Mean Anomaly* - M
- the imaginary *Eccentric Sun*, travelling on a circular path, whose centre is the centre of the ellipse, such that it is vertical (in the picture) above/below the True Sun. The Eccentric Sun is another imaginary body, which is only required as an intermediate to solve Kepler's Third Law. Its position in relation to perihelion is called the *Eccentric Anomaly* - E.
- the longitude of Perihelion - ω - provides the link between longitude and anomalies

$$M_0 = M + \omega$$ .... Eqn. 2.4

$$\lambda = \nu + \omega$$ .... Eqn. 2.5

Application of Kepler's third law reveals the connection between the Eccentric Anomaly, and the Mean Anomaly is...

$$M^{rad} = E^{rad} - e \times \sin\left(E^{rad}\right)$$ .. Kepler's Formula .. Eqn. 2.6

Appendix 1 provides the derivation of this equation in the 17C method used before calculus was common. Unfortunately, Kepler's Formula - combining an angle E together with its trigonometrical sine - is not directly

return to "Table of Contents"

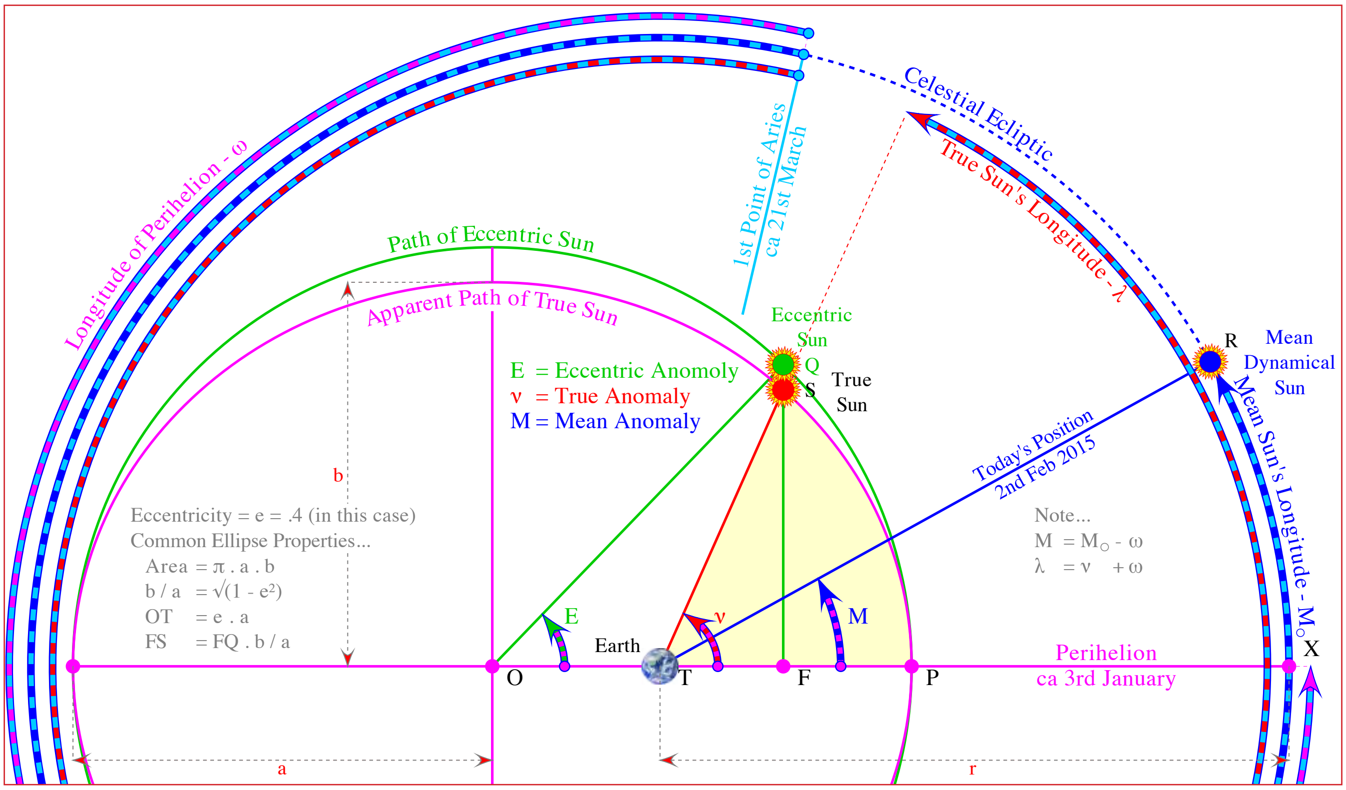

Fig. 9 : True, Dynamical & Eccentric Suns, viewed in the ***Ecliptic*** *Plane, with a vastly increased eccenticity of 0.4*

soluble. It requires an iterative solution. Application of a Newton Raphson approximation shows that - since the eccentricity of the ellipse is so near to zero - only one single iteration is required, to give the value of E

$$E^{rad} = M^{rad} - {\left(e \times \sin\left(M^{rad}\right)\right)} \Big/ {\left(e \times \cos\left(M^{rad}\right) - 1\right)} \text{... Eqn. 2.7}$$

Appendix 2, Figs 17 to 22 provides the derivation of this equation

The True Anomaly is connected to the Eccentric Anomaly by trigonometry...

$$\nu = \operatorname{atan2}\left(\sqrt{(1 - e^2)} \times \sin(E),\ \left(\cos(E) - e\right)\right) \text{....... Eqn. 2.8}$$

Appendix 2, Fig. 23 provides the derivation of this equation. There is an alternate often quoted formula, see Note 3.

CALCULATING THE RIGHT ASCENSION & DECLINATION

Knowing the Sun's Longitude and the Obliquity of the Ecliptic, it is simply a matter of solving a Spherical right angle triangle to find the Right Ascension & Declination.

Obliquity in degrees is given by... See Note 1

$$\varepsilon^{deg} = 23.69930^{deg} - 0.00013 \times \text{YYYY.................Eqn. 2.9}$$

The right-angled triangle can be solved using Napier's pentagon which is a mnemonic aid that helps to find all relations between the angles in a right spherical triangle.

The mnemonic works thus... Write the six angles of the triangle (three vertex angles, three arc angles) in the form of a circle, sticking to the order as they appear in the triangle (i.e. start with a corner angle, write the arc angle of an attached side next to it, proceed with the next corner angle, etc. and close the circle). Then cross out the 90° corner angle and replace all angles non-adjacent to it by their complement to 90° (i.e. replace, say, λ by 90° – λ). The five numbers that you now have on your paper form Napier's Pentagon.

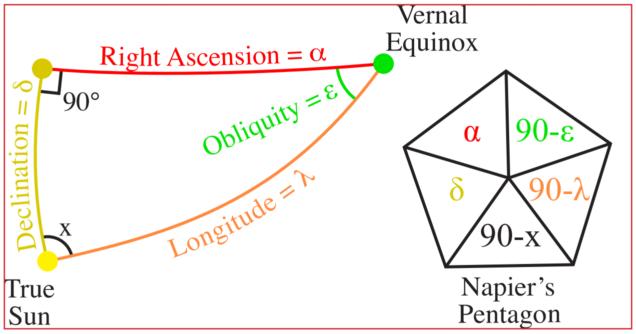

Fig. 10 : Napier's Pentagon. The careful reader will note that this illustration does not conform to the others in this paper. As shown, δ would be calculted as a +ve number. Rest assured that the trigonometry works and using the conforming 360 - α & 360 - λ will provide a negative value of δ.

For any choice of three angles, one (the middle angle) will be either adjacent to or opposite the other two angles. Then Napier's Rules hold that the sine of the middle angle is equal to:

- the product of the cosines of the opposite angles, as in Fig. 11, thus...

$$\sin(\delta) = \cos(90 - \varepsilon) \times \cos(90 - \lambda)$$

$$\partial = \sin^{-1}\left(\sin(\varepsilon) \times \sin(\lambda)\right) \quad \text{................ Eqn. 2.10}$$

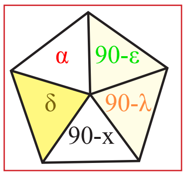

Fig. 11 : Declination

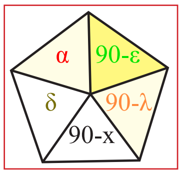

Fig. 12 : Right Ascension

- the product of the tangents of the adjacent angles, as in Fig. 12, thus...

$$\sin(90-\varepsilon) = \tan(\alpha)\times\tan(90-\lambda)$$
$$\tan(\alpha) = \cos(\varepsilon)\times\tan(\lambda)$$
$$\alpha = \operatorname{atan2}\left(\cos(\varepsilon)\times\sin(\lambda),\cos(\lambda)\right)$$

.... Eqn. 2.11

CALCULATING THE LOCAL HOUR ANGLE

All the astronomical calculations so far have related to Greenwich. In order to calculate the Sun's Altitude and Azimuth for an observer at a particular time of day and at a particular terrestrial location, we will require to find its Local Hour Angle...

The Sun's Local Hour Angle is the angle between the Sun's meridian and the Observer's meridian.

At solar noon, the LHA is zero. Following normal practice, the LHA is negative before noon and positive after noon. In this document, however, it is counted positive from noon.

Looking at Fig. 13, we can deduce the connection between LHA - **h^0** -, Right Ascension - **α^0** - Greenwich Mean Sidereal Time - GMST - and the observer's longitude - **λ_t^0**. The LHA is the innermost dotted arc. The green arrow is 360^0 - LHA^0 (and is the 'normal' definition of LHA). Working from the outer arc, it is apparent that the Green arc =...

$$\alpha^{deg} - \lambda^{deg} - GMST^{deg} = 360^{deg} - h^{deg}$$

$\therefore$

$$h^{deg} = GMST^{deg} + \lambda^{deg} - \alpha^{deg}$$ ....Eqn. 2.12

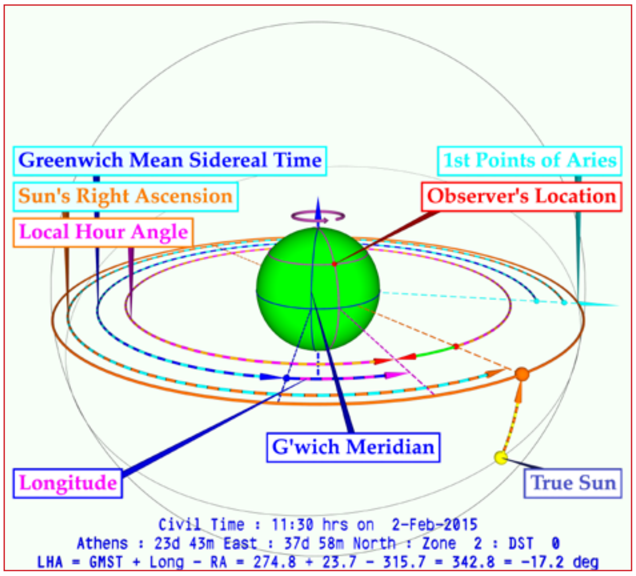

Fig. 13 : Local Hour Angle

CALCULATING THE SUN'S ALTITUDE AND AZIMUTH

All the calculations so far in this paper have related to the Celestial Sphere. Now we must introduce the position of the observer at a given terrestrial Latitude & Longitude

Fig. 14 shows the situation at a given time. Note the...

1) Equatorial Plane (olive coloured) from which are measured the...

- Sun's declination (the orange arcs) - already calculated
- Observer's Latitude (the purple arcs) - known
- Observer's location with respect to the Sun: the Local Hour Angle (the red arc) - already calculated

2) Observer's horizontal plane (greenish coloured), from which is measured ...

- Sun's Altitude (the blueish arcs) - to be found
- Sun's Azimuth (the green arc) - to be found

Fig. 15 strips away extraneous detail to show the spherical triangles involved. While Fig. 16 shows the final spherical triangle to be solved.

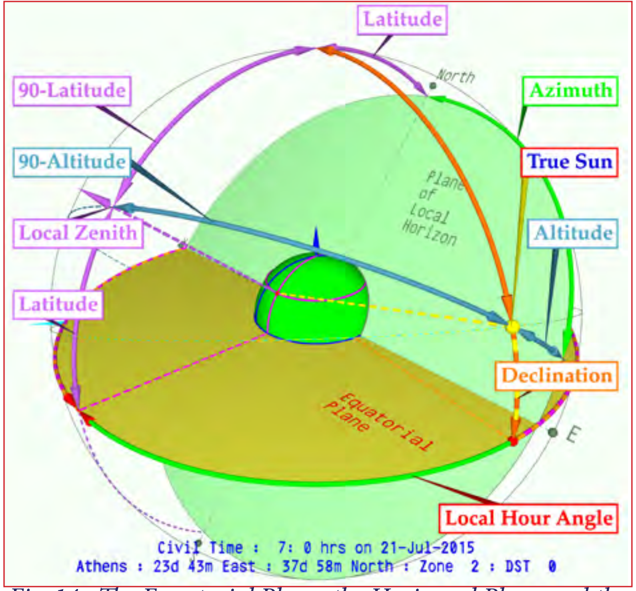

Fig. 14 : The Equatorial Plane, the Horizonal Plane and the Observer

In spherical trigonometry see Ref. 1, the spherical laws of cosines and sines state that...

$$\cos(c) = \cos(a)\times\cos(b) + \sin(a)\times\sin(b)\times\cos(C)$$
$$\sin A/\sin(a) = \sin(B)/\sin(b) = \sin(C)/\sin(c)$$

.........Eqn. 2.13

where a, b & c are the angular arc lengths, while A is the angle between arcs b & c, etc. Applying the cosine law to Fig. 16, twice...

$$\cos(90-Alt) = \cos(90-Lat)\times\cos(90-Decl) + \ldots$$
$$\sin(90-Lat)\times\sin(90-Decl)\times\cos(h)$$

.... Eqn. 2.14

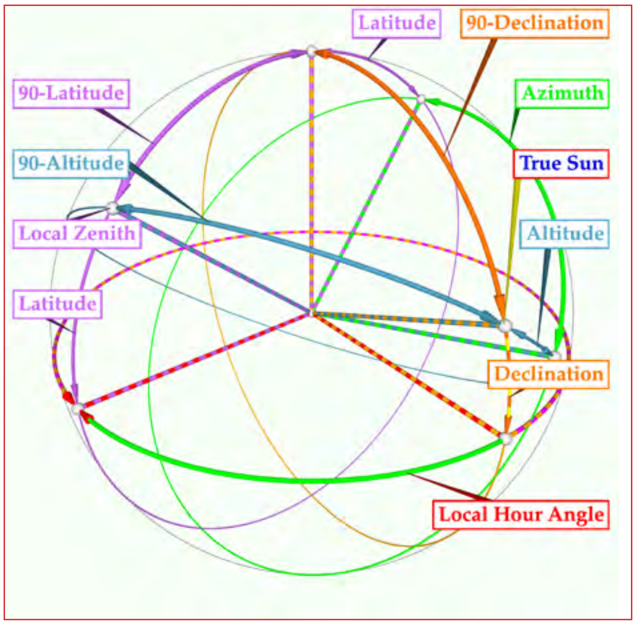

Fig. 15 : As Fig. 13. but with extraneous information removed

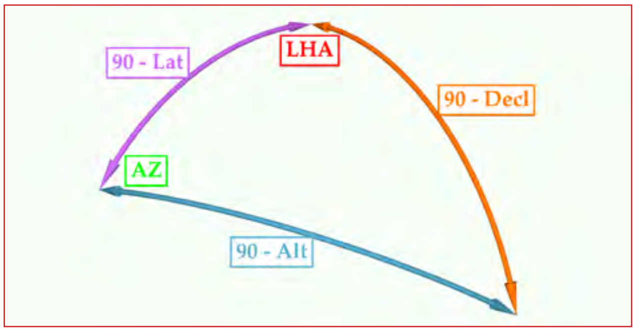

Fig. 16 : The essential spherical triangle

and

$$\cos(90-Decl) = \cos(90-Lat)\times\cos(90-Alt)+\ldots$$
$$\sin(90-Lat)\times\sin(90-Alt)\times\cos(Az)$$

.... Eqn. 2.15

Converting these to standard nomenclature gives the Sun's Altitude...

$$a^{rad} = \sin^{-1}\left(\sin(\phi)\times\sin(\delta)+\cos(\phi)\times\cos(\delta)\times\cos(h)\right)$$

.... Eqn. 2.16

$$\cos(A) = \left\{ \sin(\delta)\times\sin(\phi)\times\sin(a) \Big/ \cos(\phi)\times\cos(a) \right\}$$

.... Eqn. 2.17

Equation 2.17 provides some ambiguity to the azimuth value since (e.g.) the cosine of both 170° & 190° are the same. But if the sine law is applied..

$$\sin(A) = \cos(\delta)\times\sin(-h)/\cos(a)$$ .... Eqn. 2.18

then with no ambiguity, combining Eqn. 2.13 with Eqn. 2.14

$$A^{rad} = \operatorname{atan2}\left(\sin(A),\cos(A)\right)$$ .... Eqn. 2.19

FINDING THE TIMES OF SUNRISE AND SUNSET

Sunrise and Sunset are defined as the moment when the *apparent* centre of the Sun's disc is at zero altitude. In addition, the twilights are defined in terms of the *apparent* altitude of the centre of the Sun's disk

- civil twilight altitude 0° to -6°
- nautical twilight: altitude -6° to -12°
- astronomical twilight: altitude -12° to -18°

Apparent altitude is the term used when the altitude is corrected for the effect of atmospheric refraction. The degree of refraction is dependent on the temperature and pressure of the atmosphere. There are empirical formulae allowing its estimation, which are presented - without comment - in steps 63 - 67 of Table 1, below. But See Chapter 16 of Ref. 2 for further elaboration. Refraction can be around ½° at altitudes close to zero in temperate climates. This is approximately equal to the angular size of the whole of the Sun's disc. One cannot find the moment of sunset without knowing atmospheric conditions and then iterating through the refraction calculations.

For most gnomonists, it is sufficient to estimate in the following fashion...

- forget about refraction
- calculate the declination δ at midday
- calculate the longitude corrected gnonomical Equation of Time, $\text{EoT}_{\text{Local}}$ at midday
- put altitude = 0 into Eqn. 25.

This yields the sunrise/set hour angle to be...

$$h^{deg}_{Sunrise/set} = \pm\cos^{-1}\left(-\tan(\varphi)\times\tan(\delta_{Noon})\right)\times 180^{deg}\Big/\pi$$

.... Eqn. 2.20

Then, converting to hours & including the EoT, yields the time and azimuth of Sunrise and Sunset...

$$T^{hrs}_{Sunrise} = 12^{hrs} - \left(h^{deg}_{Sunrise/set}\Big/15\right) - EoT^{hrs}_{Local}$$ .... Eqn. 2.21

$$T^{hrs}_{Sunset} = 12^{hrs} + \left(h^{deg}_{Sunrise/set}\Big/15\right) - EoT^{hrs}_{Local}$$ .... Eqn. 2.22

$$A^{deg}_{Sunrise/set} = \pm\cos^{-1}\left(-\sin(\delta_{Noon})\Big/\cos(\phi)\right)\times 180^{deg}\Big/\pi$$

...............Eqn. 2.23

In passing, we may note that, adding together Eqns. 2.17 & 2.18, gives

$$EoT^{mins}_{Local} = 30\times\left(T^{hrs}_{Sunset} + T^{hrs}_{Sunset} - 24\right)$$ ....Eqn. 2.24

which means that, if you read the time of sunrise and sunset from your *local* newspaper, you can find the latitude corrected Equation of Time for your location. This was a trick used from Victorian times[See Ref. 3 & Note 4]. Since sunrise and sunset are usually only quoted to the nearest minute, it is somewhat surprising that this somewhat crude method gives the Equation of Time accurate to +/- 1 minute throughout the year in temperate latitudes.

return to "Table of Contents"

WORKED EXAMPLE

The Table, below, consolidates all the calculations in Parts 1 and 2. The functions that are used are given at then end of this section. Note carefully, that in some applications, these functions may not be present or called in a different manner.

