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CONTENTS

EDITORIAL

This December issue of the *Bulletin* is, alas, without the customary report on the September Newbury Meeting, another casualty of the Covid-19 saga.

Happily, however, we have no fewer than three articles from first-time *Bulletin* authors and they nicely illustrate the huge range of topics that falls under the very general heading of 'dialling'...

Anne Guest uses diagrams of sundials as the basis of artwork that uses the cyanotype process, familiar to engineers of a certain age who remember blueprints!

Geoff Thurston has written a fascinating article that is a follow-up to the opening article by Ortwin Feustel about a Graeco-Roman sundial in the British Museum. This dial features some declination arcs which do not correspond to the cusps of signs of the zodiac. Geoff convincingly argues that the answer may lie in an ancient Egyptian parapegma.

Our third new author is Pete Caldwell who designed and implemented a vertical declining sundial for the front wall of his house. It is rare to start with a declining sundial and the achievement is all the greater given that most of the project was carried out during lockdown.

Our Patron, Mark Lennox-Boyd, submitted an article about a scaphe dial proposal which particularly intrigued me. Some comments I made turned into two follow-up articles. Dennis Cowan has another Thomas Ross article, this one taking in Aberdeen. Doug Bateman describes his noon mark at the Research Establishment in Farnborough. I must mention the astonishing 're-creation' described by David Brown.

Sadly, we have two more obituaries of noted members: Michael Lowne and David Le Conte.

We wish all our readers a Happy Christmas.

Frank King

A RARE PLANAR GRAECO-ROMAN SUNDIAL

ORTWIN FEUSTEL

The photograph in Fig. 1 shows an unusual sundial in the British Museum.[1] Its inventory number is 1884,0615.1 and the entry on the British Museum website describes it as a Roman sundial incised into Parian marble; the date is given as AD 1–200. There is a biographical note recording that it was donated by the Rev. Greville John Chester who spent much of his time in Egypt where he was a collector of antiquities. There is no record of where it was found but the circumstantial evidence suggests Egypt, which was part of the Roman Empire during the period suggested. Parian marble comes from Greece so the stone would have had to be imported.

In her 1976 book *Greek and Roman Sundials*,[2] Sharon Gibbs assigns this dial catalogue number 5022G and describes it as "a unique planar model whose shadow-receiving surface lies in the equinoctial plane of latitude 32°." She adds that it is "practically impossible to determine its date." She also asserts: "No other equinoctial plane dials are known from antiquity." It is not clear how the latitude estimate was derived but 32° N is just off the north coast of Egypt.

This sundial is one of scores of Graeco-Roman sundials that have been scanned by the *Edition-Topoi* project[3] which dates it as second century BC. The *Edition-Topoi* project provides open-access tools enabling anyone to turn a sundial around in three dimensions and thereby inspect it from any angle; a measuring tool is included so it is possible to measure specific features. A brief User's Guide is given at the end of this article under 'Notes on Using the *Edition-Topoi* Collection.' It is very worthwhile trying out this facility.

Preliminary Analysis

The sundial is made from a rectangular slab of stone. The two broad faces have dials incised into them and there are also significant gnomonic markings on the two narrow side faces. In Fig. 1, the sundial is shown supported by a stand but this stand is not contemporary; it may have been made by the British Museum in the 19th century.[4] The principal visible face in Fig. 1 is shown more clearly in Fig. 2 and the principal hidden face is shown in Fig. 3.

The two faces have the same width but the lower face is not as tall as the upper face. The difference is largely accounted for by the hidden bottom face of the stone not being at right angles to the two principal faces. This also explains why the stone is displayed leaning over in Fig. 1.

Fig. 1. A Graeco-Roman planar sundial, on a 19th century stand, in the British Museum, inventory number 1884,0615.1. Creative Commons licence BY-NC-SA 3.0 DE.

Both faces have obvious hour lines and, seen out of context, a casual observer might describe each face as that of a vertical direct-south-facing sundial. If it is assumed that the holes in the faces are gnomonic centres, then this suggestion has to be rejected because the hour lines do not radiate from the holes. The hour lines in Fig. 2 appear to radiate from a point above the hole and the hour lines in Fig. 3 appear to radiate from a point below the hole. More careful inspection shows that neither set of hour lines converges on *any* point.

Both faces also have three concentric circular arcs and these really are centred on the holes. If these are to be interpreted as constant-declination curves, then the two faces must lie in the equatorial plane (as suggested by Sharon Gibbs). With any other orientation, constant-declination curves are elliptical or hyperbolic. If it is accepted that this is an equatorial sundial, then some kind of rod must have been driven through the hole with each of its two ends serving as a nodus. The path traced by the shadow of the nodus on an equatorial dial follows a circular arc whose radius depends on the nodus height and the solar declination.

With the stone parallel to the plane of the equator and the rod perpendicular to the principal faces of the stone, the rod would be polar-oriented. In consequence, the shadow of the north half of the rod would fall on the north face of the dial

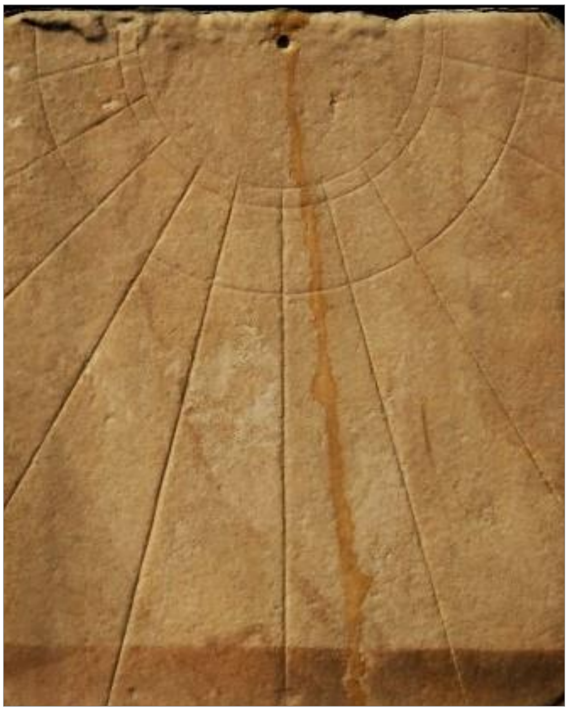

Fig. 2. The upper (north-facing) face.

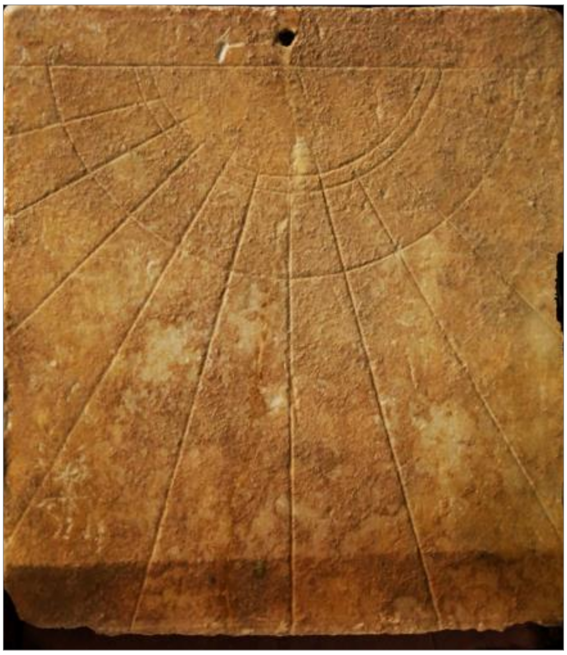

Fig. 3. The lower (south-facing) face (hidden in Fig. 1).

and indicate common hours in the summer half of the year, and the shadow of the south half of the rod would fall on the south face of the dial and indicate common hours in the winter half of the year. If there were associated hour lines they *would* radiate from the holes but the actual hour lines do *not* radiate in this way. It seems likely that the designer was unaware of the properties of a polar-oriented gnomon. The hour lines shown are *nodus-driven unequal-hours hour lines*. The putative rod should be regarded as a nodus support and not as a regular gnomon.

Noting that the rod is inclined to the horizontal, the nodus at the upper end of the rod will be above the horizontal level of the hole and the nodus at the lower end will be below the horizontal level of the hole. Accordingly, one might expect to see a horizon line on each face. In Fig. 3, the south face, the horizon line is clearly visible below the level of the hole; the perpendicular distance from the hole to the line is the horizon offset. There is no obvious equivalent horizon line in Fig. 2, on the north face, where it should be above the level of the hole. It is assumed that the top edge of the north face of the stone served as the horizon line.

As on a regular sundial, constant-declination lines cannot usefully be drawn above the horizon line and, in both Fig. 2 and Fig. 3, the circular arcs are cut off by the respective horizon lines. On the north face, where the horizon line is above the level of the hole, the circular arcs extend for more than semi-circles. On the south face, where the horizon line is below the level of the hole, the circular arcs are less than semi-circles. This is all as expected: the daylight period is longer in summer than in winter.

What Declinations are Associated with the Circular Arcs?

By using the *Edition-Topoi* measuring tool it is easy to estimate the two horizon offsets and the radii of the circular arcs. The radius of each of the six circles varies a millimetre or two about some mean. By inspection, the two mean radii of each pair of corresponding circles differ only slightly and, in the following analysis, it is assumed that the radii of the three circles on the south face match the radii of the three circles on the north face. This approach differs from that taken by Sharon Gibbs.

The principal unknowns which cannot be directly measured are the design latitude, the nodus heights and the solar declinations associated with the circles. To make progress, first note the implications of the assumption that the three north-face radii match the three south-face radii:

1. The nodus height is the same for both the north nodus and the south nodus.
2. The two horizon offsets are equal and opposite.
3. The three declinations associated with the circles on the south face are the negatives of the three declinations associated with the circles on the north face.

On many sundials there are seven constant-declination curves which serve to mark the cusps of the signs of the zodiac. From the vernal equinox to the summer solstice the relevant signs and the associated solar declinations (today) are Aries (0°), Taurus (11.5°), Gemini (20.1°) and Cancer (23.4°, which is the assumed obliquity of the ecliptic). When the declination is zero, the sun is in the plane of an equatorial dial plate, so the shadow of any nodus is at infinity. The First Point in Aries cannot therefore be indicated on either the north face or the south face of the dial, but could the three circles on the north face mark the cusps of Taurus, Gemini and Cancer? Let us investigate…

A Graphical Approach

To gain a full understanding of this sundial, it is useful to make scale drawings of its faces, and one can make considerable progress without any recourse to mathematics. Begin by noting that the simplest example of an equatorial sundial is a horizontal dial at the Earth's north pole.

Imagine a horizontal dial plate with a nodus suspended a short distance above it. Subject to two caveats, the path traced by the shadow of the nodus on a given day will be a circle. The first caveat is that the solar declination is constant during the chosen day and the second is that we are in the summer half of the year (in winter, the north pole is in darkness!).

If the same dial plate is then transported to British latitudes and set up as an equatorial dial, it will not be locally horizontal. The plane will slope upwards to the south at an angle that matches the co-latitude. Fig. 4 shows a cross-section of the arrangement in the plane of the local meridian at the moment the sun is due south.

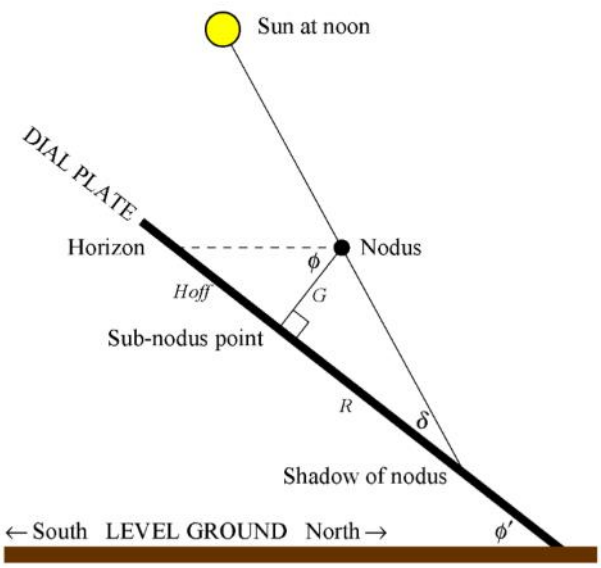

Fig. 4. Cross-section of an equatorial sundial.

The angle ϕ is the local latitude (taken in this example as 52°) and ϕ' is the co-latitude (taken as 38°). To draw this diagram, begin by drawing the line representing the dial plate; this should slope at 38° to the horizontal. Then depict the nodus as a spot a short distance from the north face of the dial plate. The perpendicular distance of the nodus from the dial plate is the nodus height, *G*, and the foot of the perpendicular is the sub-nodus point.

A horizontal line drawn in the plane of the figure from the nodus to the dial plate will intersect the dial plate on the horizon line. The distance of this intersection point from the sub-nodus point is the horizon offset, *Hoff*. Note that the nodus support slopes at an angle ϕ to the horizontal. It is polar-oriented.

Finally, draw the sun in the plane of the figure and draw a sunray from the centre of the sun through the nodus to the dial plate. The point where this line intersects the dial plate is the position of the shadow of the nodus at noon.

The angle that this sunray makes to the dial plate is shown as δ and this matches the angle that a line from the centre of the sun to the centre of the Earth makes to the plane of the equator. This is the solar declination.

If the declination were to stay constant over the course of a day, then the angle that the sunray through the nodus makes to the dial plate would be constant. The path traced by the shadow of the nodus will therefore be a circle centred on the sub-solar point. In Fig. 4, the radius of the circle is shown as *R*.

Adapting the Graphical Approach to the Sundial

Adapting Fig. 4 to show the arrangement of the British Museum sundial presents two difficulties:

- We do not know the latitude, ϕ
- We do not know the nodus height, *G*

Fortunately, we can make progress provided we know:

- The horizon offset, *Hoff*
- The radii, R_i, R_m, R_o, of the innermost, middle and outermost circles
- The associated solar declinations, δ_i, δ_m, δ_o

The linear measurements can be determined using the *Edition-Topoi* measuring tool but great care needs to be exercised. The edges of the stone have suffered wear and tear, and measuring the horizon offset on the north face is quite challenging.

We have assumed that the north and south dials have the same horizon offset and we may further assume that the distance of the hole on the north face from the top edge is the same as the distance of the hole on the south face from the top edge. By inspection, the top face of the stone appears to be perpendicular to both the north and south faces. The assumed horizon offset, *Hoff*, is the weighted average of three measurements.

The three radii, R_i, R_m, R_o, can be obtained by making many measurements of six circles and averaging.

If we provisionally associate the three circles on the north face of the dial with the solar declinations for the cusps of Taurus, Gemini and Cancer, we need to note that the obliquity of the ecliptic, ε, was not its present-day value. It is not clear when the sundial was made, but around 2000 years ago the obliquity of the ecliptic was 23.7° and we shall use that value.

In summary, the values assumed are:

- *Hoff* = 16.3 mm
- R_i = 72.0 mm, R_m = 81.5 mm, R_o = 123.5 mm
- $\delta_i = \varepsilon = 23.7°$, $\delta_m = 20.4°$, $\delta_o = 11.6°$

Note that all three declinations differ slightly from the modern values given earlier.

With Fig. 4 in mind and using these values, proceed as follows:

1. Draw a line representing the dial plate at an arbitrary angle to the horizontal.
2. Mark a dot for the sub-nodus point 16.3 mm from the upper end of the dial plate.
3. Mark three more dots for the shadow positions 72 mm, 81.5 mm and 123.5 mm from the sub-nodus point.
4. Draw a line through the sub-nodus point at right angles to the dial plate; the line can be of arbitrary length.
5. From the innermost shadow point draw a line at 23.7° to the dial plate; this line should be long enough to intersect the perpendicular line drawn at step 4.
6. Repeat step 5 for the middle and outermost shadow points, drawing lines at 20.4° and 11.6° to the dial plate.
7. Mark the three intersection points on the line drawn at step 4. These are three candidate positions for the nodus.
8. Measure the three nodus heights.

Ideally, the three heights should be the same. In reality they are about 31.6 mm, 30.3 mm and 25.4 mm, and *this variability is too large to be ignored*. It is possible that the original designer used erroneous values for the declinations but it seems more likely that the circles do not mark the cusps of the signs of the zodiac.