In the Table above, the columns are…

- i line number.
- ii name of parameter
- iii the parameters symbol, with a qualifier subscripted and its units superscripted, thus $\text{EoT}_{\text{Gnomical}}^{\text{Min}}$
- iv the worked example resulting value
- v the required formulae
- vi the Equation number from the text - numbers thus 1.nn relate to Part 1 of this series, 2.nn to this part.

Where figures are given in red bracketed italics these are the results of working this example through a precision astronomical program, $^{\text{See Ref. 4}}$.

Functions that are used in the Table are...

- **degrees** & **radians** function - may be replaced by × 180 / π or by × π / 180
- trigonometric functions, **sin**, **cos** & **tan**. In most implementations, these require input in radians: while the inverse functions asin, acos, atan output in radians. If this is not the case, many of the degree/radian conversions below can be ignored - but not in Steps 34-37, where radians must be used. Note that in traditional trigonometry **asin** was written as **sin**$^{-1}$.
- **atan2** function - this now exists in most programming languages and returns the inverse tangent function in the correct quadrant, but requires both an x and y input parameter. Irritatingly, while most scientific languages implement this as the more trigonometrically correct atan2(y,x), Microsoft Excel uses atan2(x,y).
- **int** function - this simply strips the fractional part of a number away. Note, once more, that most scientific languages implement this strictly for positive & negative number. Thus int(1.6) = 1 and int(-1.6) = -1, but once more Microsoft Excel differs: int(1.6) = 1 but int(-1.6) = -2. This difference in not of interest below, since the int function operates only on positive numbers
- **mod** function. Particularly in angular calculations, this reduces a number to lie in a particular range (e.g. from 0° to 360°). Thus mod(370°,360°) = 10° = mod(-350°, 360°). Some languages make this function into an arithmetic operator: thus, in Python, 370 % 360 = 10.

Input Observer’s Location					
1	Longitude +ve East of Greenwich	λ_t °	23.71667	The Acropolis, Athens	
2	Latitude +ve North of Equator	φ °	37.96667		
3	Time Zone +ve East of Greenwich	TZ^{hrs}	2		
Input Observer’s Date & Civil Time - (that is the Time that one reads on a clock or hears on the radio)					
4	Summer Time	DST^{hrs}	0		
5	Year	YYYY	2015	11:30 a.m 2nd February 2015	
6	Month	MM	2		
7	Day	DD	2		
8	Hour	HH	11		
9	Minute	MM	30		
Time related Parameters, Greenwich Mean Sidereal Time & the Sun’s Mean Longitude					
10	UTC Uncorrected	$\text{UTC}_{\text{uncorr}}^{\text{hrs}}$	9.5	$\text{ST}^{\text{hrs}} - \text{TZ}^{\text{hrs}} - \text{DST}^{\text{hrs}}$	1.2
11	UTC Corrected	UTC^{hrs}	9.5	$\text{mod}(\text{UTC}_{\text{uncorr}}^{\text{hrs}}, 24)$	
12		UTC °	142.5	$15 \times \text{UTC}^{\text{hrs}}$	
13	temporary value aaa is the correction to be made if the local date differs from the date at Greenwich	aaa	0	a = 0	1.3
14			0	$\text{if } (\text{UTC}_{\text{uncorr}}^{\text{hrs}} < 0) \quad a = -1$	
15			0	$\text{if } (\text{UTC}_{\text{uncorr}}^{\text{hrs}} > 24) \; a = +1$	
16	temporary value	bbb	8973.5	367 × YYYY − 730 531.5	1.6
17	temporary value	ccc	3526	int({7. × int(YYYY + [MM + 9] / 12)} / 4)	
18	temporary value	ddd	63	int(275 × MM / 9) + DD	
19	Days since Midnight	$\text{D}_{\text{today}}^{\text{days}}$	0.39583	$\text{UTC}^{\text{hrs}} / 24$	
20	Days to 0:00 am since Epoch	$\text{J2000}^{\text{days}}$	5510.5	aaa + bbb − ccc + ddd	
21	Julian Centuries$_{2000}$	$\text{T}^{\text{Jul Cent}}$	0.15088	$\text{J2000}^{\text{days}}/365\,25$	
22	Days to Now since Epoch	$\text{D}_{2000}^{\text{days}}$	5510.89583	$\text{J2000}^{\text{days}} + \text{D}_{\text{Today}}^{\text{days}}$	
23	Greenwich Mean Sideral Time	GMST^{hrs}	18.31737 *(18.31737)*	mod(6.697374558 $+ 0.065\,709\,824\,419\,08 \times \text{J2000}^{\text{days}}$ $+ 1.002\,737\,909\,35 \times \text{UTC}^{\text{hrs}}$ $+ 0.000\,026 \times \text{T}^{\text{Jul Cent 2}}$, 24)	1.4
24		GMST °	274.76059	GMST^{hrs} x 15	
25	Sun’s Mean Longitude	M_{O} °	312.26059	mod{ (GMST ° − 180 ° − UTC °), 360 ° }	1.7
26		$\text{M}_{\text{O}}^{\text{rad}}$	5.44998	M_{O} ° × π / 180	

Table - Part 1

return to "Table of Contents"

			Astronomical Facts		
27	Perihelion Longitude	ω°	283.19530	$248.545\,36 + 0.017\,196 \times YYYY$	2.2
28		ω^{rad}	4.94269	$\omega^{\circ} \times \pi / 180^{\circ}$	
29	Eccentricity	e	0.01670	$0.017\,585 - 0.438 \times YYYY / 1{,}000{,}000$	2.3
30	Obliquity	ε°	23.43735 *(23.43758)*	$23.699\,3 - 0.000\,13 \times YYYY$	2.8
31		ε^{rad}	0.40906	$\varepsilon^{\circ} \times \pi / 180^{\circ}$	
			Solving Kepler's Theorem & Sun's True Longitude		
32	Mean Anomaly	M^{rad}	0.50729	$M_O^{\ rad} - \omega^{rad}$	2.4
33	Eccentric Anomaly	E^{rad}	0.51552	$M_O^{\ rad} - \sin(M_O^{\ rad}) / \{\cos(M_O^{\ rad}) - 1 / e\} \qquad = M^{rad}$	2.7
34	2nd iteration for example only >		0.51552	$E^{rad} - [M - E^{rad} + e \times \sin(E^{rad})] \div [e \times \cos(E^{rad}) - 1]$	-
35	True Anomaly	ν^{rad}	0.52381	$2 \times \text{atan}\{\tan(E^{rad} / 2) \times \sqrt{\ } [(1 + e) / (1 - e)]\}$	2.8
36	Sun's True Longitude	λ^{rad}	5.46650	$\nu^{rad} + \omega^{rad}$	2.5
37		λ°	313.20765 *(313.70149)*	$\lambda^{rad} \times 180^{\circ} / \pi$	
			Sun's Declination, Right Ascension & the Equation of Time		
38	Sun's Declination	δ^{rad}	-0.29413	$\text{asin}\{\sin(\varepsilon^{rad}) \times \sin(\lambda^{rad})\}$	2.10
39		δ°	-16.85245 *(-16.85158)*	$\delta^{rad} \times 180 / \pi$	
40	Sun's Right Ascension	α^{rad}	-0.77365	$\text{atan2}\{ \cos(\varepsilon^{rad}) \times \sin(\lambda^{rad}), \cos(\lambda^{rad}) \}$	2.11
41		α°	315.67321	$\text{mod}(\alpha^{rad} \times 180 / \pi , 360)$	
42		α^{hrs}	21.04488 *(21.04468)*	$\alpha^{\circ} / 15$	
43	Equation of Time	EoT°	-3.41262	$GMST^{\circ} - \alpha^{\circ} - UTC^{\circ} + 180^{\circ}$	1.9
44		$EoT_{Astro}^{\ \circ}$	-3.41262	$\text{if}(EoT_X^{\circ} < -180^{\circ})\ EoT_{Astro}^{\circ} = EoT^{\circ} + 360^{\circ}$	
45			-3.41262	$\text{if}(EoT_X^{\circ} > +180^{\circ})\ EoT_{Astro}^{\circ} = EoT^{\circ} - 360^{\circ}$	
46		$EoT_{Gnomical}^{\ \circ}$	3.41262	$- EoT_{Astro}^{\ \circ}$	1.10
47		$EoT_{Gnomical}^{\ min}$	13.65049 *(13.63333)*	$4 \times EoT_{Gnomical}^{\ \circ}$	
48	Longitude Correction	σ°	-6.28333	$LON^{\circ} - TZ^{hrs} \times 15$	1.11
49		σ^{min}	-38.84648	$\sigma^{\circ} \times 4$	
50	EoT Longitude Corrected	EoT_{Local}^{min}	-38.78381	$EoT_{Gnomical}^{\ min} + \sigma^{min}$	1.12
			The Sun's Altitude & Azimuth		
51	Observer's True Hour Angle	h°	342.80405 *(342.80778)*	$\text{mod}\{ (GMST^{\circ} + \lambda_t^{\ \circ} - \alpha^{\circ}),\ 360 \}$	2.12
52		h^{rad}	5.98306	$h^{\circ} \times \pi / 180^{\circ}$	
53	Observer's Latitude	φ^{rad}	0.66264	$\varphi^{\circ} \times \pi / 180^{\circ}$	-
54	Sun's Altitude	a^{rad}	0.57333	$\text{asin}\{ \sin(\varphi^{rad}) \times \sin(\delta^{rad})$ $+ \cos(\varphi^{rad}) \times \cos(\delta^{rad}) \times \cos(h^{rad}) \}$	2.16
55		a°	32.84937	$a^{rad} \times 180^{\circ} / \pi$	
56	Sun's Zenith Distance	z°	57.15063 *(57.01570)*	$90^{\circ} - a^{\circ}$	-
57	Sun's Azimuth	$\sin A$	0.33680	$\cos(\delta^{rad}) \times \sin(-h^{rad}) / \cos(a^{rad})$	2.18
58		$\cos A$	-0.94158	$(\sin(\delta^{rad}) - \sin(a^{rad}) \times \sin(\varphi^{rad}))$ $/ (\cos(a^{rad}) \times \cos(\varphi^{rad}))$	2.17
59		A^{rad}	2.79808	$\text{atan2}(\sin A, \cos A)$	2.18
60		A°	160.31807 *(160.32)*	$\text{mod}(A^{rad} \times 180^{\circ} / \pi, 360^{\circ})$	

Table - Part 2

The Refraction Correction for the Sun's Altitude - these are empirical formulae, see Ref. 2, they are not detailed in the text.					
61	Input Temperature	$T^{\,°C}$	20	Input	-
62	Input Atmospheric Pressure	$P^{\text{millibars}}$	1020		
63	Refraction Correction	$R\,°$	0.02389	if $(a° > 15°)\ R° = 0.004\,52 \times \tan(z° \times \pi / 180) \times P^{\text{millibars}} / (273 + T^{\,°C})$	-
64			n.a.	if $(a° < 15°)\ R° = P^{\text{millibars}} \times (0.1594 + 0.0196 \times a° + 0.00002 \times a°^{2}) / \{(273 + T^{\,°C}) \times (1 + 0.505 \times a° + 0.084\,5 \times a°^{2})\}$	
65	Sun's Altitude Corrected	$a_{\text{Corr}}°$	32.82548	$a° - R°$	

Table - Part 3

Accuracies

In the calculations above, the only non-derived astronomical parameters used are the...

- length of the tropical year,
- eccentricity of the Earth's orbit,
- obliquity of the Ecliptic,
- longitude of perihelion,
- a single factor covering precession.

With this small coterie of values, it is perhaps remarkable that a relatively simple (if long) approach can yield the accuracies stated over a period of 50 years.

GMST	+/- 0.00 $^{\text{secs}}$
Right Ascension	+/- 3 $^{\text{secs of time}}$
Declination	+/- 18 $^{\text{secs of arc}}$
Equation of Time	+/- 2.2 $^{\text{secs of time}}$
Altitude	+/- 0.7 $^{\text{minutes of arc}}$
Azimuth	+/- 1.3 $^{\text{minutes of arc}}$

The stated accuracies have been derived with reference to 75,000 calculations using the 2012 edition of the US Naval Observatory's MICA program [see Ref. 4].

The above calculations are more than sufficient for most gnomonists. However, if one wishes to pursue the calculations to a greater degree of accuracy. There are a number of factors that have to be considered

- The slowing of the year's rotation, as seen in the introduction of leap seconds in the calendar.
- The fact that solar dynamics use difference time and position reference frameworks.
- The 'correct' dynamical approach calculates the Earth's longitude for a particular instant of time. Sunlight reaches the Earth some 8 minutes later. During this time the Earth has moved somewhat. This effect is called Aberration.
- We have calculated the Sun's longitude about the Ecliptic and assumed that its latitude is zero. This is not quite true.
- We have ignored the "rattling and banging" of Nutation, which varies right ascension by up to 20 $^{\text{secs of arc}}$ and obliquity by up to 10 $^{\text{secs of arc}}$. Nutation is caused by the gravitational pull of the Moon (& especially Jupiter) on the equatorial bulge of the Earth's shape.
- Our calculations relate to the centre of the Earth. Our position on the surface of the Earth varies the values of both Right Ascension & Declination.

If the reader wishes to delve deeper, Ref. 4 provides a useful outline and Ref. 2 provides the greatest depth achievable without access to serious professional astronomical computing routines. The latter are available [see Ref. 6], through the International Astronomical Union. However , their use by amateurs requires knowledge of Fortran or the "C" programming language.

APPENDIX 1 - DERIVATION OF KEPLER'S LAW

Kepler's Equation...

is the result of his 1st and 2nd Laws of Planetary Motion

i The orbit of every planet is an ellipse with the Sun at one of the two foci.

ii A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

The means of developing this formula therefore demands that we can calculate the area swept out in any given time, e.g. from Perihelion. This is the yellow shaded area in Fig. 9. Finding this area can be done by

$$M^{rad} = E^{rad} - e \times \sin\left(E^{rad}\right) \quad \text{Eqn. 2.25}$$

simple means using an old technique,[see Ref. 7.] The steps required are shown in Figs 16 to 22 below.

The next step shown in Fig. 23, not quite so easy to grasp, relates to the "equal areas during equal intervals of time". This indicates that the area just calculated is proportional to the area swept out by the Mean Dynamical Sun in the same period.

Finally, Fig. 24 shows how the true anomaly - ν - is related to the Eccentric Anomaly - E

APPENDIX 2 - DERIVATION OF NEWTON RAPHSON APPROXIMATION FOR KEPLER'S FORMULA

Kepler's Formula, Equn, 2.25, cannot be solved directly.

So an iterative solution must be sought. The Newton-Raphson method [See Ref. 8] is an efficient method, provided that one can differentiate the function con-

return to "Table of Contents"

cerned. The method states that, if an estimation E_n is obtained, a better estimation E_{n+1} may be obtained, thus...

but , rewriting Eqn. 2.25 and differentiating...

To apply the Newton Raphson formula, we make a guess to start the process and then repeatedly put Eqns. 2.23 & 227 into Eqn. 2.28...

and we repeat the process until there is negligible difference between E_n and E_{n+1}

We make our first guess. as $E_1 = M$, then...

$$E_{n+1} = E_n - \frac{fn(E_n)}{fn'(E_n)} \text{ .... Eqn. 2.26}$$

Since the eccentricity is so small, it transpires that

$$fn(E_n) = M - E_n + e \times \sin(E_n) \text{ ....Eqn. 2.27}$$

$$fn'(E_n) = e \times \cos(E_n) - 1 \text{ .... Eqn. 2.28}$$

this is the only iteration needed ! It is left to the reader to show that, for any value of M^{rad} between 0 and 2π, the difference between E_2 and E_3 is less than +/- .5 $^{\text{seconds of arc}}$, which is sufficiently precise for that which is required by the dialist. The difference between E_3 and E_4 is effectively zero.

$$\begin{aligned} E_2 &= M - \left(M - M + e \times \sin(M)\right) / \left(e \times \cos(M) - 1\right) \\ &= M - \left(e \times \sin(M)\right) / \left(e \times \cos(M) - 1\right) \text{............ Eqn. 2.30} \end{aligned}$$

Notes

1. Equations for Eccentricity, Obliquity & Longitude of Perihelion were adapted from the formulae quoted in the Astronomical Almanac[Ref. 10].
2. If one consults the Astronomical Almanacs over the years, the reader will note that the moment of Perihelion varies back & forth in an apparently random fash-

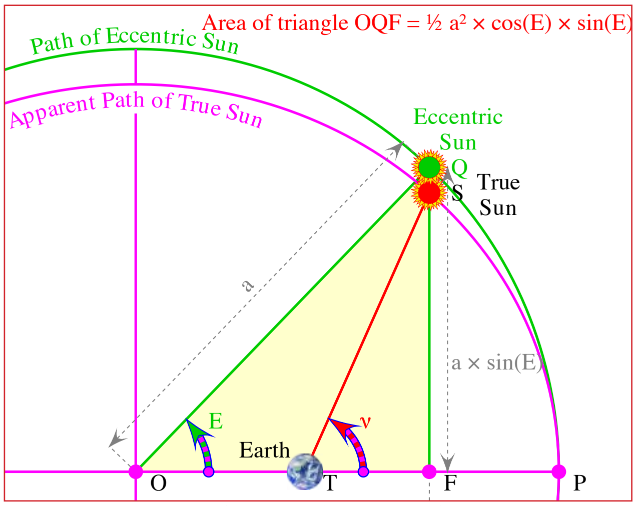

Fig. 17. Solving Kepler's Formula - Step 1

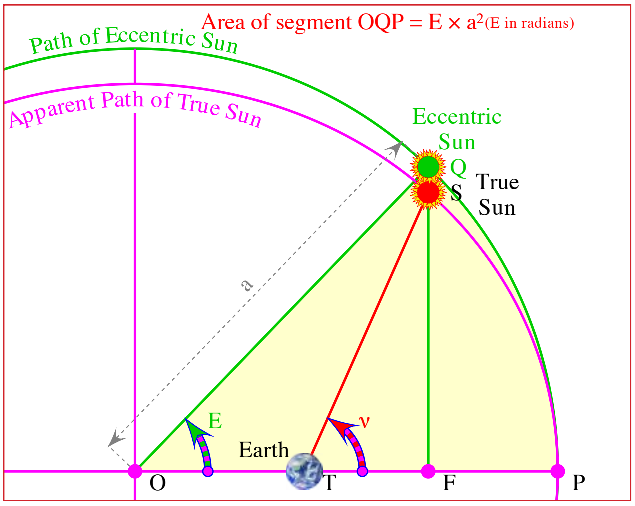

Fig. 18. Solving Kepler's Formula - Step 2

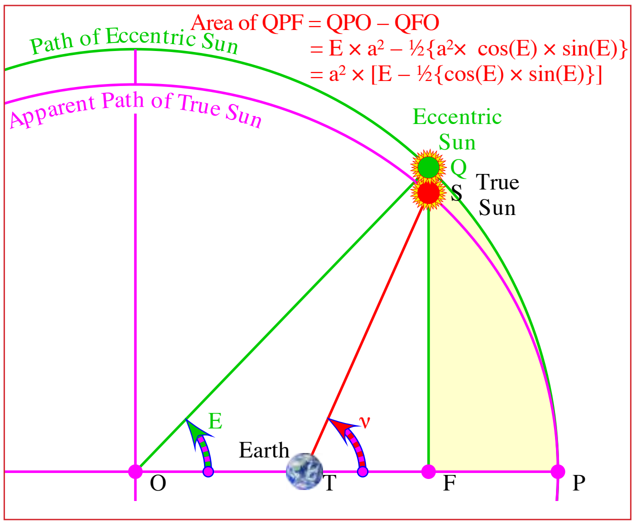

Fig. 19. Solving Kepler's Formula - Step 3

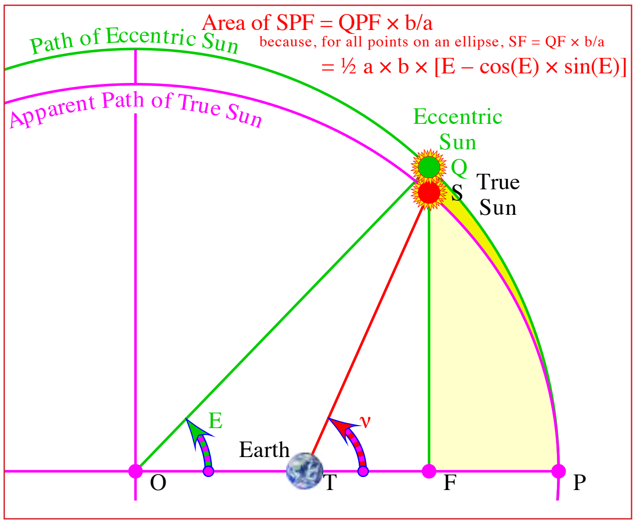

Fig. 20. Solving Kepler's Formula - Step 4

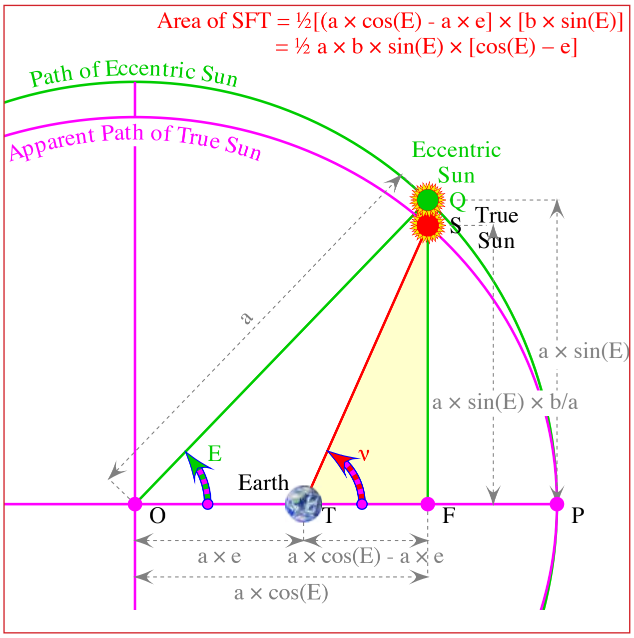

Fig. 21. Solving Kepler's Formula - Step 5

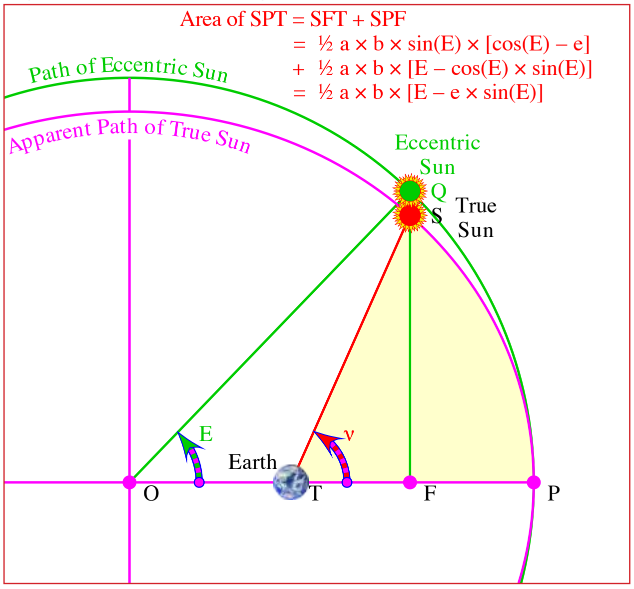

Fig. 22. Solving Kepler's Formula - Step 6.
The yellow segment is the area swept out by the True Sun

Fig. 23. is overleaf

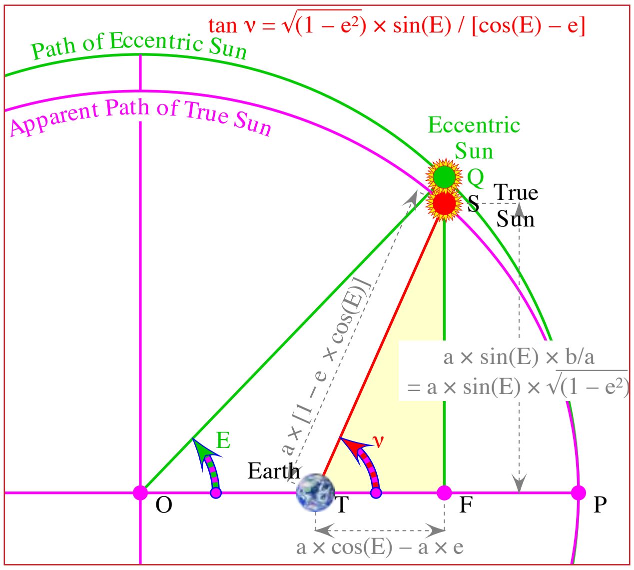

Fig. 24. Solving Kepler's Formula - Step 8

ion between Jan 2 and Jan 5th as shown below...

The table given the moment the centre of the Earth is closest to the Sun. The mean value of Perihelion - ω, as given in the equations presented in this paper, is the moment when the centre of gravity of the Earth/Moon combination is closest to the Sun. This combined mass has its centre of gravity some 1700 kms below the Earth's surface - about ¼ of the way towards the Earth's centre. As far as Keplarian physics is concerned, the calculations above relate to the unequal dumb-bell that is the Earth/Moon combination.

2013	Jan 2, 06:38	2017	Jan 4, 16:18
2014	Jan 4, 13:59	2018	Jan 3, 07:35
2015	Jan 4, 08:36	2019	Jan 3, 07:20
2016	Jan 3, 00:49	2020	Jan 5, 09:48

3. Eqn. 20 requires the atan2 function to provide an answer in the correct quadrant, (ν must be in the same quadrant as E). An alternate formulae is often published, which avoids the use of atan2, through the use of the trigonometric half-angle formulae...

   The two are functionally identical. This formula can, with some cumbersome trigonometry, be derived from Eqn. 20.

4. Ref. 1 - below - gives the formula as...

5. Spike Milligan...

$$\tan\left(\nu/2\right)=\tan\left(E/2\right)\times\sqrt{\left((1+e)/(1-e)\right)}$$ .... Eqn. 2.30

What's the Time, Eccles?

Wait, I've got it written down on a piece of paper... ... Eight o'clock.

$$2\times EOT = \text{Length of afternoon} - \text{Length of morning}$$

.... Eqn. 2.31

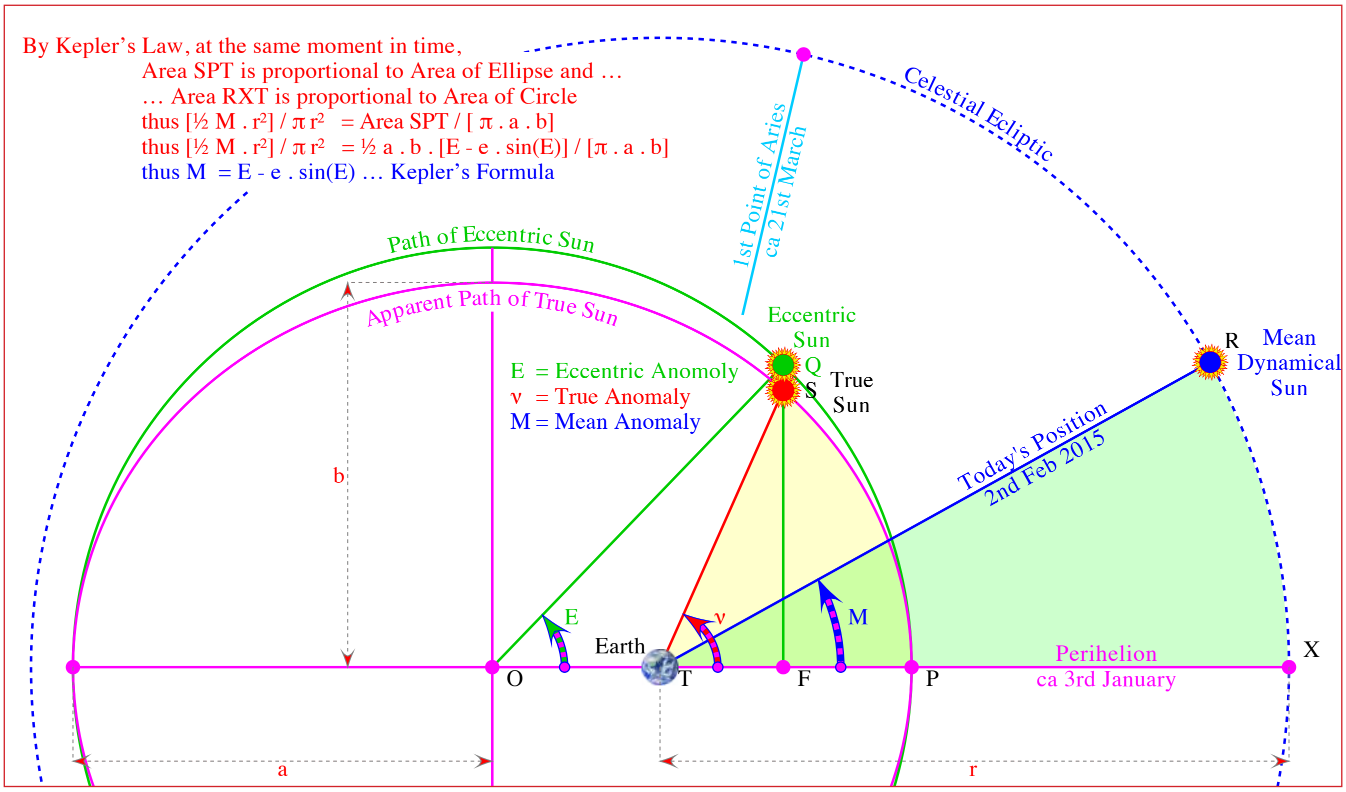

Fig. 23. Solving Kepler's Formula - Step 7.
The green segment is the area swept out by the Mean Sun. By Kepler's Law, this must equal the yellow segment.

Where did you get that?

I asked a man what the time was and he wrote it down for me. It's very nice because when people ask me the time, I can tell 'em because I've got it written down on a piece of paper.

What do you do when it's not eight o'clock?

I don't look.

So how do you know when it is eight o'clock?

I've got it written down on a piece of paper......

References

The reader is referred to the general References Part 1 of this series.

1. Wikipedia : Spherical Trigonometry http://en.wikipedia.org/wiki/Spherical_trigonometry
2. Jean Meeus: *Astronomical Algorithms:* Willman-Bell, Richmond (1998).
3. C.W.C. Barlow & G.H. Bryan: Elementary Mathematical Astronomy: 2nd Edn, Clive & Co, London (1893) see page 124.
4. *Multiyear Interactive Computer Almanac - 1800 - 2050:* US Naval Observatory: (2012). This is a high precision astronomical program, that (e.g.) provides EoT to an accuracy of 0.1 second.
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Adapted from NASS Compendium 25(4) December 2018 -publshed as 'Basic Asatronomy for Gnomonists'

return to "Table of Contents"

Basic Positional Solar Astronomy - Part 3: Fourier Derived Formulae

PREAMBLE

If the above routines are too involved for easy use, one may always use Fourier deduced trigonometric series. This study was triggered by the author's interest in derivation and quality of the Equation of Time formula given in the BSS Glossary Ref 1

$$E_a^{\text{mins}} = \begin{pmatrix} -0.000\,00.75\ldots \\ -0.001868\cos(\omega)+0.032\,077\sin(\omega)\ldots \\ +0.014\,165\cos(2\omega)+0.040\,849\sin(2\omega) \end{pmatrix} \times {}^{720}\!/_{\pi}$$

$$\omega \quad = {}^{2\pi n_d}\!/_{365}$$

$$n_d \quad = 1 \text{ at noon on 1 Jan, 32 on 1 Feb, etc.} \quad \ldots\ldots\ldots\ldots\text{Eqn 3.1}$$

If during leap years, 366 replaces 365 in the second line, this formula yields an accuracy of +48 & -36$^{\text{secs of time}}$ over the first 50 years of this century. The method described hereafter is thorough and produces results with far greater precision - for EoT, Declination and Right Ascension - than is generally required in dialing. For many, more simple formula will suffice: these are also deduced - providing some improvement over those provide in the Glossary.

The Fourier Approach

Any 'signal' that repeats with time (for example the Equation of Time, or Declination) can be approximated by the sum of a number of pure sine (or cosine) curves.

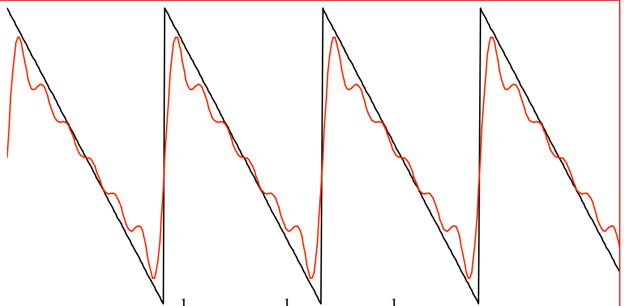
The theory states that an approximation of a function can be made...

$$f(x) \quad \approx \quad Av + \sum_{n=1}^{N} A_n \times \sin\left(n \times \left(\theta^{rad} + \varphi_n^{rad}\right)\right) \quad \ldots\ldots\ldots\ldots\text{Equ 3.2}$$

- Av = the average of the signal over an integer number of its periods
- n = harmonic number,
- A = amplitude of a particular harmonic
- θ = phase of that particular harmonic,
  (e.g. on 20th day of the year, θ = 2 x π x 20$^{\text{day}}$/365$^{\text{day}}$),
- φ = offset of the harmonic's zero point from start of computations,
  (e.g. offset of vernal equinox on 21 Mar from Jan 1
  φ ≈ 90$^{\text{day}}$/365$^{\text{day}}$ x 2 x π) ,

  See Note [1] for alternative versions of Equ 3.2

The accuracy obtained by this approximation method depends on the number of harmonics that are chosen. Bretagnon and Simon Ref 2 used 1080 terms to model the Sun's Longitude. Even, with just 6 terms, a saw tooth signal can be quite well modelled. See Fig 1. The method is widely used in many fields of industry - electronics, radio & seismic processing, to name but a few.



$$f(\theta) = \frac{1}{1}\times\sin(1\times\theta) + \frac{1}{2}\times\sin(2\times\theta) + \frac{1}{3}\times\sin(3\times\theta) + \ldots$$

$$\ldots\frac{1}{4}\times\sin(4\times\theta) + \frac{1}{5}\times\sin(5\times\theta) + \frac{1}{6}\times\sin(6\times\theta)$$

Fig. 1. This shows how even a linear periodic shape can be simulated by the sum of trigonometric components. More components, better fit.

A simple means of extracting the harmonic amplitudes and offsets from a 'signal' is illustrated in the Appendix. This method works very well if the duration of one repeating cycle is known - a 365 calendar year does not do well for most solar parameters, whereas a 365.25 cycle does better since it is closer to the length of a tropical year

However clever the Fourier approach may be in analysis, it does not cater well for the slow secular changes that are common in astronomy. As far as we are concerned, these relate to precession, the value of eccentricity, obliquity and perihelion longitude. These may by cyclical over the very long-term - but over our life time, their changes are effectively linear but small.

To overcome this problem, the following steps were followed...

1 input (EoT, Decl & RA) was calculated from MICA Ref 3 every 6 hours over a period of some 50 years from noon on 1st Jan 2000 to midnight on 1st Jan 2051 - which is exactly 50 x 365.25 days

2. for each of the three input types, the values where divided in 50 x 365.25-day cycles, each containing 1461 values.

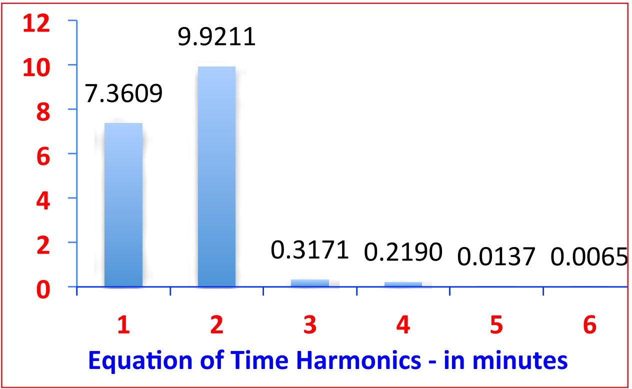


Fig. 2. The first 6 EoT harmonic amplitudes

3. for each of the 50 cycle, the first 6 harmonics were calculated using exactly the method described in Appendix 1
4. a Fourier approximation of the input was back-calculated using Equ 3.1 and compared with the input. In all cases, the last two harmonics provided little more than noise, so were discarded . See Fig 2
5. for the larger amplitude harmonics, the change in amplitude and offset was analysed over the 50 cycles and the value A_n and φ_n in Eqn 3.1 were replaced by the equation of their trend lines . For example see Fig 3. This shows how the second EoT harmonic amplitude varies over the 50 cycles and its linear trendline.

The Phase Angle & Cycle

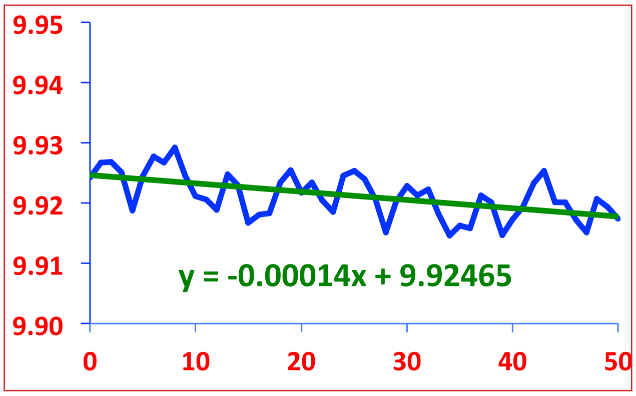


Fig. 3. The trend in the EoT 2nd harmonic amplitude over 50 years

All the formula presented need to have the date/time converted into a phase angle - θ. Each uses the Days since the 2000 Epoch (D_{2000}^{days}) as input. This value can be found in two ways - *both of which only work during this century.*

The first way uses Year/Month/Day/Hour/Minute and Time Zone, as input and uses the routines down to lines 10-20 of Table 1. (n.b. Time Zone is in hours +ve East of Greenwich).

$$\begin{aligned} bbb &= 367 \times YYYY - 730531.5 \\ ccc &= -int\Big(\big(7 \times int\big(YYYY + (MM+9)/12\big)\big) / \ 4\Big) \\ ddd &= int\big(275 \times MM / 9\big) + DD \\ D_{today} &= \big(HH + MM/60 - TimeZone\big) \ / \ 24 \\ D_{2000}^{days} &= bbb + cccc + ddd + D_{today} \qquad \text{............Equ 3.3} \end{aligned}$$

The second way uses year and ‘day-in-year’. This emulates the method used in the BSS Glossary Ref xx, and is useful if one just wishes to compute a table covering a single year. The ‘day-in-year’ = 1 for noon on 1st Jan, 32 for noon on 1st Feb, etc.

$$\begin{aligned} D_{today} &= \big(HH + MM/60 - TimeZone\big) \ / \ 24 \\ eee &= int\big((YYYY - 2000) \times 365.25\big) \\ fff &= 0 \text{ if } mod(YYYY,4) \neq 0 \\ &= -1 \text{ if } mod(YYYY,4) = 0 \text{ (leap year)} \\ D_{2000}^{days} &= eee + fff + D_{today} \qquad \text{............Equ 3.4} \end{aligned}$$

Thereafter the cyclical angle θ^{rad} is calculated thus...

$$\begin{aligned} Cycle &= int\Big(D_{2000}^{days} \ / \ 365.25\Big) \\ \theta^{rad} &= 0.0172024 \times (D_{2000}^{days} - 365.25 \times Cycle) \end{aligned}$$

............Equ 3.5

Cycle & θ^{rad} is applied in each of the routines below.