It is still possible that the innermost circle *does* mark the summer solstice. In 1898, Carolus Manitius[5] described a calendar used by the Ancient Egyptians in which the signs of the zodiac were the months of the year and the summer solstice marked the start of the year. This would justify having a summer solstice arc on a sundial.

Additionally, since Sharon Gibbs published her catalogue in 1976, some other equatorial Graeco-Roman dials have come to light. In particular, a 2015 paper by Klaus Herrman, Maria Sipsi and Karlheinz Schaldach[6] and a 2019 paper by Karlheinz Schaldach, Eduard Shehi and Klaus Hallof[7] refer to equatorial dials which have just the innermost circles on the north and south faces; it is assumed that these circles indicate the solstices.

It is possible that the other four circles mark significant dates in the Egyptian zodiacal calendar year and this topic merits a separate article which is published later in this issue of the *Bulletin*.[8]

We shall therefore proceed on the assumption that the declinations associated with the two innermost circles are ±23.7° but make no assumptions about the other declinations.

In the light of the above, let us remove the two lines drawn at step 6 but leave the line drawn at step 5 which suggested a nodus height of 31.6 mm. Continue with this revised step 6…

6. Mark in the nodus (with nodus height 31.6 mm).
7. Now draw lines from the other shadow points to the nodus.
8. Measure the angles that the new lines make to the dial plate; these are the revised solar declinations for the other two circles; they are about 21.2° and 14.4°.
9. Draw a line from the nodus to the upper end of the dial plate.
10. Measure the angle that this new line makes to the perpendicular line (the nodus support); this is the local latitude, ϕ, and it should be about 27.3°.
11. Through the lower end of the dial plate, draw a line that is parallel to the line from the nodus to the upper end of the dial plate; this represents level ground.
12. Rotate the entire drawing so that this level-ground line is horizontal. The overall appearance should be much as in Fig. 4.

From this exercise we have established the nodus height, *G*, the latitude, ϕ, and we have obtained revised values for the declinations associated with the two larger circles:

- *G* = 31.6 mm
- $\phi = 27.3°$
- $\delta_i = \varepsilon = 23.7°$, $\delta_m = 21.2°$, $\delta_o = 14.4°$

The latitude is less than the 32° suggested by Sharon Gibbs but is more convincingly in Egypt. Candidate locations might be the ancient city of Lycopolis (now Assiut) at 27.2° N or Tayu-djayet (now el Hiba) whose latitude is 28.77° N.

The declination associated with the innermost circle is still the obliquity of the ecliptic and marks the first point in Cancer. The other two declinations have been revised; δ_m is under one degree greater than the declination for the first point in Gemini and could simply reflect a setting-out error, but δ_o is nearly three degrees larger than the declination for the first point in Taurus and this cannot easily be dismissed as an error.

Marking out the Hour Lines

Figs 5 and 6 are working drawings of the north and south faces. Using the *Edition-Topoi* measuring tool one can estimate that both faces are about 290 mm wide but their heights are different. The height of the north face is about 346 mm and the height of the south face is about 319 mm because the face of the bottom of the stone is angled; it is

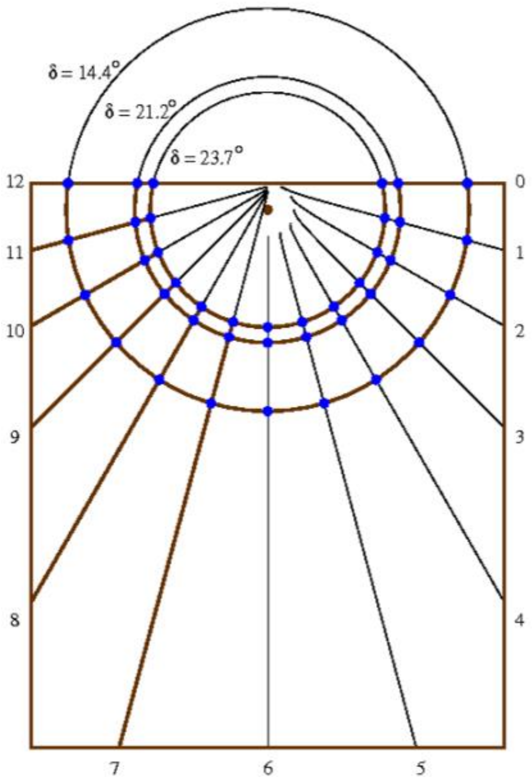

Fig. 5. Working drawing of the north face.

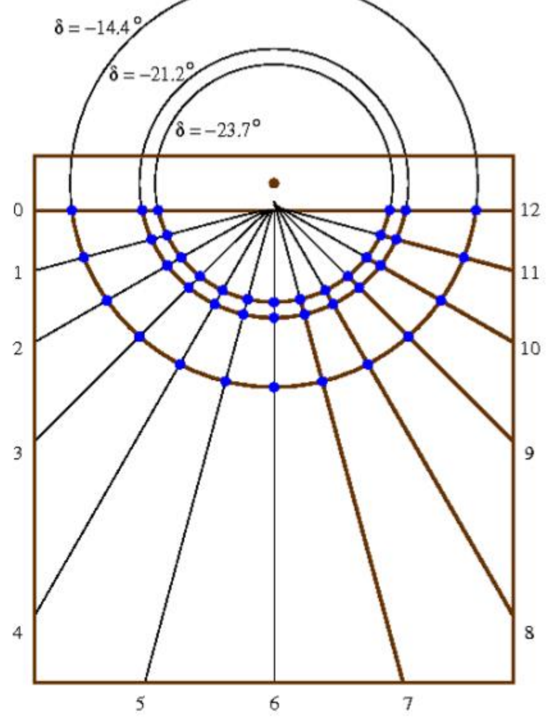

Fig. 6. Working drawing of the south face.

roughly horizontal when the stone as a whole is aligned in an equatorial plane.

The hole is marked in both faces a distance *Hoff* (16.3 mm) down from the centre of the top edge. The horizon line on the south face is drawn a further 16.3 mm down. On each face the three circles are drawn with radii 72.0 mm, 81.5 mm and 123.5 mm. The incised portions of the circles are drawn in brown and the portions above the two horizon lines are drawn in black. These portions are not incised into the stone.

The blue spots mark the daytime unequal-hours hour points on the three circles. On each brown arc there are 13 hour points which mark the hours from sunrise (hour zero) to sunset (hour 12). The spots have to be equally spaced in time, and in the special case of the constant-declination circles on an equatorial dial this equates to equal angular spacing too. By using a protractor, it can be seen that the summer solstice circle on the north face extends for 206°. One twelfth of this is about 17° and the spots should be spaced at 17° intervals.

The three spots associated with each hour line are close to being collinear and the original designer could reasonably have assumed that the hour lines really were straight. In both figures, the brown hour lines for hours 7, 8, 9, 10 and 11 have been drawn as straight lines through the hour points on the innermost and outermost circles. In all cases these brown lines appear to run through the centres of the spots on the middle circles. As on the actual sundial, these lines have been extended outwards to the margins of the stone and, as black lines, they have also been extended inwards to the meridian lines. It is clear that they do not converge on single points. On the actual sundial the hour lines are not extended inwards but in the photographs of the north and south faces in Figs 7 and 8 the hour lines, highlighted in green, have been extended inwards to the meridian lines.

The pattern of convergence on the south face is remarkably close to the pattern of convergence of hour lines 7 to 11 in Fig. 6. The pattern of convergence on the north face is not quite so convincing but it is at least plausible that the original delineation involved dividing each of the six circular arcs into 12 parts and marking the hour points rather as the blue spots have been used in Figs 5 and 6.

The designer was almost certainly familiar with Graeco-Roman dials that were set out on hollow spherical or conical surfaces. Such dials normally incorporate three circular arcs parallel to the plane of the equator. The arcs mark the solstices and the equinoxes; the summer solstice arc extends for more than a semi-circle, the equinoctial arc is a semi-circle and the winter solstice arc extends for less than a semi-circle. Almost certainly the arcs were divided into 12 parts and equivalent hour points joined up to form hour lines. The circular arcs on the British Museum dial are also all parallel to the plane of the equator; the major novelty is that they are in two sets of co-planar triplets.

In Figs 5 and 6, hour lines 1 to 6, drawn as black lines, have been constructed strictly mathematically. Instead of drawing just three circular arcs, imagine drawing a very large number of circular arcs centred on the two holes and mark 13 spots on each. When the spots for a given unequal hour are joined, the result is a curve that is almost straight for most of its length but, when extended inside the summer

Fig. 7. Hour lines on the north face.

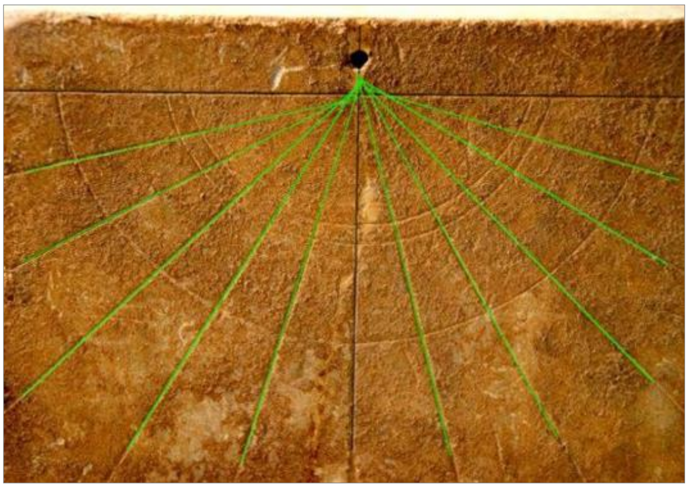

Fig. 8. Hour lines on the south face.

(or winter) solstice arc, it starts to curve very noticeably. In the case of the south face, the lines converge on the point where the horizon line and the meridian line intersect. In the case of the north face, there is no such convergence. Here the lines again curve but then abruptly stop on some limit circle.

This limit circle corresponds to a declination that matches the local co-latitude. Outside the Arctic and Antarctic regions such a declination is not possible but on the Arctic Circle the co-latitude matches the obliquity of the ecliptic and the sun has this declination on the day of the summer solstice when the constant-declination circle is the limit circle. The blue hour spots would be at 30° intervals and an unequal hour would correspond to exactly two common hours. Unequal hours have no meaning for declinations greater than the co-latitude.

The East Face

An oblique view of the east face of the stone can be seen in Fig. 1 and this face is shown in more detail in Fig. 9. A working drawing is shown in Fig. 10. There is a hole for a nodus and a centreline running downwards from the hole to the base of the stone. This centreline is crossed at right-angles by five tick marks.

Viewed with modern eyes, this is a somewhat minimalist vertical direct-east-facing dial and the centreline is the equinoctial line. On the day of an equinox, unequal hours and equal hours are the same, 60 modern minutes, and almost certainly the tick marks indicate the first five hours after sunrise on the day of an equinox. Assuming this to be the case, their positions along the centreline from the hole should be given by:

$$Position = G'.\tan(15.h)$$

Fig. 9. The east face.

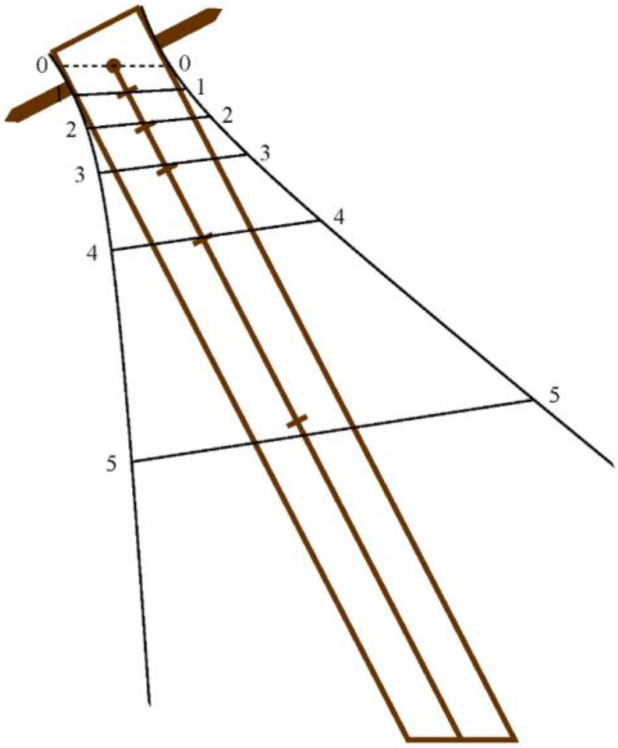

Fig. 10. Working drawing of the east face.

where G' is the nodus height and h is the number of hours since sunrise.

In this, G' is unknown but the distances to the five positions can be measured and each can be divided by tan(15.h) to give a candidate estimate for G'. The five results can be averaged to give an estimate for G'. The same arrangement is found on the west face (where the tick marks indicate the last five hours before sunset), enabling five additional results to be derived so the value of G' can be refined. The best estimate for G' is 47 mm.

If this were a common-hours sundial, the hour lines would be straight and parallel and they would all be at right-angles to the equinoctial line. In Fig. 10, parts of the winter and summer solstice curves have been added in black and mathematically correct unequal-hours hour lines have been marked in, also in black. These are not parallel and are not at right-angles to the equinoctial line. Hour line zero is of course horizontal and, thereafter, the lines gently increase in slope.

The lines do not exactly go through the tick marks but the discrepancies are small. One notes that at hour 3 the position of the tick mark should be G'.tan(15 × 3) and since tan (45) = 1 the distance to the hour 3 tick mark should match the nodus height.

Fig. 10 also shows a speculative profile for the nodus used on the north and south faces.

The west face is close to being the mirror image of the east face.

Summary of Dimensions

Many of the dimensions below differ slightly from the corresponding dimensions given by Gibbs but, for the most part, the differences are small.

Primary dimensions measured using *Edition-Topoi*, horizon offset and three radii:

- $Hoff$ = 16.3 mm
- R_i = 72.0 mm, R_m = 81.5 mm, R_o = 123.5 mm

Assumed obliquity of the ecliptic:

- $\varepsilon = 23.7°$

Derived dimensions, nodus height, latitude and three declinations:

- G = 31.6 mm
- $\phi = 27.28$
- $\delta_i = 23.7°$, $\delta_m = 21.2°$, $\delta_o = 14.4°$

Secondary dimensions estimated using *Edition-Topoi*:

- Height of north face, h_n = 346 mm
- Height of south face, h_s = 319 mm
- Width of stone, w = 290 mm

Dimensions applicable to the east and west faces:

- Thickness of stone: th = 40 mm
- Distance of nodus from the top of the stone, 17 mm
- East face tick marks: 11.90 mm, 26.66 mm, 49.24 mm, 81.09 mm, 170.62 mm
- West face tick marks: 13.28 mm, 28.62 mm, 48.49 mm, 81.64 mm 170.61 mm

Derived dimension:

- Nodus height, G' = 47 mm

Some Essential Mathematics

As might be expected of an equatorial sundial, the mathematical analysis is very simple. From the two right-angled triangles in Fig. 4:

$$\tan(\delta) = G/R$$

$$\tan(\phi) = Hoff/G$$

For the innermost circle, we have $G = R_i.\tan(\delta_i)$. We can measure R_i = 72 mm and assume that δ_i = 23.7. Hence G = 72.tan(23.7) = 31.6 mm.

For the latitude, ϕ, we have $\phi = \arctan(Hoff/G)$. We can measure $Hoff$ = 16.3 mm so ϕ = arctan(16.3/31.6) = 27.3°.

For the middle and outermost circles we note that $\delta_m = \arctan(G/R_m)$ and that $\delta_o = \arctan(G/R_o)$. We can measure R_m = 81.5 mm and R_o = 123.5 mm, and determine δ_m = arctan(31.6/81.5) and δ_o = arctan(31.6/123.5), giving δ_m = 21.2° and δ_o = 14.4°.

For marking the blue spots, consider the innermost circle in Fig. 5 and note that the point where it intersects the horizon line (the top edge of the north face) is the position of the blue spot for hour point zero. Now draw two lines from the sub-nodus point (the brown spot), one to the centre of the horizon line and the other to the blue spot. The lengths of these lines are $Hoff$ and R_i. Call the angle between them k and note that

$$k = \arccos(Hoff/R_i) = \arccos(16.3/72) = 76.9°.$$

From the figure, the total angular extent of the brown arc is 360 – 2 × 76.9 = 206.2°. Dividing this by 12 gives 17.2° as the angular separation of adjacent spots.