Equation of Time

The Equation of Time may be estimated thus:

$$\begin{aligned} Amp_1^{mins} &= 7.36303 - Cycle \times 0.00009 \\ Amp_2^{mins} &= 9.92465 - Cycle \times 0.00014 \\ \varphi_1^{rad} &= 3.07892 - Cycle \times 0.00019 \\ \varphi_2^{rad} &= -1.38995 + Cycle \times 0.00013 \\ EoT_1^{mins} &= Amp_1 \times \sin\big(1 \times (\theta + \varphi_1)\big) \\ EoT_2^{mins} &= Amp_2 \times \sin\big(2 \times (\theta + \varphi_2)\big) \\ EoT_3^{mins} &= 0.31730 \times \sin\big(3 \times (\theta - 0.94686)\big) \\ EoT_4^{mins} &= 0.21922 \times \sin\big(4 \times (\theta - 0.60716)\big) \\ EoT^{mins} &= 0.00526 + EoT_1 + EoT_2 + EoT_3 + EoT_4 \end{aligned}$$

...........Eqn 3.6

This yields the Equation of Time to +/- 3 $^{\text{seconds of time}}$ from 2000 to 2050. Dropping the fixed and the fourth term (EoT_4) reduces the accuracy to +/- 16 $^{\text{seconds of time}}$.

Additional simplification of the above routine yields..

$$\begin{aligned} EoT^{mins} &= 7.36 \times \sin(\theta + 3.08) + \ldots \\ & \quad 9.92 \times \sin(2 \times \theta - 2.78) \qquad \text{........................}Eqn.\ 3.7 \end{aligned}$$

This has errors of +/- 34 $^{\text{seconds of time}}$, which makes it adequate for most gnomonical purposes.

1st harmonic overtone. The amplitude factor of 7.3630^{min} in the term EoT_1 primarily represents the eccentricity effect, which cycles once per year, with perihelion as origin. The offset angle of ...

3.07892^{rad} = 176° = 176 x 365.25/360^{days} = 179^{days} which is the time of mean aphelion after 1st Jan.

2nd harmonic overtone. The amplitude factor of 9.92465^{min} in the term EoT_2 represents the major component of the obliquity effect, which cycles twice per year, with the equinox as origin. The offset angle of ...

1.38995^{rad} = 80° = 80 x 365.25/360^{days} = 81^{days} which is the time of mean vernal equinox after 1st Jan.

3rd and 4th harmonic overtones. These are mostly due to the fact that the obliquity effect is essentially tangential rather than sinusoidal (see Equ 28)

The error bands for the 4, 3 & 2 harmonic estimations are given in Fig 4.

Declination

The analysis for Declination was more complex,. Fig 5 shows the plot of 1st harmonic amplitude against Cycle.

This shows a linear downward trend, together with a sinusoidal shape. To investigate this harmonic, first, the linear trend was extracted, leaving a normal sine curve. Second, this sine curve which was subject to another Fourier analysis, which showed an interesting 18-year recurrence, which means that it might be

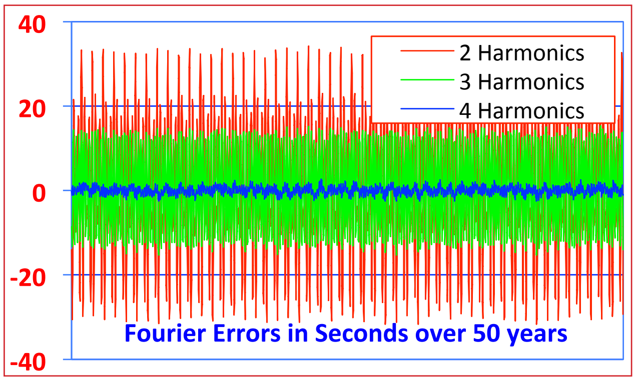


Fig. 4. 50 year trend in the EoT 2nd harmonic amplitude.

related to the Saros eclipse cycle. In turn, this suggests that it represents one of the lunar nutational effects, varying the Earth's obliquity - and hence the declination. This is to be expected.

The Declination of the Sun may be estimated thus:

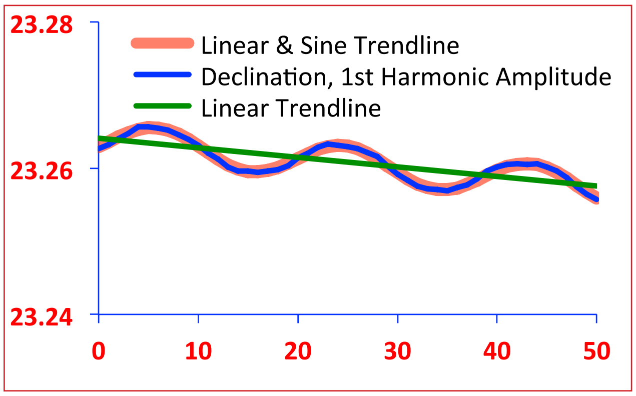


Fig. 5. The trend in the Declination 1st harmonic amplitude over 50 years

$$Amp_1^{\deg} = 23.2639 - Cycle \times 0.000131\ldots$$
$$\ldots + 0.0024 \times \sin(Cycle \times 0.335 - 0.4)$$
$$\varphi_1^{rad} = -1.38819 + Cycle \times 0.000135$$
$$\delta_1^{\deg} = Amp_1 \times \sin\left(1 \times \left(\theta^{rad} + \varphi_1^{rad}\right)\right)$$
$$\delta_2^{\deg} = 0.380897 \times \sin\left(2 \times \left(\theta^{rad} - 0.720483\right)\right)$$
$$\delta_3^{\deg} = 0.171178 \times \sin\left(3 \times \left(\theta^{rad} - 0.347175\right)\right)$$
$$\delta_4^{\deg} = 0.008067 \times \sin\left(4 \times \left(\theta^{rad} - 0.272216\right)\right)$$
$$\delta^{\deg} = 0.37657 + \delta_1 + \delta_2 + \delta_3 + \delta_4 \ldots\ldots\ldots\ldots\ldots \text{ Equ 3.8}$$

In the formula above, the first line of the equation shows the linear trend of Amp$_1$: the second line, the sinusoidal trend.

This yields Declination to +/- 30 $^{\text{seconds of arc}}$ from 2000 to 2050. Dropping the fourth term (δ4) reduces the accuracy to +/- 52 $^{\text{seconds of arc}}$. Dropping the third term (δ3) reduces the accuracy to +/- 11 $^{\textbf{minutes}\text{ of arc}}$.

Additional simplification of the above routine yields..

$$\delta^{deg} = 0.377 + 23.264 \times \sin(\theta - 1.388) \ldots$$
$$\ldots + 0.381 \times \sin(2 \times \theta - 1.44) \quad \ldots\ldots\ldots\ldots\ldots\text{Eqn 3.9}$$

This has errors of +/- 21 $^{\text{minutes of arc}}$, which makes it adequate for most gnomonical purposes.

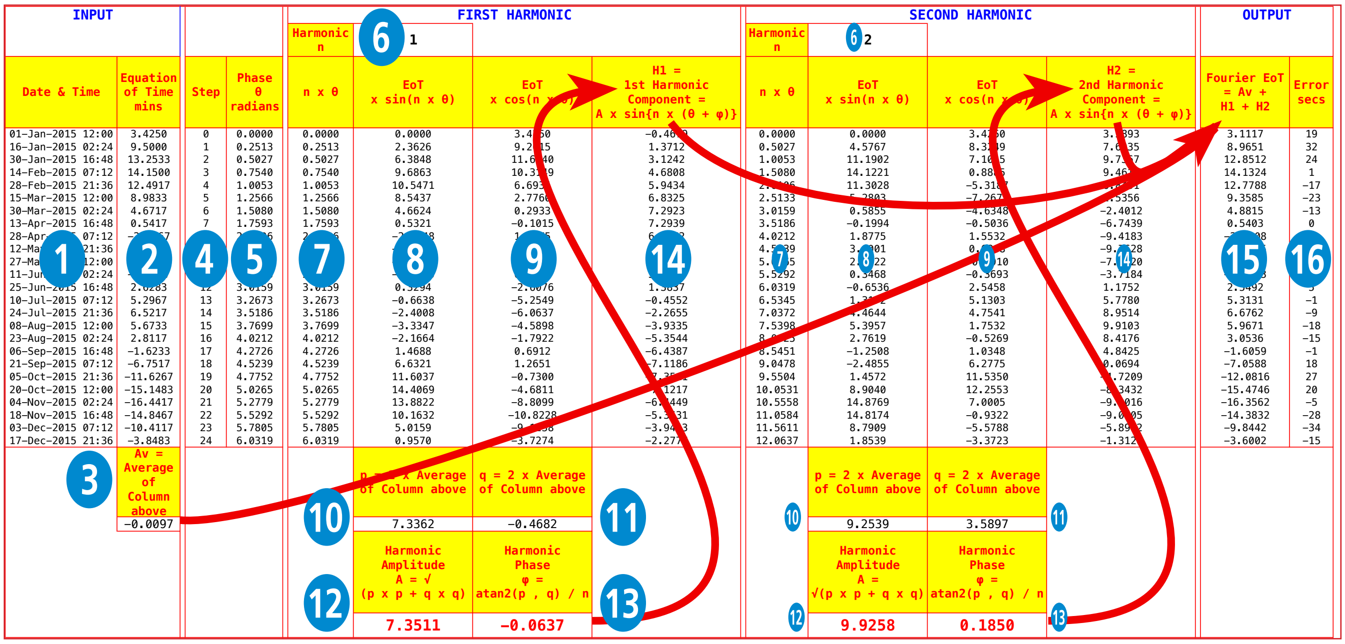
Right Ascension

Eqn 1.9, from Part 1 of this series, can be rearranged as follows...

$$\alpha^{hrs} = GMST^{\,hrs} - UTC^{hrs} + 12^{hrs} + EoT^{hrs}_{gnomonical} \ldots\ldots\ldots\text{Eqn. 3.10}$$

GMST, to the level of the accuracy of this study, is linear.

Thus GMST - UTC +12 is also linear. From which, we can deduce that the Right Ascension of the Sun may be estimated by...



INPUT				FIRST HARMONIC				SECOND HARMONIC				OUTPUT	
				Harmonic n	1			Harmonic n	2				
Date & Time	Equation of Time mins	Step	Phase θ radians	n x θ	EoT x sin(n x θ)	EoT x cos(n x θ)	H1 = 1st Harmonic Component = A x sin{n x (θ + φ)}	n x θ	EoT x sin(n x θ)	EoT x cos(n x θ)	H2 = 2nd Harmonic Component = A x sin{n x (θ + φ)}	Fourier EoT = Av + H1 + H2	Error secs
01-Jan-2015 12:00	3.4250	0	0.0000	0.0000	0.0000	[illegible]	[illegible]	0.0000	0.0000	[illegible]	[illegible]	3.1117	19
16-Jan-2015 02:24	9.5000	1	0.2513	0.2513	2.3626	[illegible]	1.3712	0.5027	4.5767	[illegible]	[illegible]	8.9651	32
30-Jan-2015 16:48	13.2533	2	0.5027	0.5027	6.3848	[illegible]	3.1242	1.0053	11.1902	[illegible]	[illegible]	12.8512	24
14-Feb-2015 07:12	14.1500	3	0.7540	0.7540	9.6863	[illegible]	4.6808	1.5080	14.1221	[illegible]	[illegible]	14.1324	1
28-Feb-2015 21:36	12.4917	4	1.0053	1.0053	10.5471	[illegible]	5.9434	[illegible]	11.3028	[illegible]	[illegible]	12.7788	-17
15-Mar-2015 12:00	8.9833	5	1.2566	1.2566	8.5437	[illegible]	6.8325	2.5133	[illegible]	[illegible]	[illegible]	9.3585	-23
30-Mar-2015 02:24	4.6717	6	1.5080	1.5080	4.6624	0.2933	7.2923	3.0159	0.5855	-4.6348	-2.4012	4.8815	-13
13-Apr-2015 16:48	0.5417	7	1.7593	1.7593	0.5321	-0.1015	7.2939	3.5186	-0.1994	-0.5036	-6.7439	0.5403	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.0212	1.8775	1.5532	-9.4183	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5.5292	[illegible]	-0.3693	-3.7184	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6.0319	-0.6536	2.5458	1.1752	[illegible]	[illegible]
10-Jul-2015 07:12	5.2967	13	3.2673	3.2673	-0.6638	-5.2549	-0.4552	6.5345	[illegible]	5.1303	5.7780	5.3131	-1
24-Jul-2015 21:36	6.5217	14	3.5186	3.5186	-2.4008	-6.0637	-2.2655	7.0372	[illegible]	4.7541	8.9514	6.6762	-9
08-Aug-2015 12:00	5.6733	15	3.7699	3.7699	-3.3347	-4.5898	-3.9335	7.5398	5.3957	1.7532	9.9103	5.9671	-18
23-Aug-2015 02:24	2.8117	16	4.0212	4.0212	-2.1664	-1.7922	-5.3544	[illegible]	2.7619	-0.5269	8.4176	3.0536	-15
06-Sep-2015 16:48	-1.6233	17	4.2726	4.2726	1.4688	0.6912	-6.4387	8.5451	-1.2508	1.0348	4.8425	-1.6059	-1
21-Sep-2015 07:12	-6.7517	18	4.5239	4.5239	6.6321	1.2651	-7.1186	9.0478	-2.4855	6.2775	0.0694	-7.0588	18
05-Oct-2015 21:36	-11.6267	19	4.7752	4.7752	11.6037	-0.7300	[illegible]	9.5504	1.4572	11.5350	[illegible]	-12.0816	27
20-Oct-2015 12:00	-15.1483	20	5.0265	5.0265	14.4069	-4.6811	[illegible]	10.0531	8.9040	12.2553	[illegible]	-15.4746	20
04-Nov-2015 02:24	-16.4417	21	5.2779	5.2779	13.8822	-8.8099	[illegible]	10.5558	14.8769	7.0005	[illegible]	-16.3562	-5
18-Nov-2015 16:48	-14.8467	22	5.5292	5.5292	10.1632	-10.8228	[illegible]	11.0584	14.8174	-0.9322	[illegible]	-14.3832	-28
03-Dec-2015 07:12	-10.4117	23	5.7805	5.7805	5.0159	[illegible]	[illegible]	11.5611	8.7909	-5.5788	[illegible]	-9.8442	-34
17-Dec-2015 21:36	-3.8483	24	6.0319	6.0319	0.9570	-3.7274	[illegible]	12.0637	1.8539	-3.3723	[illegible]	-3.6002	-15
	Av = Average of Column above				p = 2 x Average of Column above	q = 2 x Average of Column above			p = 2 x Average of Column above	q = 2 x Average of Column above			
	-0.0097				7.3362	-0.4682			9.2539	3.5897			
					Harmonic Amplitude A = √(p x p + q x q)	Harmonic Phase φ = atan2(p , q) / n			Harmonic Amplitude A = √(p x p + q x q)	Harmonic Phase φ = atan2(p , q) / n			
					7.3511	-0.0637			9.9258	0.1850			

Fig. 6. Example Spreadsheet. Follow the blue numbers. The yellow boxes show the formulae to be used in each column.

$$\alpha^{hrs} = \begin{pmatrix} 18.6974 & +3.8198 & \times\ \theta \\ & +24.00051 & \times\ Cycle \\ & -\ EoT^{mins}_{Gnomonical}\ /\ 60 & \end{pmatrix} \bmod 24 \text{...... Eqn 3.11}$$

This yields the Sun's Right Ascension to +/- 4 $^{\text{seconds of time}}$ from 2000 to 2050, if 4 harmonic terms are used for EoT. This reduces to +/-17 secs of time is 3 harmonics are used, and to +/- 35 seconds of time if equation 3.5 is used

APPENDIX

Fig 6 is a simple spreadsheet example, just looking at 25 Date/EoT pairs spread evenly every 14.6 (= 365 / 25) days over a year. The input was taken from the MICA program $^{\text{Ref 3}}$. Just the first and second harmonics were calculated and the output was generated as the sum of those two components, together the average value.

Fig 7 compares the black line, generated by 2 harmonics calculated from just those 25 points marked with the 'x's. The fit is close but not visually exact. Fig 8 shows the very close match achieved by using 365 input values to generate four harmonics.

Note 1

Fourier series can be quoted as a sum of sine &/or cosine curves thus...

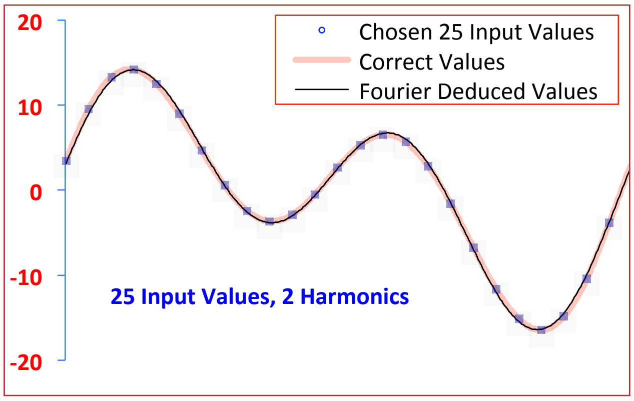


Fig. 7. The black line shows the EoT generated for 25 'correct' points. The pink line shows 365 'correct' points. Note how close the black line follows the pink curve.

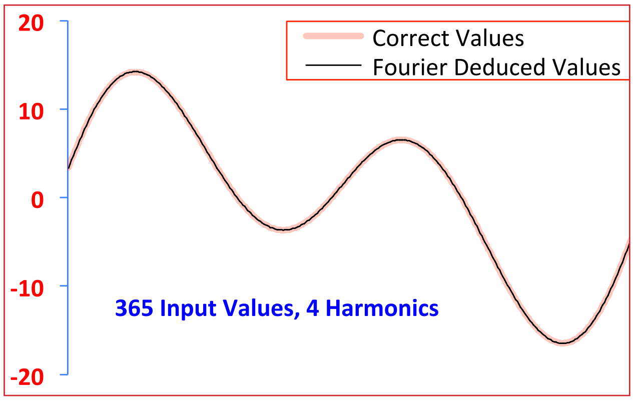


Fig. 8. As more input points and more harmonics are used, the estimation gets better and better.

$$\sum_{1}^{N} a_n \times \sin\left(n \times \theta + b\right)$$

$$\sum_{1}^{N} a_n \times \cos\left(n \times \theta - b'\right) \text{where } b' = \pi/_{2} - b$$

$$\sum_{1}^{N} q \times \sin\left(n \times \theta\right) + r \times \cos\left(n \times \theta\right)$$

$$\text{where } q = a_n \times \cos\left(b\right)\ \&\ \mathrm{r} = a_n \times \sin\left(b\right)$$

$$\text{....................Eqn 3.12}$$

All are trigonometrically the same.

Note 2

The following Microsoft Excel function macros can be copied into a module in Excel's Visual Basic Editor. Then, on a spreadsheet, they can be called by filling in a formula, such as

=EoT(YYYY,MM,DD,HH,MM,SS)

=Decl(YYYY,MM,DD,HH,MM,SS)

=RA(YYYY,MM,DD,HH,MM,SS)

YYYY = Year, MM = Month, etc. Note that Date and Time must be UTC.