Notes on Using the *Edition-Topoi* Collection

1. Key the following URL into your favourite browser: *http://repository.edition-topoi.org/collection/BSDP/object/272*
2. This takes you Dialface ID 272 which is the identifier of the British Museum sundial.
3. Scroll down to 3D Models and click on ObjID262 under the icon.
4. This takes you to the initial view of the 3D model.
5. Try dragging across it in different directions. You can steer it around.
6. Try using the tools on the left. You can change the size and make measurements.
7. By default, the appearance is of a *papier maché* version of the dial but you can use the colour palette to change this.

ACKNOWLEDGEMENTS

I am most grateful for the valuable comments made by BSS member Geoff Thurston and for additional help provided by all the members of the Editorial Team.

REFERENCES

1. The relevant entry on the British Museum website is: *https://www.britishmuseum.org/collection/object/G_1884-0615-1* The full citation is Keil, Maria, Image of Dialface ID 271 / 272 / 273 / 274, London, Inventory Nr. 2546; Reg: 1884,0615.1, 2015, Ancient Sundials, Edition Topoi, DOI: 10.17171/1-1-2945.
2. Sharon L. Gibbs: *Greek and Roman Sundials*, Yale University Press (1976), pp.84, 363-5.
3. The *Edition-Topoi* website is *http://repository.edition-topoi.org* and further details are given in the Notes above.
4. Private communication from Thomas Kiely, Cyprus Curator, Department of Greece and Rome, British Museum.
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READERS' LETTERS (1)

David Young

I was deeply saddened by the passing of David Young who had been my close friend for over 25 years. During that time, we had collaborated on many sundial projects including the Horniman Trail. Some dials were easy whilst others were tricky but David's good sense of humour kept us on an even keel and always made the experiences fun.

One memorable sundial request we had was from the Mudchute Farm Community Charity in East London. On touring the site, we discovered two old cartwheels together with a pole which we decided could be used to make a quirky equatorial dial. Having cobbled these components together and painted on the hour numbers, all that was needed now was a plinth of suitable dimensions. To our sheer disbelief, a discarded cable spool was found on a nearby building site which meant that we could complete our project without having had any outlay.

I have cherished the memory of this sundial project ever since.

John Moir

Can I make a correction to the obituary in the September *Bulletin* for the late David Young who was a work colleague of mine?

He was the Departmental Superintendent of the Department of Physics at Queen Mary College (University of London), and he was a popular and well-respected member of the technical staff.

I in fact left QMC and worked elsewhere, but many years later, when I became interested in sundials, I visited David and his wife at their home in Chingford. They made me very welcome and it was a pleasure to catch up with them.

Julian Greenberg

Pronunciation of 'gnomon'

On a recent socially-distanced visit to my parents, I was updating my father on the progress of the sundial I am making. I was talking about the gnomon, pronouncing it in the normal way ('know-monn'), when he asked why I was not pronouncing it properly. I checked the OED and it is of course pronounced in English as expected – *noʊmɒn*.

However, in Greek it is written γνώμων which does seem to swap the vowels round, sounding much closer to 'nomm-own'. I actually prefer that! But I could not admit that to my father, of course. But can any readers shed any light on this and put an end to our argument?

Pete Caldwell

IN THE FOOTSTEPS OF THOMAS ROSS

Part 33: The Aberdeen City Sundials

DENNIS COWAN

Aberdeen is well known today as the United Kingdom's oil capital, but in 1890 Thomas Ross knew none of this. He would have seen it as a fishing port and eventually as a source of three sundials, possibly including Scotland's oldest. In volume 5 of *The Castellated and Domestic Architecture of Scotland*[1] he mentioned the following examples:

"*The town-house of Aberdeen was erected in 1730, and on the front of it there was a plain metal dial* [Fig. 1] *which was transferred to the new building when the old one was taken down about twenty years ago* [around 1870]. *The gilt gnomon issues from a radiant sun, and is of wrought-iron, ornamented as shown on sketch. Along the top of the dial is the motto UT UMBRA SIC FUGIT VITA. We are indebted for a sketch and photograph of this dial to Mr. John Morgan of Rubislaw House.*"

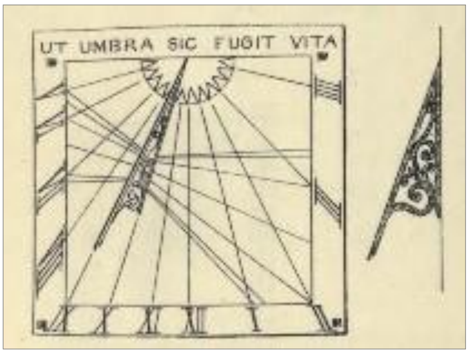

Fig. 1. Ross's sketch of the sundial and gnomon at Aberdeen Town House.

The Municipal Building, as the new building is known today, stands in the centre of Aberdeen at the eastern end of its most famous thoroughfare, Union Street, and has several uses including the Sheriff Court. One part of the building is used as the Tolbooth Museum and it is here that the vertical sundial is situated (Fig. 2).

It is in fine condition and has probably undergone a restoration in recent times. As can be seen in Fig. 3, the

Fig. 2. Location view of the Town House sundial.

Fig. 3. The Town House sundial today. The original fixing holes can be seen.

original fixing holes indicate that it has probably been moved slightly to the left. The dial face is dark with golden furniture and an ornamental gnomon. It has Roman numerals from 5 am to 4 pm with a one-minute scale and it declines around 25 degrees to the east.

There are seven declination lines; the BSS Fixed Dial Register notes that there are no reports of a nodus and there is no obvious one in Ross's sketch, but there clearly is one today as can be seen in Fig. 4. The tip of the gnomon has been broken off at some point and has been repaired. Gatty translates the Latin motto as "*as a shadow so life doth fly*".[2]

Ross notes above that the sundial was originally on the Town House which was demolished and replaced by the current building, where the sundial was resited. I think that it may possibly have been at a lower height on the original building as it is fairly high in its current position and

Fig. 4. Close-up of the Town House gnomon showing the nodus and the repair.

Fig. 6. The two sundials at roof level on Andrew Begg's shop in Upper Kirkgate today.

difficult to read at that height, especially so the seven declination lines.

This sundial may not be the original one though, as the Royal Commission on the Ancient and Historical Monuments of Scotland tell us "*a sundial was provided for the Town House in 1598, and in 1733-4 a payment was made for a fine peuther*[3] *dial and for cutting, calculating, painting and gilding the same*", almost certainly the current dial.[4] The 1598 sundial would have been the earliest dated Scottish one that we know of today. I wonder what became of it?

Nearby in Upper Kirkgate, Andrew Begg's shop has two sundials sitting on corbels at roof level. Ross shows only one of them and comments simply:

"*This dial* [Fig. 5], *for which we are indebted to Mr. Keith, jun., stands on a house in Upper Kirkgate, and occupies a similar position to the last mentioned*." [5]

Fig. 5. Ross's sketch of one of the Upper Kirkgate sundials.

Fig. 7. The left-hand sundial at Upper Kirkgate.

Fig. 8. The south-east face of the right-hand sundial at Upper Kirkgate.

There are two sundials in Upper Kirkgate, one at each side of the building, which today houses a shop on the ground floor (Fig. 6). The presumption is that this is the same building that the sundials were on in 1892, but it is not necessarily so as the roofline is certainly different today.

Both sundials are stone cubes each with two dial faces. The left-hand sundial faces south-east and south-west and is

Fig. 9. The north-east face of the right-hand sundial at Upper Kirkgate.

complete with both gnomons (Fig. 7). Both faces have the date 1694, the south-east face showing 3 am to 2 pm whilst the south-west face shows 10 am to 9 pm. They are both quite accurate.

The right-hand sundial faces both south-east (Fig. 8) and north-east and both faces have the initials WTMG with the date 1694. These details take up much of the north-east face with little left over for the dial furniture (Fig. 9). The south-east dial shows 3 am to 2 pm, has its gnomon and is accurate. The north-east dial meanwhile has lost its gnomon and shows 3 am to 9 am. The numerals on all four dial faces are Arabic.

However, we have seen that the sundials are both dated 1694 but the building is later than that, probably late 18th century, so if the date is authentic they must have been mounted elsewhere before they were moved here. Unfortunately I have been unable to find any reference to an earlier location for them.

Moving a mile or so northwards we come to Old Aberdeen and the University, founded in 1494, and it is here at King's College Chapel that we see our next sundial. According to Ross:

Fig. 11. The King's College Chapel sundial seen at roof level at the top of the buttress.

"*There is a dial here about 3 feet square, formed of a metal plate set on the face of one of the buttresses of the chapel at a height of about 25 feet from the ground. It appears to be an original part of the structure, which was founded in 1494, and in that case it is probably the earliest example of a sundial known in Scotland.*"

Evidently Ross never managed to see these three Aberdonian sundials as sketches of the first two here mentioned were supplied by others and he did not provide a sketch of this sundial at the chapel.

Like the Town House sundial, it is dark with golden furniture but its ornamental gnomon has lost its upper part (Fig. 10). There are Arabic numerals and hour, half hour and quarter hour lines. It is in poor condition and is situated high on one of the buttresses at roof level (Fig. 11), but I can't agree with Ross here as I am convinced that the dial is not as early as he suggests. If it were so, it would be the oldest dial in Scotland by some 129 years. It just does not have that feel and I fear that it was probably added later.

Fig. 10. Close-up of the King's College Chapel sundial today.

So there we have it. Two would-be contenders for Scotland's oldest sundial, but one lost and the other unlikely. Pity.

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2. Mrs Alfred Gatty: *Book of Sun-Dials* - George Bell & Sons, London (1900).
3. Peuther = old Scots spelling of pewter.
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5. The last sundial referred to here by Ross was at East Calder in Midlothian, which I have yet to find. It may no longer exist.

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WILD CLOCKS

ANNE GUEST

I am a visual artist and my artwork is inspired by the natural world. I am particularly interested in the relationship between plants, insects and birds. I use the cyanotype process to make my work and I have recently started to incorporate diagrams of sundials into my artwork (Fig. 1).

Fig. 1. 'Time after time'. 28 × 28 cm. Cyanotype with gold leaf. Anne Guest, 2020.

Background

Migratory birds serve key functions in the interconnected systems that keep nature healthy, including pollination and seed dispersal of crops for human and livestock consumption and control of the numbers of insects.

Birds use the sun as a navigation aid like a compass to migrate across vast distances which allows them to make progress towards a goal without having to reassess their position constantly. In a local environment birds use solar clues such as shadows cast by hedges for orientation which shows both direction of the sun and time of day – like a sundial.

Flowers respond to the sun in different ways. Many flowers will follow the track of the sun throughout the day much like the shadow from the gnomon showing the direction of the sun on the face of a sundial. Others open and close to match the intensity of the sun. Some flowers face east in order to warm up more rapidly in the early morning which results in a significant increase in the number of pollinator visits.

I use the cyanotype process to make artwork in which photosensitive chemicals are exposed to the ultraviolet rays of the sun to create an image. To incorporate sundials into my work about plants, insects and birds follows as a logical extension.

The Cyanotype Process

The cyanotype is a photographic printing technique that does not use a camera. It was introduced in 1842 by John Herschel.[1] A year later Anna Atkins created a series of books using the cyanotype process to document plants.[2]

The cyanotype process was once extensively used for engineering drawings and is the origin of the term 'blueprint'. Technical blueprints were used by engineers and architects for reproducing technical and specification drawings rapidly and accurately. The process is characterised by white lines on a blue background, a negative of the original. The blueprint was eventually superseded by cheaper and simpler copying processes.

The cyanotype image is made by applying ultraviolet-sensitive chemicals to paper. The paper is exposed to sunlight to develop and then washed in running water to fix the image.

There are various formulae for creating the cyanotype solution but they all involve mixing various quantities of two chemicals: ammonium citrate and potassium ferricyanide. These are in powder form and are prepared by combining with distilled water. I currently use a variation of the classic cyanotype formula. These solutions are stored away from light and mixed together in equal quantities when ready to use. Once mixed, they again have to be stored in the dark.

I apply the combined chemicals to 300 gsm alkaline-free watercolour paper evenly with a sponge and allow it to dry for an hour or so in a darkened room. The paper has to be strong enough to be able to withstand thorough washing in water following exposure. I find the best results are when the coated paper is used within 24–48 hours.

To create the imagery I use a combination of digitally prepared and printed transparencies of vintage sundial

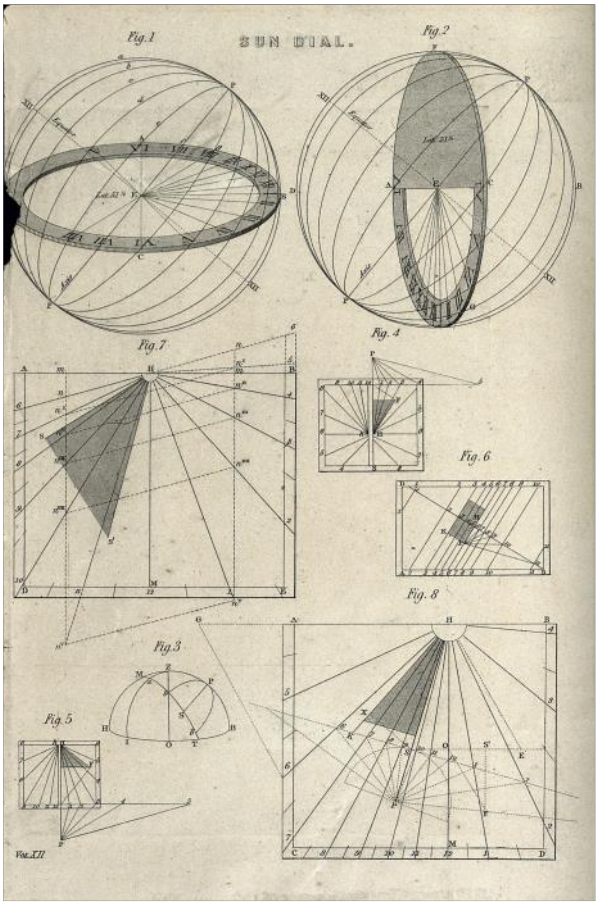

Fig. 2. Diagrams for setting-out a sundial. Engraving c.1861. Credit: Wellcome Collection. Attribution 4.0 International (CC BY 4.0).

diagrams, birds and insects. I then place this transparency onto the prepared paper and add real flowers or plants. I place this on a flat surface outside in the sun and put a piece of glass on top to hold everything firmly in place. Anything that stops the light reaching the sensitised paper will appear white when developed and everything else will be blue.

It is important to me that I use properly delineated diagrams of sundials and I select them from royalty-free sources via the Internet; an example is shown in Fig. 2.[3]

The sun provides a convenient source of ultraviolet light with parallel rays, and exposure times vary according to the intensity and orientation of the sun, amount of cloud and ambient temperature. Exposure times can vary from 30 minutes upwards. The time can be calculated scientifically using a step wedge test but you soon get a good idea of timings with practice using a particular combination of chemical formula and paper.

Once it has been in the sun for the required amount of time the image is developed by holding the paper under running water for at least 6 minutes. It is a great feeling to see the image appear. It takes up to 24 hours for the paper to dry completely and only then can I add the gold leaf detail (Figs 3 and 4).

I apply the gold leaf detail by hand once the cyanotype image has dried completely. This is a delicate process which involves applying a special glue which dries until tacky – approximately 10 minutes – and then I apply the gold leaf, rubbing away any excess. I then apply a clear shellac varnish to prevent the gold metal tarnishing. I use the gold leaf to represent the sun; in some pictures I create a disc (Fig. 3) and in others I make a circular dotted line (Fig. 4). I also sometimes apply the gold leaf to highlight specific elements of the picture, for example the bees in Fig. 5.

I chose to make these cyanotypes at this size and shape in order to reflect the more familiar circular horizontal sundial.

Fig. 3. 'Time matters'. 28 × 28 cm. Cyanotype with gold leaf. Anne Guest, 2020.

Fig. 4. 'Time and again'. 28 × 28 cm. Cyanotype with gold leaf. Anne Guest, 2020.

Fig. 5. 'Time memory'. 28 × 28 cm. Cyanotype with gold leaf. Anne Guest, 2020.

It is a fascinating process which has an element of serendipity in that I never quite know what the image will look like. Combining images of sundials with plants, insects and birds highlights how the rhythms of life are reliant on the cycle of the sun, hence the title of this series of works: Wild Clocks.