To avoid errors in copying these out, they can be found as text files at Ref 4

```
` ****************************************
‘ EoT Macro
Function EoT(The_Year, The_Month, The_Day, The_Hour, The_
Minute, The_Second)
bbb = 367 * The_Year - 730531.5
ccc = Int((7# * Int(The_Year + (The_Month + 9) / 12)) / 4)
ddd = Int(275 * The_Month / 9) + The_Day
D2000 = bbb - ccc + ddd + (The_Hour + The_Minute / 60 +
The_Second / 3600) / 24
Cycle = Int(D2000 / 365.25)
Theta = 0.0172024 * (D2000 - 365.25 * Cycle)
Average = 0.00526
Amp1 = 7.36303 - Cycle * 9e-05
Amp2 = 9.92465 - Cycle * 0.00014
Phi1 = 3.07892 + Cycle * -0.00019
Phi2 = -1.38995 + Cycle * 0.00013

EoT1 = Amp1 * Sin(1 * (Theta + Phi1))
EoT2 = Amp2 * Sin(2 * (Theta + Phi2))
EoT3 = 0.3173 * Sin(3 * (Theta - 0.94686))
EoT4 = 0.21922 * Sin(4 * (Theta - 0.60716))

EoT = Average + EoT1 + EoT2 + EoT3 + EoT4
End Function

` ****************************************
‘ Declination Macro
Function Decl(The_Year, The_Month, The_Day, The_Hour,
The_Minute, The_Second)
bbb = 367 * The_Year - 730531.5
ccc = Int((7# * Int(The_Year + (The_Month + 9) / 12)) /
4)
ddd = Int(275 * The_Month / 9) + The_Day
D2000 = bbb - ccc + ddd + (The_Hour + The_Minute / 60 +
The_Second / 3600) / 24
Cycle = Int(D2000 / 365.25)
Theta = 0.0172024 * (D2000 - 365.25 * Cycle)
Amp1 = 23.2639 - Cycle * 0.000131 + 0.0024 * Sin(Cycle *
0.335103 - 0.4)
Amp1 = 23.2639 - Cycle * 0.000131 + 0.0024 * Sin(Cycle *
0.335 - 0.4)
Phi1 = -1.38819 + Cycle * 0.000135
Decl1 = Amp1 * Sin(1 * (Theta + Phi1))
Decl2 = 0.380897 * Sin(2 * (Theta - 0.720483))
Decl3 = 0.171178 * Sin(3 * (Theta - 0.347175))
Decl4 = 0.008067 * Sin(4 * (Theta - 0.272216))
Decl = 0.37657 + Decl1 + Decl2 + Decl3 + Decl4
End Function
```

```
‘ ****************************************
‘ Right Ascension Function
Function RA(The_Year, The_Month, The_Day, The_Hour, The_
Minute, The_Second)
bbb = 367 * The_Year - 730531.5
ccc = Int((7# * Int(The_Year + (The_Month + 9) / 12)) / 4)
ddd = Int(275 * The_Month / 9) + The_Day
D2000 = bbb - ccc + ddd + (The_Hour + The_Minute / 60 +
The_Second / 3600) / 24
Cycle = Int(D2000 / 365.25)
Theta = 0.0172024 * (D2000 - 365.25 * Cycle)
Average = 0.00526
Amp1 = 7.36303 - Cycle * 9e-05
Amp2 = 9.92465 - Cycle * 0.00014
Phi1 = 3.07892 + Cycle * -0.00019
Phi2 = -1.38995 + Cycle * 0.00013

EoT1 = Amp1 * Sin(1 * (Theta + Phi1))
EoT2 = Amp2 * Sin(2 * (Theta + Phi2))
EoT3 = 0.3173 * Sin(3 * (Theta - 0.94686))
EoT4 = 0.21922 * Sin(4 * (Theta - 0.60716))

EOT_hrs = (Average + EoT1 + EoT2 + EoT3 + EoT4) / 60
RA = (18.6974 + 3.8198 * Theta + 24.00051 * Cycle -
EOT_hrs)
RA = RA - (24 * (RA \ 24))
If RA < 0 Then RA = RA + 24
End Function
```

References

1. BSS Glossary - as of Sept 2014.
2. Jean Meeus: *Astronomical Algorithms:* Willman-Bell, Richmond (1998) - quoted on page 166.
3. *Multiyear Interactive Computer Almanac - 1800 - 2050:* US Naval Observatory: (2012). This is a high precision astronomical program, that (e.g.) provides EoT to an accuracy of 0.1 second.
4. http://www.precisedirections.co.uk/Sundials

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return to "Table of Contents"

Code to do the Calculations

Calculations involving the Equation of Time vary from straightforward to somewhat complex - in the case of analemmas. This chapter provides five sets of Python code. *They are included here for reference and study only.* If the reader wants to use the code, it may be downloaded from:

https://github.com/kevinkarney/Equation-of-Time

The five sets of code are.

i) EoT and other essential solar parameters with a good degree of accuracy. . See 'Table of Contents' on page ii.

ii) EoT and the Solar Declination using easy to code Fourier routines. See 'The Fourier Method' on page 30.

iii) produce a spreadsheet readable output file needed to create a EoT Table. See 'The Problem with Equation Tables' on page 51.

iv) produce a spreadsheet readable output file with all the data needed to draw an analemmatic sundial any plane surface. See 'Delineating an Analemma' on page 43.

v) various subroutines that are used by all of the above. For example, to convert dates in to Julian days and vice versa.

The four sets of code on Github contain their own subroutines and will run any Python-3 interpreter, of which there are many

For those interested in detailed Solar Astronomy, a Python solution to Meeus' calculations can also be found on Github. See 'Meeus' Algorithms' on page 32.

The code is written in the easy-to-learn Python language and is available for anyone. The files can be opened and should run in virtually any Python interpreter, of which there are many.

The code available on Github contains many lines of code to produce intermediate results for use in study or debugging. These lines are omitted here for sake of brevity.

For those new to the Python language....

- code blocks are, not delineated by curly brackets {},but by the tab key or 4 spaces. A code block is ended if the next line contains one less tab.
- 'def' indicates the beginning of a function.
- the colon indicates the end of line of a function definition and if, while, or repeat statements.
- a number without a decimal point is an integer.
- x % y is the same as x mod y
- != is the same as not =
- # indicates a comment (shown in green in the following code.

Subroutine Titles are in **`Red`**

Input required to run these routines are in **`Magenta`**.

Comments are in `Green`

See ‘Table of Contents’ on page ii.

```
from math import degrees,radians,tan,sin,acos,cos,floor,atan2,sqrt,asin,pi
# --------------------------------------------------------------------------------------
# EQUATION OF TIME
# --------------------------------------------------------------------------------------
# Python Code written by Kevin Karney, Winter 2024
# Should work on all releases of Python
# Free for anyone to use without any guarantees!
# NOTA BENE
# where Time is input, it is local STANDARD time (i.e. no Daylight saving).
# Hence Time Zone occurs in many routines to correct to UTC,

# The code performs calculates the essential Solar parameters
# for a given location and date
#     calculated by subroutine Sun_JD(JD)
#     its output is an array comprising:
#          EoT, Longitude Corrected EoT,
#          Right Ascension, Declination
#          Solar Altitude, Azimuth
#          approx time of Sunrise, Sunset
#     Main routines are
#          Sun(Year,Month,Day,Hour,Longitude,Latitude,Zone) which calls Sun_JD(JD)
#          Sun_Year(Year,Longitude,Zone) which gives values for noon over a year
#     output is a tab-delimited which can be pasted into any spreadsheet

# There are a number of service routines
#     Julian(Year,Month,Day,Hour,Zone) which gives the Julian Day
#     Get_Calendar_Date(The_JD,Zone) which converts between Julian Day and normal calendar 
values
#     EoT_Dec_MMSS(The_EoT) which formats decimal minutes to mins and second
#     Dec_Deg_DMS(The_Degs) which formats decimal degrees to degrees, mins and secs
#     Dec_Hrs_HMS(The_Hrs) which formats decimal hours to degrees, mins and secs

# ------------------------------------------------------------------------------------------
# What is Wanted
# ------------------------------------------------------------------------------------------
Year_Calc      = False   # if True,  provides values for a whole year at noon
                         # if False, provides a single set of calcs for specified date and time
rounder        = 5       # rounds output to these number of decimals to results
# ------------------------------------------------------------------------------------------
# Location Input
# ------------------------------------------------------------------------------------------
Place          = ‘Athens’
Longitude      = 23.71667 # Degrees : +ve East of Greenwich
Latitude       = 37.96667 # Degrees : +ve East of Greenwich
Zone           = 2         # Hrs     : +ve East of Greenwich
Year           = 2025
Month          = 2
Day            = 13
Hour           = 12        # Local Standard Clock Time (no DST)

def Calculate():

    # -------------------------------------------------------
    # This is routine called from the last line of this code
    # It selects the task required
    # -------------------------------------------------------
    if Year_Calc == True:
        print (‘Year Result from Sun routine’)
        Sun_Year(Year,Longitude,Zone)
        print (“\rThis output can copied and pasted into any spreadsheet”)
    else:
        Results = Sun(Year,Month,Day,Hour,Longitude,Latitude,Zone)
        print (‘Location                = ‘,Place)
        print (‘Lat : Long : Zone       = ‘,Dec_Deg_DMS(Latitude),’ : ‘,Dec_Deg_DMS(Longitude),’ 
: ‘,Zone)
        print (‘EoT                     = ‘,round(Results[0],rounder),EoT_Dec_MMSS(Results[0
]))
        print (‘EoT Longitude Corrected = ‘,round(Results[1],rounder),EoT_Dec_MMSS(Results[1]))
        print (‘Right Ascension         = ‘,round(Results[2],rounder),Dec_Hrs_HMS (Results[2]))
        print (‘Solar Declination       = ‘,round(Results[3],rounder),Dec_Deg_DMS (Results[3]))
        print (‘Altitude                = ‘,round(Results[4],rounder),Dec_Deg_DMS (Results[4]))
        print (‘Azimuth                 = ‘,round(Results[5],rounder),Dec_Deg_DMS (Results[5]))
        print (‘approx Sunrise          = ‘,round(Results[6],rounder),Dec_Hrs_HMS (Results[6]))
        print (‘approx Sunrise Az       = ‘,round(Results[7],rounder),Dec_Deg_DMS (Results[7]))
        print (‘approx Sunset           = ‘,round(Results[8],rounder),Dec_Hrs_HMS (Results[8]))
        print (‘approx Sunset Az        = ‘,round(Results[9],rounder),Dec_Deg_DMS (Results[9]))
```

return to "Table of Contents"

```
def Sun(Year,Month,Day,Hour,Longitude,Latitude,Zone):

    JD = Julian (Year,Month,Day,Hour,Zone)
    Ans = Sun_JD(JD)
    return Ans

def Sun_Year(Year,Longitude,Zone):

    # --------------------------------------------------------------
    # Routine to Calculate...
    #      Equation of Time,Longitude Corrected Equation of Time,
    #      Right Ascension, Declination, Altitude and Azimuth
    # It contains much of the same code as routine Sun
    # --------------------------------------------------------------
    JD     = Julian(Year,1,1,12,Zone)
    print (‘Date ‘ + ‘\t’ + ‘EoT ‘+ ‘\t’ + ‘Long Corr EoT ‘+ ‘\t’ + ‘RA ‘+ ‘\t’ + ‘Decl ‘ +
‘\t’ + ‘Alt ‘+  ‘\t’ + ‘Az ‘  + ‘\r’ )
    Days_in_Year = 366 if Year % 4 == 0 else 365
    for i in range(Days_in_Year):
        Answer                = Sun_JD(JD)
        EoT_min               = Answer[0]
        EoT_Corr_min          = Answer[1]
        Right_Ascension_hrs = Answer[2]
        Declination_deg       = Answer[3]
        Altitude_deg          = Answer[4]
        Azimuth_deg           = Answer[5]
        print (Get_Calendar_Date(JD,Zone)[6] + ‘\t ‘ + str(round(EoT_min,rounder))+ ‘\t’ +
str(round(EoT_Corr_min,rounder))+ ‘\t ‘ + str(round(Right_Ascension_hrs,rounder))+ ‘\t ‘
+ str(round(Declination_deg,rounder))+ ‘\t ‘ + str(round(Altitude_deg,rounder))+ ‘\t ‘ +
str(round(Azimuth_deg,rounder))  + ‘\r’ )
        JD += 1

Calculate()


print (‘\rDone’)
```

See also ‘General Service Routines’ on page 226.
in particualr Sun_JD which does the main astronomical calculation

FOURIER EOT AND DECLINATION

See ‘The Fourier Method’ on page 30.

```
from math import degrees,radians,tan,sin,acos,cos,floor,atan2,sqrt,asin,pi
# -------------------------------------------------------------------------------------
# EQUATION OF TIME & DECLINATION BY FOURIER
# -------------------------------------------------------------------------------------
# Python Code written by Kevin Karney, Winter 2024
# Should work on all releases of Python
# Free for anyone to use without any guarantees!
#
# NOTA BENE
# where Time is input, it is local STANDARD time (i.e.no Daylight saving).
# Hence Time Zone occurs in many routines to correct to UTC,

# The code quickly calculates noon EoT,Longitude Corrected EoT and Declination
#     calculated by subroutine EoT_Decl_JD_Fourier(JD)
#     Output is a 2 value array of Longitude Corrected EoT and Declination
#     Subservient routines are
#         EoT_Decl_Fourier(Year,Month,Day,Hour,Longitude,Zone)
#         Year_Output_Fourier(Year,Longitude,Zone)
#     output is a tab-delimited which can be pasted into any spreadsheet
#
# There are a number of service routines
#     Julian(Year,Month,Day,Hour,Zone) which gives the Julian Day
#     EoT_Dec_MMSS(The_EoT) which formats decimal minutes to mins and second
#     Dec_Deg_DMS(The_Degs) which formats decimal degrees to degrees, mins and secs

# -----------------------------------------------------------------------------------------
# What is Wanted
# -----------------------------------------------------------------------------------------
Year_Calc       = False  # if True,  provides values for a whole year at noon
                         # if False, provides a single set of calculations
rounder         = 5      # rounds output to these number of decimals to results
# -----------------------------------------------------------------------------------------
# Location Input
# -----------------------------------------------------------------------------------------
Place           = ‘Athens’
Longitude       = 23.71667 # Degrees : +ve East of Greenwich
Latitude        = 37.96667 # Degrees : +ve East of Greenwich
Zone            = 2        # Hrs     : +ve East of Greenwich

Year            = 2024
Month           = 2        # not required for Year calculation or analemma
Day             = 13       # not required for Year calculation or analemma
Hour            = 12       # not required for Year calculation or analemma

def Calculate():

    # -----------------------------------------------------
    # This is routine called from the last line of this code
    # It selects the task required
    # -----------------------------------------------------
    if Year_Calc == True:
        print (‘Year Results from Fourier routines’)
        Year_Output_Fourier(Year,Longitude,Zone)
        print (“\rThis output can copied and pasted into any spreadsheet”)
    else:
        print (‘Results from EoT_Decl_Fourier routines’)
        Results = EoT_Decl_Fourier(Year,Month,Day,Hour,Longitude,Zone)
        print (‘Place                     = ‘,Place)
        print (‘Longitude                 = ‘,Longitude)
        print (‘Zone                      = ‘,Zone)
        print (‘Year                      = ‘,Year)
        print (‘Month                     = ‘,Month)
        print (‘Day                       = ‘,Day)
        print (‘Local Standard Hour       = ‘,Hour)
        print (‘EoT                       = ‘,round(Results[0],rounder))
        print (‘                          = ‘,EoT_Dec_MMSS(Results[0]))
        print (‘EoT Longitude Corrected = ‘,round(Results[1],rounder))
        print (‘                          = ‘,EoT_Dec_MMSS(Results[1]))
        print (‘Solar Declination        = ‘,round(Results[2],rounder))
        print (‘                          = ‘,Dec_Deg_DMS (Results[2]))
```

```
def EoT_Decl_JD_Fourier(JD):

    Days_from_2000 = JD - 2451545.0
    Index = (4 * Days_from_2000) % 1461
    Theta = 0.004301 * Index # = 2 pi /1461
    # Equation of Time
    EoT1 = 7.3529 * sin(1 * Theta + 6.2085)
    EoT2 = 9.9269 * sin(2 * Theta + 0.3704)
    EoT3 = 0.3337 * sin(3 * Theta + 0.3042)
    EoT4 = 0.2317 * sin(4 * Theta + 0.7158)
    EoT  = 0.019 + EoT1 + EoT2 + EoT3 + EoT4
    Long_Corr = 4 * (Zone * 15 - Longitude)
    EoT_Corr = EoT + Long_Corr
    # Declination
    Aver           = 0.3747
    Decl1          = 23.2802 * sin(1 * Theta + 4.8995)
    Decl2          = 0.422 * sin(2 * Theta + 4.8324)
    Decl3          = 0.2034 * sin(3 * Theta + 4.8995)
    Decl4          = 0.0415 * sin(4 * Theta + 4.8465)
    Decl_deg       = Aver + Decl1 + Decl2 + Decl3 + Decl4
    return EoT,EoT_Corr,Decl_deg

def EoT_Decl_Fourier(Year,Month,Day,Hour,Longitude,Zone):

    JD = Julian(Year,Month,Day,Hour,Zone)
    return EoT_Decl_JD_Fourier(JD)

def Year_Output_Fourier(Year,Longitude,Zone):

    # ---------------------------------------------------------------
    # Routine to Calculate the output of EoT_Decl_JD_Fourier routine
    #      at local noon over a whole year
    #      The output is tab delimited text which can be cut and pasted
    #      directly into a spreadsheet.
    # ---------------------------------------------------------------
    JD         = Julian(Year,1,1,Hour,Zone)
    Days_in_Year    = 366 if Year % 4 == 0 else 365
    print (‘Date ‘ + ‘\t’ + ‘EoT ‘+ ‘\t’ + ‘Long Corr EoT ‘+ ‘\t’ + ‘Declination ‘ + ‘\r’ )
    for i in range(Days_in_Year):
        EoT_min,EoT_Corr_min,Declination_deg = EoT_Decl_JD_Fourier(JD)
        print (Get_Calendar_Date(JD,Zone)[6] + ‘\t ‘ + str(round(EoT_min,rounder))+ ‘\t’ +
str(round(EoT_Corr_min,rounder))+ ‘\t ‘ + str(round(Declination_deg,rounder)))
        JD += 1
```

Subroutines Julian(Year,Month,Day,Hour,Zone) : EoT_Dec_MMSS(The_EoT): Dec_Deg_DMS(The_Degs) : can be found at the end of this chapter

```
Calculate()

print (‘\rDone’)
```

See also ‘General Service Routines’ on page 226.

Equation Tables

See ‘Displaying the Equation of Time’ on page 91. & ‘The Problem with Equation Tables’ on page 51.

```
from math import degrees,radians,tan,sin,acos,cos,floor,atan2,sqrt,asin,pi
# -----------------------------------------------------------------------------------
# EQUATION OF TIME TABLES
# -----------------------------------------------------------------------------------
# Python Code written by Kevin Karney, Winter 2024
# Should work on all releases of Python
# Free for anyone to use without any guarantees!
#
# NOTA BENE
# where Time is input, it is local STANDARD time (i.e. no Daylight saving).
# Hence Time Zone occurs in many routines to correct to UTC

# Calculates the values needed for a Equation table
#     it returns either a table, averaged over a leap cycle which starts on 1st March on a leap year
#     Output is a spreadsheet-readable text file with dates and values
#
# There are 3 number of routines that are called
#     Sun_JD(JD) which performs the astronomical calculations
#     Julian(Year,Month,Day,Hour,Zone) which gives the Julian Day
#     Get_Calendar_Date(The_JD,Zone) which converts between Julian Day and normal calendar values
# ---------------------------------------------------------------------------------------

# ---------------------------------------------------------------------------------------
# Location Input
# ---------------------------------------------------------------------------------------
Place          = ‘Athens’
Longitude      = 23.71667 # Degrees : +ve East of Greenwich
Latitude       = 37.96667 # Degrees : +ve East of Greenwich
Zone           = 2         # Hrs     : +ve East of Greenwich
Year           = 2024
Table_Fineness = 1         # 0 = every minute      (average over leap cycle)
                           # 1 = every half minute (average over leap cycle)
                           # 2 = every day         (average over leap cycle)

# n.b. Latitude is not required in this routine, but is a necessary parameter
# for parts of rountine Sun(JD), which are not used in this application.
# Enter any value.

def Calculate(Year):

    # This is called by the last line of the code.
    # Build an empty Matrix to store the results
    Matrix_W = 24 # 2 columns per month
    Matrix_H = 31
    Matrix = [[‘’ for x in range(Matrix_W)] for y in range(Matrix_H)]
    Month_Names = [‘January’,’February’,’March’,’April’,’May’,’June’,’July’,’August’,’September’,’October’,’November’,’December’]

    if Year % 4 != 0:
        print (“Stopped, for an Equation Tables, Specified Year must be a Leap Year...”)
        return

    # Calculations are done from 1st March on a Leap Year
    # until the next 29th Feb, 1461 days later
    JD_Start = Julian(Year  , 3,  1, 12, Zone)
    JD_End   = Julian(Year+4, 2, 29, 12, Zone)

    All_Days   = []
    All_Months = []
    All_EoTs   = []

    # Build arrays with days of months, month name & EoTs
    JD = JD_Start
    for i in range(365):
        All_Days  .append(Get_Calendar_Date(JD,Zone)[2])
        All_Months.append(Get_Calendar_Date(JD,Zone)[1])
        All_EoTs  .append((Sun_JD(JD)[1]+Sun_JD(JD+365)[1]+Sun_JD(JD+2*365)[1]+Sun_JD(JD+3*365)[1])/4)
        JD += 1
    # Get Values for 29th Feb at end of cycle
    All_Days.append(29)
    All_Months.append(2)
    All_EoTs.append(Sun_JD(JD_End)[1])

    # rotate the calculated values 60 days backwards to bring 1st Jan to start of array
    All_Days  [:] = All_Days  [-60:366] + All_Days  [0:-60]
    All_Months[:] = All_Months[-60:366] + All_Months[0:-60]
    All_EoTs  [:] = All_EoTs  [-60:366] + All_EoTs  [0:-60]
```

```
    # build values for table÷
    Days,Months,EoTs = [],[],[]
    Last_EoT = 999
    for i in range(366):
        This_Day       = All_Days[i]
        This_Month     = All_Months[i]
        if Table_Fineness == 0:
            This_EoT      = int(round(All_EoTs[i],0))
        elif Table_Fineness == 1:
            This_EoT      = (round(2*All_EoTs[i],0))/2
        else:
            This_EoT      =  All_EoTs[i]
            Mins          = int(This_EoT)
            Secs          = int(round(60*(This_EoT - Mins),0))
            if Secs == 60:
                Secs = 0
                Mins += 1
            xx = “0” if Secs < 10 else “”
            yy = “0” if Mins < 10 else “”
            This_EoT      = “ “ + yy + str(Mins)+ “:”+ xx + str(Secs)

        # Select the days when the appropriate value changes
        # or 1st Month
        if This_Day  == 1 or This_EoT != Last_EoT:
            Days  .append(This_Day)
            Months.append(This_Month)
            EoTs  .append(This_EoT)
            Last_EoT = This_EoT

    # Build a 24 column matrix (2 cols per month) to store the results
    The_Month  = 0
    Last_Month = 0
    Row        = -1
    Max_Row    = 1
    This_Day   = -1
    for i in range(len(EoTs)):
        This_Month    = Months[i]
        This_Day      = Days[i]
        This_EoT      = EoTs[i]
        if This_Day == 1:
            Row       = 0
            Kol_Day   = (This_Month-1) * 2
            Kol_Val  = Kol_Day + 1
        if This_Day == 1:
            Matrix[Row][Kol_Day] = This_Day
            Matrix[Row][Kol_Val] = This_EoT
        else:
            if This_EoT != Last_EoT:
                Matrix[Row][Kol_Day] = This_Day
                Matrix[Row][Kol_Val] = This_EoT
        Max_Row   = max(Max_Row,Row)
        Row      += 1
        Last_EoT = This_EoT
        Max_Row   = max(Max_Row,Row)

    # Write Output File
    Table_Filename             = ‘Table.txt’
    File                       = open(Table_Filename,’w’)
    Year_String                = “ for Year “ + str(Year)
    Year_String                = “ for Years “  + str(Year) + “ - “ + str(Year+3)
    File.write(“Equation-of-Time Table - Longitude Corrrected - averaged over a leap cycle
starting 1st March “ + Year_String  + ‘\r’)
    File.write(“Place”     + ‘\t’ + Place          + ‘\r’)
    File.write(“Longitude” + ‘\t’ + str(Longitude) + ‘\r’)
    File.write(“Zone”      + ‘\t’ + str(Zone)      + ‘\r’)
    File.write(‘\r’)
    if Table_Fineness < 2:
        File.write(“To read this Table, find the date that is less than or equal to today’s
date,” + ‘\r’)
        File.write(“then read the corresponding value of the Longitude Corrected Equation of
Time in minutes” + ‘\r’)
    else:
        File.write(“EoT Values are in mm:ss”)
    File.write(‘\r’)
```

```
    # Write Column Headings
    String1 = ‘’
    String2 = ‘’
    for i in range(12):
        String1 = String1 + str(Month_Names[i]) + ‘\t\t’
        String2 = String2 + “Day” + ‘\t’+ “EoT” + ‘\t’
    String1 = String1[:len(String1) - 1] # delete last tab mark
    String2 = String2[:len(String2) - 1] # delete last tab mark
    File.write(String1 +’\r’)
    File.write(String2 +’\r’)

    # Write Table Data
    for i in range(Max_Row):
        String = ‘’
        for j in range(24):
            String += str(Matrix[i][j]) + ‘\t’
        String = String[:len(String) - 1] # delete last tab mark
        File.write(String +’\r’)
    File.close()
    print (“Please find output in file “ + Table_Filename)
    print (“which should be in the same folder as this program.”)
    print (“The file may be opened in any spreadsheet.”)

Calculate(Year)


print (‘\rDone’)
```

See also ‘General Service Routines’ on page 226.
in particualr Sun_JD which does the main astronomical calculation

ANALEMMA PLOTTING

‘Delineating an Analemma’ on page 43.

```
from math import degrees,radians,tan,sin,acos,cos,floor,atan2,sqrt,asin,pi
# -----------------------------------------------------------------------------------
# ANALEMMAS
# -----------------------------------------------------------------------------------
# Python Code written by Kevin Karney, Winter 2024
# Should work on all releases of Python
# Free for anyone to use without any guarantees!
#
# NOTA BENE
# where Time is input, it is local STANDARD time (i.e.no Daylight saving).
# Hence Time Zone occurs in many routines to correct to UTC,

# The code provides the coordinates required to plot an Analemma on any plane surface
#     a full analemma or days-lengthening or days-shortening analemma can be chosen
#     calculated by subroutine Calculate
#     four main subroutines are called
#         Analemmas         (File)
#         Declination_Lines (File)
#         each of which calls subroutine
#            Sol(.....) which produces the x-y coordionates of the nodus shadow on the plane
#     Output is a spreadsheet-readable text file with the x-y coordinates
#
# There are a number of service routines
#     Julian(Year,Month,Day,Hour,Zone) which gives the Julian Day
#     Get_Calendar_Date(The_JD,Zone) which converts between Julian Day and normal calendar
values

# There a number of self explanatory print routines used by the Analemma routines

# ---------------------------------------------------------------------------------------
# What is Wanted
# ---------------------------------------------------------------------------------------
Which_Analemma = 2       # 0 = Full Analemma :
                         # 1 = Daylight Increasing (Winter and Spring)
#                        # 2 = Daylight Shortening (Summer and Autumn)
# ---------------------------------------------------------------------------------------
# Location Input
# ---------------------------------------------------------------------------------------
Place          = ‘Athens’
Longitude      = 23.71667 # Degrees : +ve East of Greenwich
Latitude       = 37.96667 # Degrees : +ve East of Greenwich
Zone           = 2         # Hrs      : +ve East of Greenwich

Year           = 2024

Hour_Start             = 11
Hour_End               = 14      #   e.g. will draw analemmas from 11 a.m. to 2 p.m.
Analemma_Minute_Inc    = 30      #   e.g 15 if you want analemmas every 15 minutes
Declination_Increment  = 2       #   e.g 5 if you want declination points every 5 minutes
Want_Declination_Lines = True
Mean_or_Solar          = True    # if False, will produce results for traditional straight-
line solar sundial
# -----------------------------------------------------
# Sizes are unitless: Output the same as input
Dial_Plate_Width       = 16
Dial_Plate_Height      = 16
Nodus_Height           = 5
Nodus_x                = 4   # Shift Nodus away from physical centre of the Dial Plate in
x-direction
Nodus_y                = 4   # ditto in y-direction (+ve Up)
Zenithal_Dist          = 60  # Degrees :  0 = Horizontal, 90 = Vertical
Gnomonic_Decl          = 50  # Degrees :  0 = due South,  90 = due West,
                             #          180 = due North, 270 = due East
# -----------------------------------------------------
Declination_Days  = [1,10,20]  # e.g. lines on 1st, 10th, 20th of the month
Special_Days      = [“12-May”] # use for birthdays, etc
Days_in_Year      = 366 if Year % 4 == 0 else 365
#==========================================================================================
L                 = radians(Latitude)
D                 = radians(Gnomonic_Decl)
Z                 = radians(Zenithal_Dist)
P                 = sin(L) * cos(Z) – cos(L) * sin(Z)   * cos(D)
# X0 & Y0 are the coodinates from the dial Plate centre of ...
# ...the foot of a polar stylus passing through the nodus
# i.e. the centre of the traditional dial
X0                = Nodus_Height * cos(L)   * sin(D) / P
Y0                = Nodus_Height * (sin(L) * sin(Z) + cos(L) * cos(Z) * cos(D)) / P
#==========================================================================================
```

```
def Calculate():

    # -----------------------------------------------------
    # This is routine called from the last line of this code
    # this should write output file to same folder as this progranm
    if Which_Analemma == 0:
        Analemma_Filename      = ‘Analemma whole year.txt’
    elif Which_Analemma == 1:
        Analemma_Filename      = ‘Analemma days increasing.txt’
    else:
        Analemma_Filename      = ‘Analemma days decreasing.txt’

    File = open(Analemma_Filename,’w’)
    Print_Super_Header(File)
    Find_Solstices_and_Equinoxs(Year)
    Analemmas(File)
    Declination_Lines(File)
    File.close()
    print (“Please find output in file ‘” + Analemma_Filename+”’,”)
    print (“which should be in the same folder as this program.”)
    print (“The file may be opened in any spreadsheet.”)

def Find_Solstices_and_Equinoxs(Year):

    # ----------------------------------------------------------
    # Finds the Julian Days for the Equinoxs & Solstices
    # for the specified Year, by looping through the dates that surround
    # surround that event
    # It calls routine ‘Julian’ & ‘Sun(JD)’
    # The output is an array of the four Julian Days
    # ----------------------------------------------------------
    global Dates
    Dates = []
    # Find Winter Solstice for previous Year
    JD    = Julian(Year-1,12,18,12,Zone)
    Last  = 25
    for i in range(10):
        Decl = Sun_JD(JD)[3]
        if Decl > Last :
            Dates.append(int(JD))
            break
        Last=Decl
        JD+=1
     # Find Spring Equinox for Year
    JD = Julian(Year,3,16,12,Zone)
    Last = -10
    for i in range(10):
        Decl = Sun_JD(JD)[3]
        if Decl > 0:
            Dates.append(int(JD))
            break
        Last=Decl
        JD+=1
     # Find Summer Solstrice for Year
    JD = Julian(Year,6,16,12,Zone)
    Last=20
    for i in range(10):
        Decl = Sun_JD(JD)[3]
        if Decl < Last :
            Dates.append(int(JD))
            break
        Last=Decl
        JD+=1
    # Find Autumnal Equinox for Year
    JD = Julian(Year,9,16,12,Zone)
    Last = 10
    for i in range(40):
        Decl = Sun_JD(JD)[3]
        if Decl < 0:
            Dates.append(int(JD))
            break
        Last=Decl
        JD+=1
    # Find Winter Solstice for Year
    JD = Julian(Year,12,18,12,Zone)
    Last=-20
    for i in range(10):
        Decl = Sun_JD(JD)[3]
        if Decl > Last :
            Dates.append(int(JD))
            break
        Last=Decl
        JD+=1
    return Dates
```

return to "Table of Contents"

```
def Analemmas(File):

    # ----------------------------------------------------------
    # Draw the Analemmas
    # Loop over the Hours requested in the day, then each day in the year
    # It calls routine ‘Shadow’ (& various output print formatting rountines)
    # ----------------------------------------------------------
    global Dates
    Print_Analemma_Header(Hour_Start,File)

    Start_Minute = Hour_Start * 60
    End_Minute   = Hour_End   * 60
    for Minute in range(Start_Minute,End_Minute+1,Analemma_Minute_Inc) :
        Hour           = Minute / 60.
        Minute_in_Hour = Minute % 60
        Time_Text = str(int(Hour)) + ‘:’ + (‘0’ if Minute_in_Hour < 10 else ‘’)+ str(Minute_in_
Hour) + ‘ hh:mm’
        Hr = str(int(Hour))
        Min = str(Minute_in_Hour)
        Print_Analemma_Sub_Header(Hr,Min,File)

        if Which_Analemma == 0:
            Start = Dates[0]
            End   = Dates[4]
        elif Which_Analemma == 1:
            Start = Dates[0]
            End   = Dates[2]
        else:
            Start = Dates[2]
            End   = Dates[4]
        for JD in range(Start,End) :
            Inc_Dec = “I” if JD >= Dates[0] and JD < Dates[2] else “D”
            Answer = Get_Calendar_Date(JD,Zone)
            # Find the Shadow Point
            q = Shadow(Inc_Dec,Answer[0],Answer[1],Answer[2],int(Hour),Minute_in_Hour,Longi-
tude,Latitude,Zone,File)

def Declination_Lines(File):

    global Dates
    # ----------------------------------------------------------
    # Draw the Declination Lines
    # First: Loop over the Days in a Year day, looking for the days on which
    # a declination line is requested  -
    # Second: Loop over Hours during the day
    # It calls routine ‘Shadow’ (& various output formatting rountines)
    # ----------------------------------------------------------

    # Don’t try Declination Lines if only a Single Analemma
    if not (Want_Declination_Lines or Hour_Start != Hour_End) :
        return

    if Which_Analemma == 0:
        Start = Dates[0]
        End   = Dates[4]
    elif Which_Analemma == 1:
        Start = Dates[0]
        End   = Dates[2]
    else:
        Start = Dates[2]
        End   = Dates[4]

    Print_Declination_Line_Header(File)
    Last_Date = “xx-xxx”
    for JD in range(Start,End) :
        Year,Month,Day,Hour,Minute,Second,Date_Text = Get_Calendar_Date(JD,Zone)
        if Day in Declination_Days or Date_Text in Special_Days:
            Minute_Start       = Hour_Start * 60
            Minute_End         = Hour_End   * 60
            for Minute in range (Minute_Start,Minute_End+1,Declination_Increment):
                Hour_in_Day    = int(Minute / 60)
                Minute_in_Hour = Minute % 60
                if Date_Text != Last_Date:
                    Print_Declination_Lines_Sub_Header(Date_Text,File)
                Last_Date = Date_Text
                # Find the Shadow Point
                Inc_Dec = “-”
                q = Shadow(Inc_Dec,Year,Month,Day,Hour_in_Day,Minute_in_Hour,Longitude,Lati-
tude,Zone,File)
```

```
def Shadow(Inc_Dec,The_Year,The_Month,The_Day,The_Hour,The_Minute,The_Longitude,The_Lati-
tude,The_Time_Zone,File):
    # --------------------------------------------------------------
    # This is the Gnonomic Heart of the program
    # from Robert Sagot and Denis Savoie of Commission des Cadrans Solaires
    # Quoted in Meeus, Astronomical Algorithms - Chapter 58
    # It calls routine Sun to provide EoT & Decl
    # --------------------------------------------------------------
    My_Decimal_Hour    = The_Hour + The_Minute/60.
    Answer   = Sun(The_Year,The_Month,The_Day,The_Hour,The_Longitude,The_Latitude,The_Time_Zone)
    EoT_min,EoT_Corr_min,Decl_deg = Answer[0],Answer[1],Answer[3]
    # Noon EoT & Decl used to estimate time of sunrise/set
    Answer = Sun(The_Year,The_Month,The_Day,12,The_Longitude,The_Latitude,The_Time_Zone)
    EoT_min,EoT_Corr_min_Noon,Decl_deg_Noon = Answer[0],Answer[1],Answer[3]

    # Traditional Solar Time Sundial requested
    if Mean_or_Solar  == False :
        EoT_Corr_min       = 4 * (Zone * 15 - Longitude)
        EoT_Corr_min_Noon  = 4 * (Zone * 15 - Longitude)

    Decl_radians       = radians(Decl_deg)
    Decl_radians_noon  = radians(Decl_deg_Noon)
    # ---------------------------------------------------------
    # Find Times of Sun Rise/Set
    # ---------------------------------------------------------
    HA_Sunrise_degrees = degrees(-acos(-tan(L) * tan(Decl_radians_noon)))
    Sunrise_hour       = 12. + HA_Sunrise_degrees/15. - EoT_Corr_min_Noon/60.
    Sunset_hour        = 12. - HA_Sunrise_degrees/15. - EoT_Corr_min_Noon/60.
    # ---------------------------------------------------------
    # Only Continue between Sun Rise and Sun Set
    # ---------------------------------------------------------
    if My_Decimal_Hour >= Sunrise_hour and My_Decimal_Hour <= Sunset_hour:
        H  = radians(((My_Decimal_Hour - EoT_Corr_min/60. - 12.) * 15.))

        Q1 = sin(D) * sin(Z) * sin(H)
        Q2 = (cos(L) * cos(Z) + sin(L) * sin(Z) * cos(D)) * cos(H)
        Q3 = P * tan(Decl_radians)
        Q  = Q1 + Q2 + Q3
        # ---------------------------------------------------------
        # Only Continue if Sun is in front of the surface of the dial plate
        # Reference Meeus Astronomical Algorithms Chapter 58
        # ---------------------------------------------------------
        if Q  > 0 :
            Nx1 = cos(D) * sin(H)
            Nx2 = sin(D) * (sin(L) * cos(H) - cos(L) * tan(Decl_radians))
            Nx = Nx1 - Nx2

            Ny1 = cos(Z)  * sin(D) * sin(H)
            Ny2 = (cos(L) * sin(Z) - sin(L) * cos(Z) * cos(Decl_radians)) * cos(H)
            Ny3 = (sin(L) * sin(Z) + cos(L) * cos(Z) * cos(D)) * tan(Decl_radians)
            Ny = Ny1 - Ny2 - Ny3
            # ---------------------------------------------------------
            # Find Coordinates of Shadow from nodus foot
            # Reference Meeus Astronomical Algorithms Chapter 58
            # ---------------------------------------------------------
            x = (Nodus_Height * Nx / Q)
            y = (Nodus_Height * Ny / Q)
            # ---------------------------------------------------------
            # Find Coordinates of Shadow from plate centre
            # ---------------------------------------------------------
            xx = x + Nodus_x
            yy = y + Nodus_y
            On_Plate = ‘ ’ if ((xx >= -Dial_Plate_Width/2 and xx <= Dial_Plate_Width/2) and
(yy >= -Dial_Plate_Height/2 and yy <= Dial_Plate_Height/2)) else ‘Off Plate’

            # Write the output to file
            File.write(Inc_Dec + ‘\t’ + str(The_Year) +’\t’ + str(The_Month) + ‘\t’ +
str(The_Day) + ‘\t’ + str(The_Hour) + ‘\t’ + str(The_Minute) + ‘\t’ + str(round(xx,3)) +’\t’+
str(round(yy,3))+’\t’+ On_Plate +’\r’)
            return
        else:
            # Sun is Behind the Plate
            File.write(“-” + ‘\t’ + str(The_Year) +’\t’ + str(The_Month) + ‘\t’ + str(The_Day) +
‘\t’ + str(The_Hour) + ‘\t’ + str(The_Minute) + ‘\t\t\t’ + ‘Behind’+’\r’)
            return
    else:
        # Sun is below Horizons
        File.write(“-” + ‘\t’ + str(The_Year) +’\t’ + str(The_Month) + ‘\t’ + str(The_Day) + ‘\t’
+ str(The_Hour) + ‘\t’ + str(The_Minute) + ‘\t\t\t’ + ‘Night’+’\r’)
        return
```