And to use the sun to create an image of a sundial adds a sense of completeness to these images.

I use *Cyanotype – The Blueprint in Contemporary Practice* as my go-to reference book and I thoroughly recommend it if you are interested in learning anything further about this process.[4]

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Anne Guest is an artist who lives and works in Worcestershire, UK. More about Anne and her art and how to purchase or commission artwork can be found at *www.anneguest.co.uk*.

Postcard Potpourri 53 Hampton Court Palace

Peter Ransom

This is a postcard of the original dial on the south terrace at Hampton Court Palace. The dial is now in store (SRN 2119) with, according to the Fixed Dial Register, an undelineated replica (SRN 6163) on show. This original dial is a double horizontal dial, made by Thomas Tompion in 1690.

The postcard is unused and on the reverse it mentions that it is photogravure by The Rembrandt Intaglio Printing Co., Ltd., London. This company was formed in 1895 by Storey Brothers of Lancaster and moved to London in 1926. The company was bought by the Sun Engraving Co. in 1932 and renamed Rembrandt Photogravure Ltd., so this dates the card to between 1926 and 1932.

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A SCAPHE DIAL FOR HOLKER HALL
…and Calculating it with Corrections

MARK LENNOX-BOYD

I designed the Holker dial,[1] a scaphe (see Fig. 1), in 1992. I have designed two more scaphes since then and as far as I am aware no other member of the BSS has made one. I have for some time thought it would be a good idea to make a pair of large ones for some suitable place like Holker, about 1000 mm in diameter, each with corrections for the equation of time and displacement from the Greenwich meridian, one of them for days longer and the other for days shorter, as I explain later in this article. Perhaps not surprisingly no one has commissioned or asked me to make these objects and I decided during lockdown to make a model demonstrating the idea. I originally described the Holker dial as the 'Holker Bowl' for I did not then know the term 'scaphe', but I had seen examples of them in photographs of portable dials like those beautiful ivory objects from Nuremberg. Ortwin Feustel has written about them in an article cited below.[2] The portable scaphes gave me the idea for Holker. In the 1980s my wife had given me for Christmas a Sinclair ZX Spectrum (at a cost of £99.99 I think) and I discovered BASIC and the opportunity it provided to design sundials. It was with this machine that my designs of that time were made.

Fig. 1. Members checking the accuracy of the Holker dial during the Grange-over-Sands BSS Conference in May 2009. Photo: John Davis.

Fig. 2. The Holker dial model. As I have explained this was made in 1992 and is a little worn. I have glued a new protractor on the western rim. This was to enable me to check the original reasoning and calculation when I prepared this article.

First, I must describe the calculations for the Holker dial. It is not possible to design a scaphe on a plan. The only way is to make a model as in Fig. 2 and provide calculations for setting it out. The model was made from glued blocks of Accoya® and turned by a local craftsman Luke Tatham of Lancaster Woodcraft. Note that the bowl is formed from a segment of a sphere. I chose the dimensions by eye. The radius of the sphere from which the model derives is 250 mm and the depth of the segment 50 mm. To my eye it looks right. The actual Holker dial is of Burlington slate 1800 mm in diameter, scaled up from the model so that the depth of the segment is one fifth of the radius of the sphere. The obvious solution to the calculation was to provide the distance from the centre of the bowl along its curved surface and the horizontal angle, the azimuth towards various intersection points which would be joined with spline curves. An example from the model of the Holker dial will make this clearer (see Fig. 2). Consider the intersection on the winter solstice curve with the 10 am or 2 pm hour curve. This point is calculated as DIST = 12.85 cm measured along the curve with a flexible rule from the centre of the nodus base, at an ANGL = 27.61°, the azimuth shown by the protractor drawn round the bowl. I have a flexible aluminium ruler 30 cm long purchased on the Internet, and I made an accurate paper protractor of a dimension to fit around the circumference of the bowl.

With one end of the ruler on the centre of the nodus base the angle can be measured, and a short line drawn with the ruler at about the position of an intersection; the distance is then measured with the ruler and the point of the intersection is determined. When the engravers at Burlington slate did their work, they had the model and calculations to its scale for an explanation, and the same calculations to the scale of the Burlington slate they worked on.

It occurred to me that it might be helpful if I explained the computer programs that I have used for the scaphes and place them on record in the *Bulletin*. I am now occasionally using QB64 in a personal computer. I am most certainly not a skilled programmer or mathematician and the program really is very simple provided you recall basic trigonometry, but of course my program requires an understanding of the most simple elements of QBasic. It is very easy to learn and there are plenty of tutorials on the Web. When I started playing with my Spectrum I devised and proved what I call here the 'y' formula.

Look at Fig. 3 which represents a projection of the tip of the nodus (N) onto a plane dial and a scaphe at a declination d from the equinox. You will see that 'y' is the distance from the point where the gnomon slant intersects with the base plane to any declination curve point on the plane dial, given hour line angle (X) and declination (d). The 'y' formula was calculated by me. It would no doubt be possible to make calculations using the formulae in the *BSS Sundial Glossary*,[3] edited so well by John Davis, and Frank King has provided me with a much simpler and more elegant proof,[4] but I use my 'y' formula which I proved nearly 30 years ago and with which I am so familiar. In the formulae and programs that follow I have used the preferred symbols for *N, X* and *A* but δ becomes d and ϕ becomes f, and I have introduced other symbols. I have not given proofs for the various formulae. They look involved but are fairly straightforward to prove. I have used the values for the EoT and declination provided by Gianni Ferrari as set out in the *Glossary*.[5] A program must of course be composed into QB64 and then run. It will provide values for ANGL and DIST for different values of hour angle. Most readers who do programming of this kind will use Excel, but if anyone is keen to see the actual QB64 programs I used I would be delighted to email copies of them to any interested person, with some other comments.

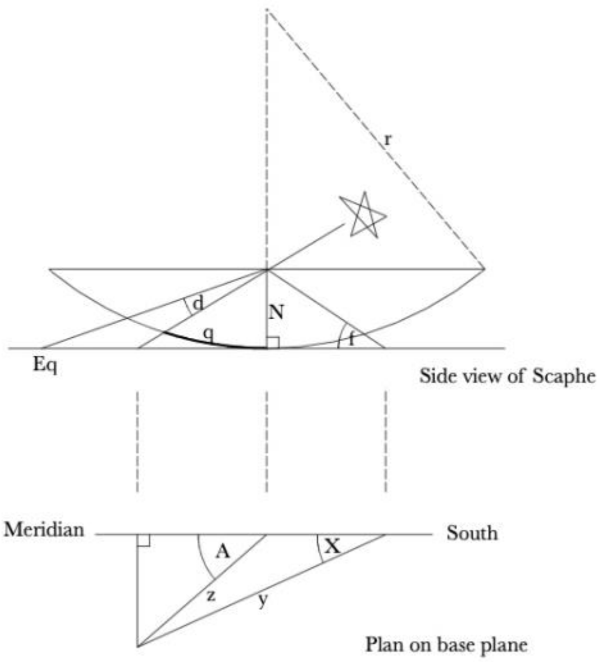

Fig. 3. A sketch for the geometry. A is the azimuth and q (in bold) is the distance along the curve from the nodus base point.

First, the basic formula:

$$X = \arctan\{\sin(f).\tan(h)\}$$

Then:

$$y = \frac{N}{\sin(f).\{\tan(d).\sqrt{1-\cos^2(f).\cos^2(X)}+\cos(f).\cos(X)\}}$$

Which leads to:

$$A = \arctan\left\{\frac{y.\sin(X)}{y.\cos(X)-\frac{N}{\tan(f)}}\right\}$$

And then:

$$z = \sqrt{y^2.\sin^2(X)+\left\{y.\cos(X)-\frac{N}{\tan(f)}\right\}^2}$$

And then:

$$q = \frac{\left\{\arctan\left(\frac{z}{N}\right)-\arcsin\left\{\frac{(r-N)}{r}.\sin\left[180-\arctan\left(\frac{z}{N}\right)\right]\right\}\right\}}{360}.2.\pi.r$$

But this cannot be used because QB64 does not accept arcsin, and so mindful that:

$$\tan(j) = \frac{\sin(j)}{\sqrt{1-\sin^2(j)}}$$

We must insert a new function j:

$$j = \left\{\frac{(r-N)}{R}.\sin\left[180-\arctan\left(\frac{z}{N}\right)\right]\right\}$$

And the final formula:

$$q = \frac{\left\{\arctan\left(\frac{z}{N}\right)-\arctan\left[\frac{(j)}{\sqrt{1-(j)^2}}\right]\right\}.2.\pi.r}{360}$$

When the program I composed is run it will give some of the values required, and also produce garbage when the program seeks to calculate values for points beyond the surface of the bowl, and you also need formulae which will provide the azimuths at sunrise and sunset of the time and declination curves. For the time curves you can develop a simple program to give A values in terms of h, namely:

$$A = \arctan\{\sin(f).\tan(h)\}$$

And for A in terms of d you must use:

$$A = \arccos\left\{\frac{\sin(d)}{\cos(f)}\right\}$$

Both these expressions derive from the formulae in the equations section of the *Glossary*, when the altitude a = 0.