See also ‘General Service Routines’ on page 226.
in particualr Sun_JD which does the main astronomical calculation

return to "Table of Contents"

```
from math import degrees,radians,tan,sin,acos,cos,floor,atan2,sqrt,asin,pi
# ----------------------------------------------------------------------------------
# EQUATION OF TIME SERVICE ROUTINES
# ----------------------------------------------------------------------------------
# Python Code written by Kevin Karney, Winter 2024
# Should work on all releases of Python
# Free for anyone to use without any guarantees!
# NOTA BENE
# where Time is input, it is local STANDARD time (i.e. no Daylight saving).
# Hence Time Zone occurs in many routines to correct to UTC,

# Herein
#     Sun_JD(JD) whicg provides all tye astronomical details used in Kepler’s Method
#     Julian(Year,Month,Day,Hour,Zone) which gives the Julian Day
#     Get_Calendar_Date(The_JD,Zone) which converts between Julian Day and normal calendar
values
#     EoT_Dec_MMSS(The_EoT) which formats decimal minutes to mins and second
#     Dec_Deg_DMS(The_Degs) which formats decimal degrees to degrees, mins and secs
#     Dec_Hrs_HMS(The_Hrs) which formats decimal hours to degrees, mins and secs

def Sun_JD(JD):

    # -----------------------------------------------------
    # This does the main astronomical calculations
    # It returns the EoT, Longitude Corrected EoT & Declination
    # but these may be changed to any other parameter that has been calculated
    # -----------------------------------------------------
    X = JD -.5
    UTC_hrs = ((X-int(X)) * 24) % 24 # Extract UTC_hrs from Julian Day
    D0                  = JD - 2451545.
    T                   = D0 / 36525

    GMST_deg            = 280.46061837 + 360.98564736629 * D0   + 0.000387933 *T**2 -
T**3/38710000.
    GMST_deg            = GMST_deg % 360
    GMST_hrs            = GMST_deg / 15.
    LMST_Hrs            = GMST_hrs + Longitude / 15.

    Mean_Longitude_hrs  = GMST_hrs + 12. - UTC_hrs
    Mean_Longitude_deg  = Mean_Longitude_hrs * 15
    #-------------------
    # These values obtained from the Astronmical Almanac vols between 2000 and 2023
    Perihelion_deg      = 282.938     + 1.7     * T
    Eccentricity        = 0.016708617 - 0.00004 * T
    Obliquity_deg       = 23.43929111 - 0.013   * T
    Obliquity_rad       = radians(Obliquity_deg)
    #-------------------
    Mean_Anomaly_deg     = Mean_Longitude_deg - Perihelion_deg
    Mean_Anomaly_rad     = radians(Mean_Anomaly_deg)
    E0 = Mean_Anomaly_rad
    E1 = E0 + (Mean_Anomaly_rad + Eccentricity*sin(E0)- E0)/(1 - Eccentricity*cos(E0))
    # p.s. second Newton Raphson iteration not really needed
    E2 = E1 + (Mean_Anomaly_rad + Eccentricity*sin(E1)- E1)/(1 - Eccentricity*cos(E1))
    Eccentric_Anomaly   = E2
    True_Anomaly_rad     = atan2(sqrt(1 - Eccentricity**2) * sin(Eccentric_Anomaly), (cos(Eccen-
tric_Anomaly)- Eccentricity))
    True_Anomaly_deg     = degrees(True_Anomaly_rad)
    True_Long_deg        = True_Anomaly_deg + Perihelion_deg
    True_Long_rad        = radians(True_Long_deg)
    Eccent_Effect_deg   = True_Long_deg - Mean_Longitude_deg
    Eccent_Effect_min   = 4 * Eccent_Effect_deg
    #--------------------
    Right_Ascension_rad = atan2(cos(Obliquity_rad) * sin(True_Long_rad),cos(True_Long_rad)) %
(2*pi)
    Right_Ascension_deg = (degrees(Right_Ascension_rad)) % 360.
    Right_Ascension_hrs = Right_Ascension_deg / 15.
    Declination_rad     = asin(sin(Obliquity_rad) * sin(True_Long_rad))
    Declination_deg     = degrees(Declination_rad)
    #--------------------
    EoT_deg             = Right_Ascension_deg - Mean_Longitude_deg
    if EoT_deg >  180.: EoT_deg = EoT_deg-360.
    if EoT_deg < -180.: EoT_deg = EoT_deg+360.
    EoT_min             = 4 * EoT_deg
    Obliq_Effect_min    = EoT_min - Eccent_Effect_min
    Long_Corr           = 4 * (Zone * 15 - Longitude)
    EoT_Corr_min        = EoT_min + Long_Corr

    Solar_Noon_hrs      = 12 + EoT_Corr_min/60
    Hour_Angle_hrs      = GMST_hrs + Longitude/15. - Right_Ascension_hrs
    Hour_Angle_rad      = radians(Hour_Angle_hrs * 15.)
    Latitude_rad        = radians(Latitude)
    Altitude_rad        = asin((sin(Latitude_rad) * sin(Declination_rad) + cos(Latitude_rad) *
cos(Declination_rad) * cos(Hour_Angle_rad)))
    Altitude_deg        = degrees(Altitude_rad)
```

```
    a                   =-cos(Declination_rad) * cos(Latitude_rad) * sin(Hour_Angle_rad)
    b                   = sin(Declination_rad) - sin(Latitude_rad) * sin(Altitude_rad)
    Azimuth_rad         = atan2(a,b)
    Azimuth_deg         = degrees(Azimuth_rad)%360
    q_hrs               =(degrees(acos(-tan(Latitude_rad) * tan(Declination_rad)))) / 15
    SR_hrs              = Solar_Noon_hrs - q_hrs
    SS_hrs              = Solar_Noon_hrs + q_hrs
    r_deg               = degrees(acos(-sin(Declination_rad) / cos(Latitude_rad)))
    SRA_deg             = 180 - r_deg
    SSA_deg             = 180 + r_deg
    return EoT_min,EoT_Corr_min,Right_Ascension_hrs,Declination_deg,Altitude_deg,Azimuth_de-
g,SR_hrs,SS_hrs

f def Julian(Year,Month,Day,Hour,Zone) :
    #=============================================
    # ROUTINE TO GET JULIAN DAY FROM DATE & TIME
    # Reference: Astronomical Algorithms 2nd Edition 1998 by Jean Meeus - Page 60-61
    YY                  = Year
    MM                  = Month
    UTC_hrs             = Hour - Zone
    if UTC_hrs<0:  Day -=1
    if UTC_hrs>24: Day +=1
    UTC_hrs             = UTC_hrs % 24
    if Month   <= 2: Month,Year = Month + 12,Year - 1
    a           = int(Year / 100)
    b           = 2 - a + int(a / 4)
    c           = int(365.25 * Year) ;
    d           = int(30.6001 * (Month + 1)) ;
    Julian_Day  = b + c + d + Day + 1720994.5 + (UTC_hrs)/24.
    return Julian_Day

def Get_Calendar_Date(The_JD,Zone) :
    #========================================================
    # ROUTINE TO GET CALENDAR DATE AND TIME FROM JULIAN DAY
    # Reference: Practical Astronomy with your Calculator 3rd Edn : Duffet Smith - Page 8
    Month_List = [‘Jan’,’Feb’,’Mar’,’Apr’,’May’,’Jun’,’Jul’,’Aug’,’Sep’,’Oct’,’Nov’,’Dec’]
    JDD                = The_JD + .5
    III                = int(JDD)
    FFF                = JDD - III
    if III > 2299160 :
        AA             = int((III - 1867216.25) / 36524.25)
        BB             = III + 1 + AA - int(AA / 4)
    else :
        BB             = III
    CC                 = BB + 1524
    DD                 = int((CC - 122.1) / 365.25)
    EE                 = int(365.25 * DD)
    GG                 = int((CC - EE) / 30.6001)
    Day_inc_Frac       = CC - EE + FFF - int(30.6001 * GG)
    Dayo               = int(Day_inc_Frac)
    Hour               = 24 * (Day_inc_Frac - Dayo) + Zone
    Houro              = int(Hour)
    Minute             = 60. * (Hour - Houro)
    Minuteo            = int(Minute)
    Second             = 60. * (Minute-Minuteo)
    if round(Second,3) == 60.:
        Second = 0
        Minuteo += 1
    if Minuteo == 60:
        Minuteo = 0
        Houro +=1
    if GG < 13.5 :
        Montho         = GG - 1
    else :
        Montho         = GG - 13
    if Montho > 2.5 :
        Yearo          = DD - 4716
    else :
        Yearo          = DD - 4715
    Txt = str(Dayo) + ‘-’ + Month_List[Montho-1]
    X = The_JD-.5
    UTC_hrs = ((X-int(X)) * 24) % 24 # Extract UTC_hrs from Julian Day
    Txt = str(Year) + ‘-’ + Month_List[Montho-1] + ‘-’ + str(Dayo) + ‘ ‘ + str(Houro) + ‘hrs’
    return Yearo,Montho,Dayo,Houro,Minuteo,Second,Txt
```

```
def EoT_Dec_MMSS(The_EoT):

    # -----------------------------------------------------
    # Routine to convert Decimal Minutes to Minutes & Seconds
    Sign = ‘+’
    if The_EoT < 0 : Sign = ‘–’
    M0   = abs(The_EoT)
    M1   = int(M0)
    S0   = 60 * (M0 – M1)
    return Sign + ‘%02.0f’ % M1 + ‘:’ + ‘%02.1f’ % S0 + ‘ mm:ss’

def Dec_Deg_DMS(The_Degs) :

    # -----------------------------------------------------
    # Routine to convert Decimal Degrees to Degrees,Minutes & Seconds
    D0 = abs(The_Degs)
    Sign = ‘+’
    if (The_Degs < 0) : Sign = ‘–’
    D1 = int(D0)
    M0 = 60. * (D0 – D1)
    M1 = int (M0)
    S0 = 60. * (M0 – M1)
    return Sign + str(round(D1,2)) + u’° ‘ +str(round(M1,2)) + “’ “  + str(round(S0,2))  + “\””

def Dec_Hrs_HMS(The_Hrs) :

    # -----------------------------------------------------
    # Routine to convert Decimal Hours to Hours,Minutes & Seconds
    D0 = abs(The_Hrs)
    Sign = ‘+’
    if (The_Hrs < 0) : Sign = ‘–’
    D1 = int(D0)
    M0 = 60. * (D0 – D1)
    M1 = int (M0)
    S0 = 60. * (M0 – M1)
    return Sign + ‘%.02d’ % D1 + u’:’ +’%.02d’ % M1 + u’:’ + ‘%2.1f’ % S0 + u’ hh:mm:ss’
```

An Unrealised Heliochronometer

Kevin Karney

Preamble

A very, very wealthy gentleman bought a large estate somewhere in Norfolk. Therein, he planned to build a private observatory. He employed a full time engineer with a sophisticated machine shop to make the mechanisms. In the observatory garden, it was suggested to place a 'unique' sundial.

The design criteria was to produce a dial that:

- accurate to half a minute
- maintain that accuracy for 500 years
- to work in any location around the world
- to be 'interesting', elegant, robust and educational
- entirely mechanical and optical

What a brief!

If the author had been older and wiser, he would have realised the impossibility of such a dream.

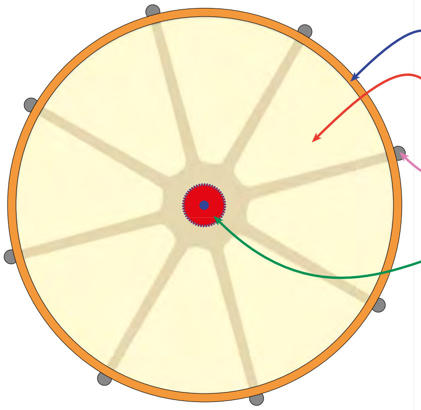
The gentleman, however, appears to have been involved in the relocation of Russian chemical weapons scientists from Russia to America. He seemed to have rather suddenly lost his source of income, left the house in a hurry and vanished.

So no observatory and no dial.

Below are the concept design drawings.

Nota Bene. The atmospheric refraction mechanism proposed for this project is theoretically incorrect and almost certainly unnecessary.

Pictorial Description



Version 6 - December 2006

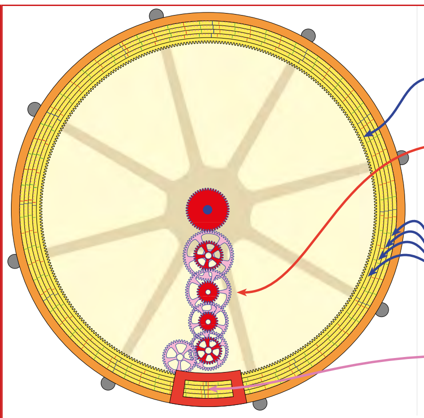
Conceptual Design for a Heliochronometer

Step 1

A Cylindrical Casing
whose
floor is parallel
to the Equator.

A handwheel, turning
1/4 revolutions per day
is connected to
a central Cog

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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Conceptual Design for a Heliochronometer

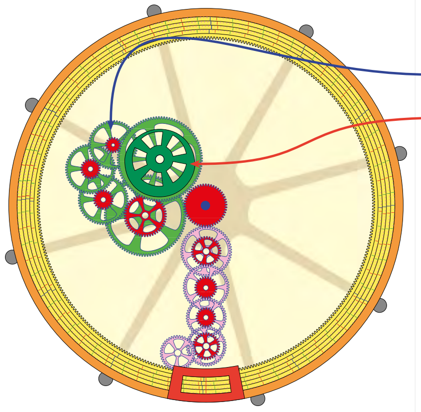
Step 2

Mounted inside the casing
is the Date Reading Ring.

A set of gears rotates
the ring
once per Calendrical Year
of 365.25 days.
A small instrument adjustment is required
for Century years with no leap day

There are four Day Scales:
3 with 365 graduations,
1 with 366 graduations.
The appropriate date
is set in the Date Sight

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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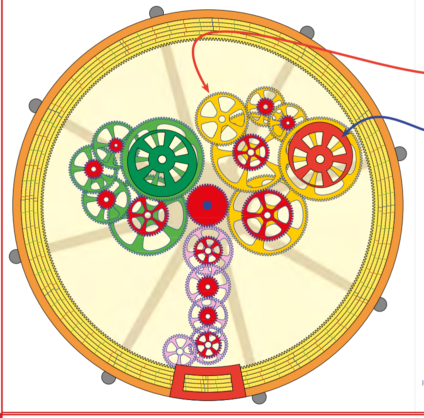
Conceptual Design for a Heliochronometer

Step 3

A set of Gears connected
to the central cog drives
a Cam, which rotates
once per Anomalystic year
of 365.259652 days.

The shape of the Cam
reflects
the Eccentricity Error
in the Equation of Time

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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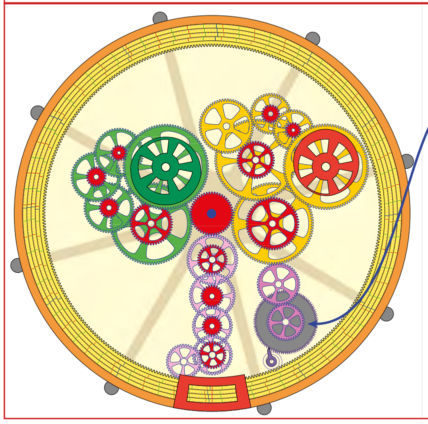
Conceptual Design for a Heliochronometer

Step 4

Another set of Gears
connected to the central
cog drives a second
Cam, which rotates
twice per Tropical year
of 365.242201 days.

The shape of the Cam
reflects
the Obliquity Error
in the Equation of Time

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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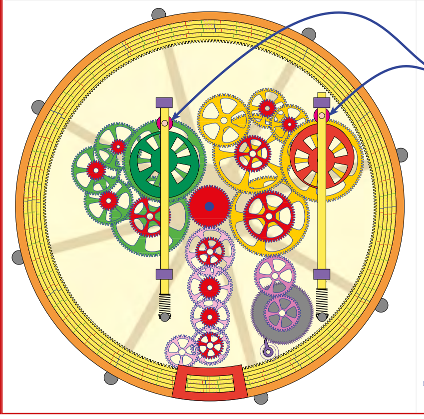
Conceptual Design for a Heliochronometer

Step 5

A Pawl & Ratchet
prevents the gears from
turning in the reverse
direction - hence reducing
the backlash error in the
gear trains.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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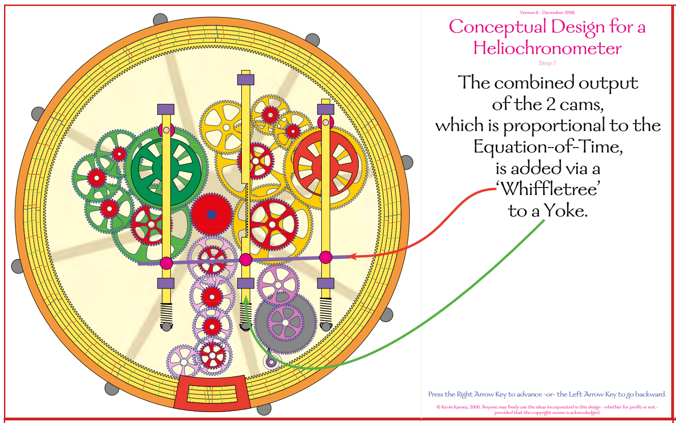
Conceptual Design for a Heliochronometer

Step 6

Two Cam Followers
ride on the cams.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.

return to "Table of Contents"



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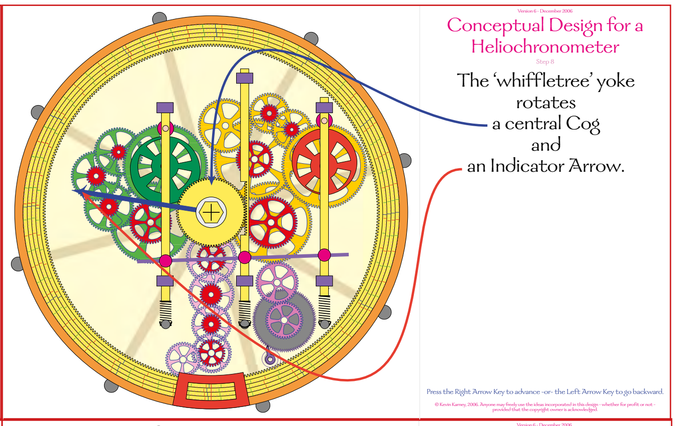
Conceptual Design for a Heliochronometer

Step 7

The combined output
of the 2 cams,
which is proportional to the
Equation-of-Time,
is added via a
'Whiffletree'
to a Yoke.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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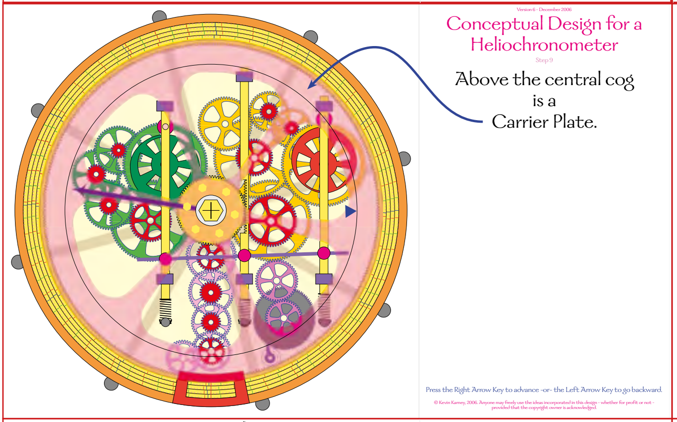
Conceptual Design for a Heliochronometer

Step 8

The 'whiffletree' yoke
rotates
a central Cog
and
an Indicator Arrow.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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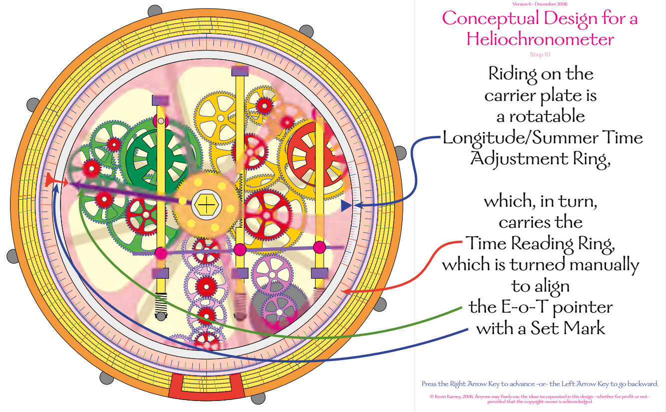
Conceptual Design for a Heliochronometer

Step 9

Above the central cog
is a
Carrier Plate.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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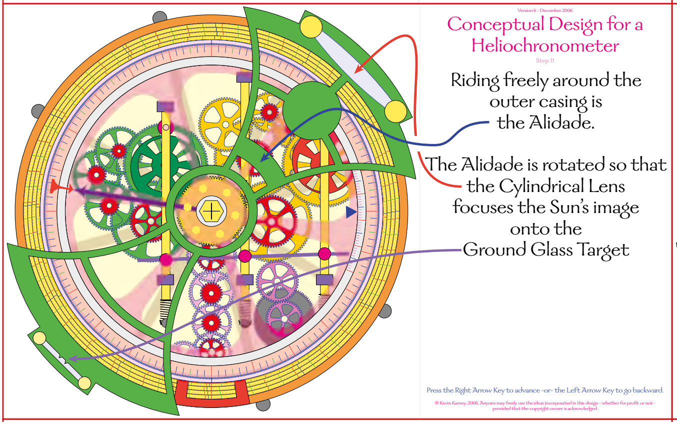
Conceptual Design for a Heliochronometer

Step 10

Riding on the
carrier plate is
a rotatable
Longitude/Summer Time
Adjustment Ring,

which, in turn,
carries the
Time Reading Ring,
which is turned manually
to align
the E-o-T pointer
with a Set Mark

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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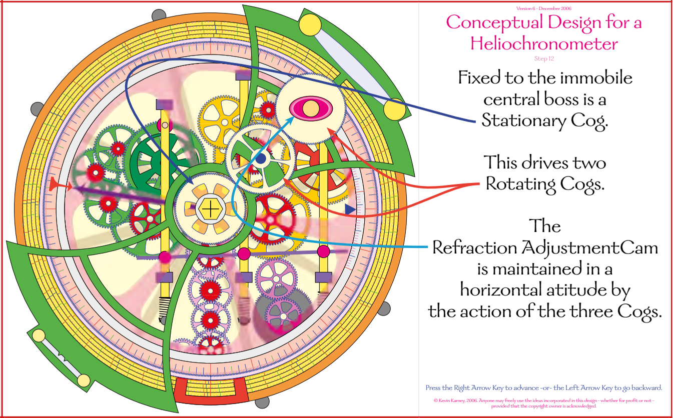
Conceptual Design for a Heliochronometer

Step 11

Riding freely around the
outer casing is
the Alidade.

The Alidade is rotated so that
the Cylindrical Lens
focuses the Sun's image
onto the
Ground Glass Target

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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Conceptual Design for a Heliochronometer

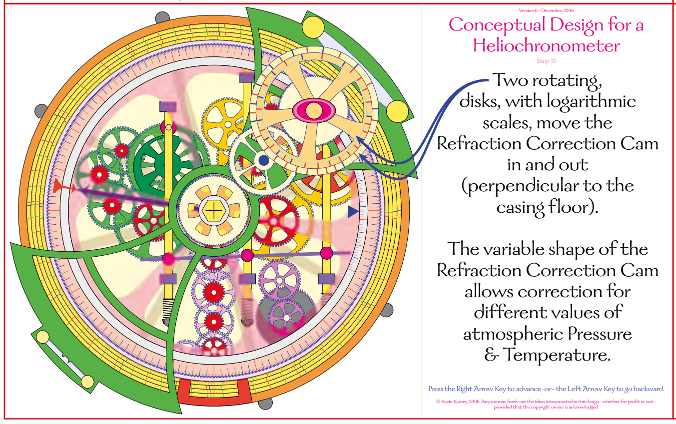
Step 12

Fixed to the immobile
central boss is a
Stationary Cog.

This drives two
Rotating Cogs.

The
Refraction AdjustmentCam
is maintained in a
horizontal atitude by
the action of the three Cogs.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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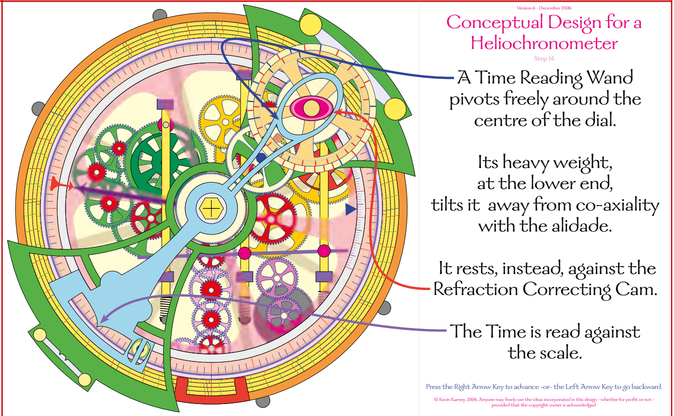
Conceptual Design for a Heliochronometer

Step 13

Two rotating
disks, with logarithmic
scales, move the
Refraction Correction Cam
in and out
(perpendicular to the
casing floor).

The variable shape of the
Refraction Correction Cam
allows correction for
different values of
atmospheric Pressure
& Temperature.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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Conceptual Design for a Heliochronometer

Step 14

A Time Reading Wand
pivots freely around the
centre of the dial.

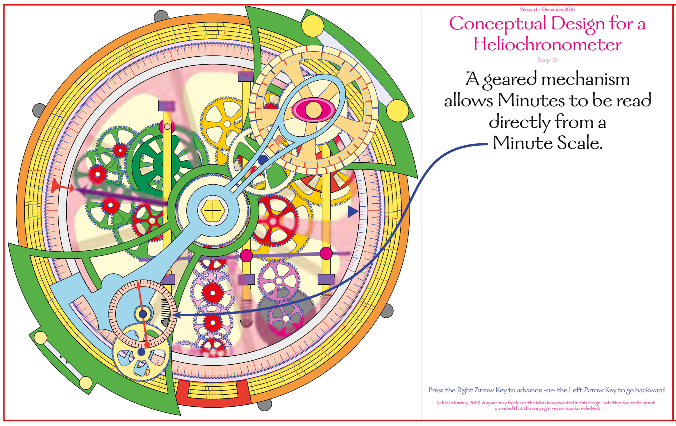
Its heavy weight,
at the lower end,
tilts it away from co-axiality
with the alidade.

It rests, instead, against the
Refraction Correcting Cam.

The Time is read against
the scale.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.

return to "Table of Contents"



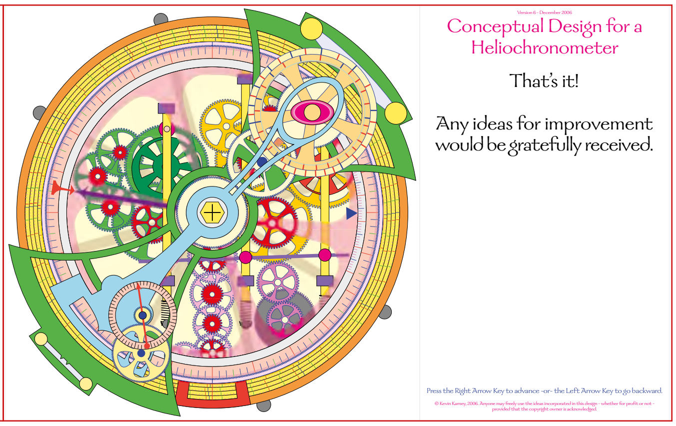
Version 6 - December 2006

Conceptual Design for a Heliochronometer

Step 15

A geared mechanism allows Minutes to be read directly from a Minute Scale.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.



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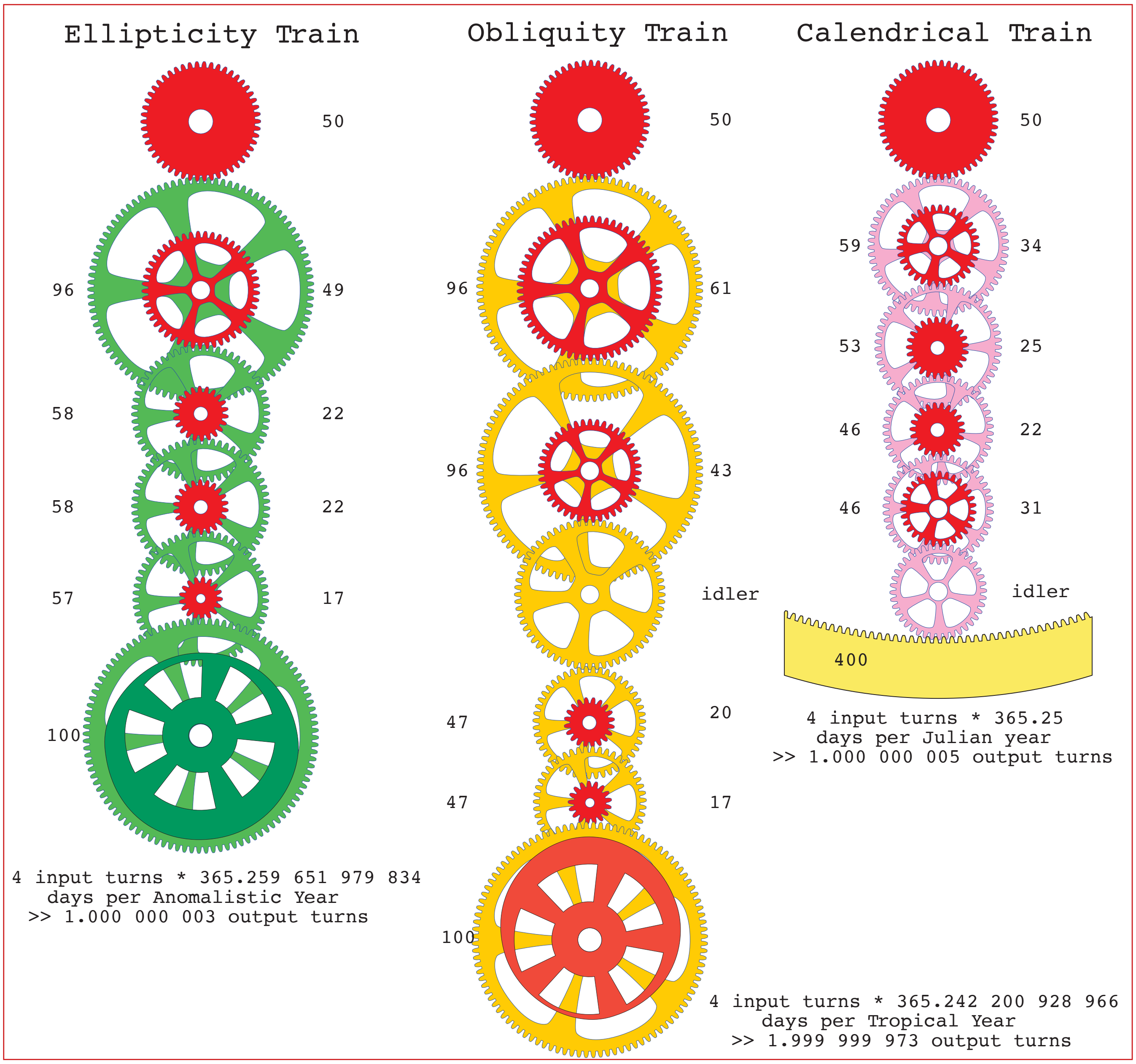
Conceptual Design for a Heliochronometer

That's it!

Any ideas for improvement would be gratefully received.

Press the Right Arrow Key to advance -or- the Left Arrow Key to go backward.

GEAR CHAINS



4 input turns * 365.259 651 979 834 days per Anomalistic Year >> 1.000 000 003 output turns

4 input turns * 365.242 200 928 966 days per Tropical Year >> 1.999 999 973 output turns

4 input turns * 365.25 days per Julian year >> 1.000 000 005 output turns

return to "Table of Contents"

Reading List & Available Videos

The author has extensively used three major resources, which are articles in....

i) Bulletins of the British Sundial Society;

ii) Compendia of the North American Sundial Society;

iii) Journals of the Antiquarian Horological Society.

Access to the resources of these societies may require membership in order to freely obtain their articles.

There is a searchable open index to the contents of i) and ii) at ...

https://gnomoni.ca/IndexSciathericus/

The list below just contains those books in the author's library that have helped devise this book. Those high-lighted in yellow are the most important.

Generally, references have been given in text, rather than being hoarded at chapter or book ends.

Astronomical Algorithms - 2nd Edition	Jean Meeus	Willmann-Bell Inc	1998	0-943396-61-1
Practical Astronomy with your Calculator - 3rd Edition	Peter Duffett-Smith	Cambridge Univ. Press	1988	0-521-35699-7
Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac	-	H.M's Stationary Office	1961	0-11-880578-9
The Astronomical Almanac for the Year 2023 (or any other year)	-	U.S. Govt Publishing Office	2022	978-0-7077-46333
The History and Practice of Ancient Astronomy	James Evans	Oxford Univ. Press	1998	0-19-509539-1
Admiralty Manual of Navigation - Vol III. Chap IX - The Sun's Apparent Orbit.	-	H.M's Stationary Office	1954	-
Ptolemy's Almagest	G.J. Toomer	Princeton Univ. Press	1998	0-691-002606
The Silent Voice of Time - Sciatherics 1	Frederick W. Sawyer III	lulu.com	2022	-
A Magic Shadow Show - Sciatherics 2				
Ray of Sunlight - Sciatherics 3				
Born of Light - Sciatherics 4				
More to Say - Sciatherics 5				
Sundials - Design, Construction and Use	Denis Savoie	Springer Praxis	2009	978-0-387-09801-2
Sundials - History, Art, People, Science	Mark Lennox-Boyd	Frances Lincoln	2005	978-0-7112-2494-0
Dutch Light: Christiaan Huygens and the Making of Science in Europe	Hugh Aldersey-Williams	Picador	2020	978--1-5098-9335-1
The Bagnold Sun-Compass - History & Utilization	Kuno Gross	Herstelling und Verlag	2011	978-384-233702-2
The Most Important Clock in America - The David Rittenhouse Astronomical & Musical Clock	Ronald R. Hoppes	American Philosophical Soc.	2009	978-1-60618-992-4
Some Outstanding Clocks over Seven Hundred Years: 1250-1950	A. Alan Lloyd	Leonard Hill(Books) Ltd	1958	-
The Mastery of Time	Dominique Fléchon	Flammarion	2011	978-2-08-020080-8
Masterpieces of English Furniture & Clocks	R.W. Symonds	B.T. Batsford Ltd	1986	1-85170-068-4
Thomas Tompion: his Life & Works	R.W. Symonds	B.T. Batsford Ltd	1951	-
Jens Olsen's Clock	Otto Mortensen	Technological Institute Copenhagen	1957	-

return to "Table of Contents"

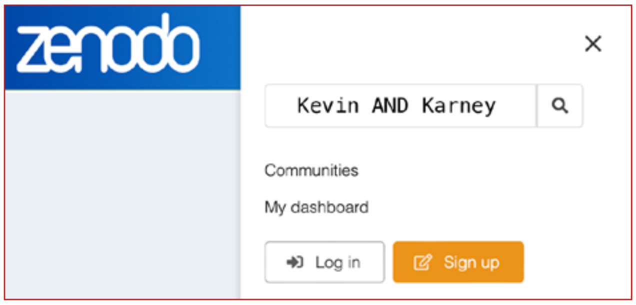
AVAILABLE VIDEOS

Zenodius c. 325 – c. 270 BC was the first known head librarian of the Great Library of Alexandria. In his honour, CERN has created its document archive, which is freely available to all.

All but one of the Videos are stored on CERN's Zenodo document archive. To access the library go to..

https://zenodo.org

Click on the three bars and sign up. It is free to use.



Kevin AND Karney

Type in Kevin AND Karney and press Enter.

You will then get access to all the author's documents and videos (including a .pdf of this book)

Select the document or video you want, which you can download. Or you can wait until it appears in the viewer which may take some time, if the file is large.

Subject	Comment	Where
00 - The Equation of Time - Æquātiō Diērum	This book in .pdf form. In this document, the table of contents and cross references are all hyper-linked	Zenodo
01 - Eccentricity Effect with Exaggerated Eccentricity	A year-long view of half of the celestial sphere cut through the Ecliptic	
02 - Eccentricity Effect with True Eccentricity		
03 - The Equation of Time (zero eccentricity)	A year-long view of the rotating celestial sphere, looking at the Sun	
04 - The Equation of Time		
07 - Single Tusi Couple	Using Tusi Couples to generate the EoT	
06 - EoT by Tusi Couple		
07 - Goodman Tusi Couple		
08 - EoT from 2xGoodman Tusi and Pulley Adder		
09 - EoT from 2xNormal Tusi and Pulley Adder		
10 - Duscovich EoT mechanism		
11 - Lusby Taylor - Exaggerated	Lusby Taylor's geometrical method of producing the EoT	
12 - Lusby Taylor - True		
13 - Earth EoT & Analemma with Cams	Cams used to generate both EoT & Analemma	
14 - Mars EoT & Analemma with Cams		
15 - Greubel Forsey EoT	A novel watch design to produce the EoT	
16 - Rittenhouse Method - but Exaggerated	Demonstrating Rittenhouse's method of incorporating a 3rd harmonic	
17 - Rittenhouse Method with Indicator		
18 - John Goodman describing his Tusi mechanism		
Bury St Edmunds Dial	https://www.youtube.com/watch?v=_W4nDPnv5RI	YouTube

return to "Table of Contents"

return to "Table of Contents"