Note that I used seven values for declination corresponding to the divisions of the zodiac. These seven values calculated points for making the time curves which were therefore spline curves drawn by eye through the seven points.

If you have followed up to this far it will be clear that calculating a scaphe is not very difficult, and it is scarcely much harder to calculate a scaphe incorporating corrections for the EoT and displacement from Greenwich. Of course, two dials are needed, and an example of a model dial for days between the summer and winter solstices is shown in Fig. 4. So, this model is for the second half of the year only, because of course the corrections for the first half of the year between the winter and summer solstices are different. When the declination is 23.44° at the summer solstice the correction for the EoT and displacement combined is 13.77 minutes, calculated from values for EoT in the *Glossary* combined with the value for the displacement from Greenwich, converted into hours. h is thereby adjusted with this correction. The same reasoning applies to other values for time adjusted according to the EoT with displacement at the different declination points. All these lines 14 to 22 contain nine values for declination so that more accurate spline curves can be drawn when the points have been plotted on the scaphe.

Another program, not illustrated, must be composed with appropriate values for EoT and displacement combined for the other half year.

One qualification must be made. It is not possible to calculate the azimuths at times of sunrise and sunset because the formulae mentioned earlier work for apparent solar values of time and not clock time. However, there is an easy way of estimating these values reasonably accurately by means of the wonderful program ShadowsPro.[6] Draw two vertical sundials, one east- and one west-facing with added value of longitude for Holker of 2° 59′ 00″ W. Amend them to draw the analemmas for the June to December half curve and the azimuths for every degree, including the correction for the EoT and displacement. You can then from a print judge with reasonable accuracy the azimuths of the analemma and solstice curves at sunrise and sunset.

Fig. 4. The Holker dial model with corrections, just made. It shows corrections for the equation of time and displacement from Greenwich, and so reads GMT. Note that the equilateral triangle surrounding the astronomical sun symbol points down to indicate the dial is read only when the days are getting shorter.

It is easy to get the positive and negative signs wrong in any calculation and ShadowsPro can provide a visual check to the designs. I drew a plane dial with the right coordinates and inclined about 15° so that the winter solstice and horizon were visible. Amended with the analemma half curves and corrections and compared with a printout of an unamended design I was able to judge that the analemma curves looked right to my eye.

I expect I shall continue to dream that one day someone will commission me to design two scaphes for a suitable public location. In my mind's eye I see them made in slate and standing on two truncated limestone pyramids, perhaps flanking a great lawn leading to a fine landscape beyond: see Fig. 5. I pity any skilled engraver entrusted with the task of making them. The solution must be to improve the programming and use a computer-controlled 3D engraver to do the work. I hope that Frank King will have something to say about 3D engraving.[7]

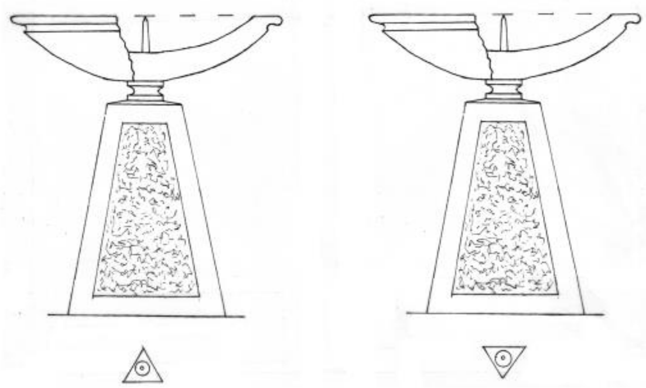

Fig. 5. A sketch to illustrate my idea. The image on the left has a triangle pointing up to indicate that the dial is read only when the days are getting longer.

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THE SIGNIFICANCE OF THE DECLINATION ARCS ON A BRITISH MUSEUM SUNDIAL

GEOFFREY THURSTON

The British Museum sundial numbered 1884,0615.1 has been described and analysed by Ortwin Feustel earlier in this journal.[1] The dial faces are inscribed with three arcs centred on the foot of the perpendicular gnomon with radii of 72, 81.5 and 123.5 mm. The inner arc is also the inner boundary of the hour lines and may reasonably be assumed to represent a solstitial declination. Based on this hypothesis and the obliquity of the ecliptic in the period 500 BC to 1 BC, the arcs must represent declinations of 23.70°, 21.20° and 14.35°, but what is the significance of the last two declinations that justified their inclusion on the dial faces? What follows is an attempt to answer this question.

The Egyptian civil calendar was based on a year of twelve months of thirty days plus an additional five days so that each year lasted exactly 365 days. The tropical year lasts about 365¼ days so the Egyptian year began one day early every four years and the months retrogressed through the seasons in a cycle of 1460 years[2] so that any fixed dates such as religious festivals would move through all the seasons with the passage of the centuries.

Consequently, the Egyptian calendar was of limited use to farmers whose livelihoods depended on anticipating seasonal changes in the weather. What the farmer needed to know was the progress of the sun around the ecliptic (the apparent orbit of 365¼ days against the background of zodiacal stars) since this determines the seasons of the year. Unfortunately, we cannot directly observe the sun's location in the zodiac because the adjacent stars are all invisible in the glare of the sun. However, there are other indications that can be used to track the sun's progress.

Consider those bright stars which lie just to the east of the sun on any given day. They will initially appear when the sky darkens after sunset but after a few days they will no longer be visible as the sun draws nearer to them in its eastward progress. They will then remain unobservable until the sun has moved sufficiently far past them that they become visible when they rise just before sunrise as illustrated in Fig. 1. These disappearances and reappearances of stars and asterisms were used to track the progress of the sun and thereby the seasons of the tropical year. Instead of an unreliable calendar, the Egyptians relied upon observations of the heavens to track the progress of the sun along its ecliptic path.

We cannot calculate with confidence the exact historical moments when stars set or appeared just before sunrise because these events depend upon many unknowns including the location of the observer, their visual acuity, the clarity of the atmosphere, the brightness of the star and its proximity to the ecliptic. However, these events were regularly observed and recorded and, fortunately, there are extant records of these observations known as parapegmata and one of the most useful was compiled by Geminos in the first century AD from earlier observations by Eudoxos, Euktemon and others. Fig. 2 is a fragment from a German translation[3] of the original Greek.

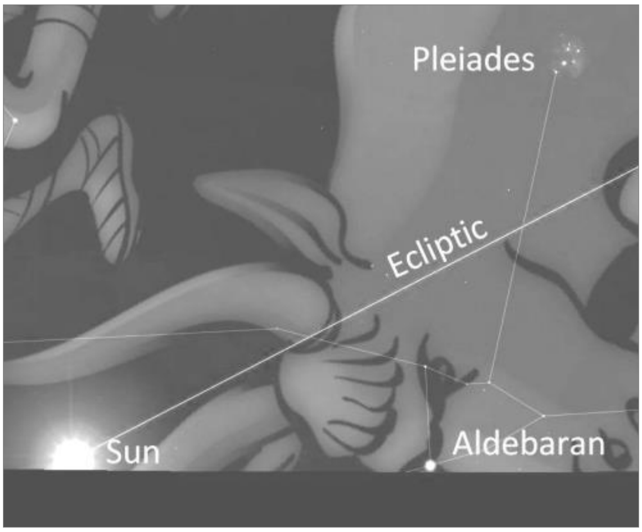

Fig. 1. The Pleiades risen before sunrise (generated using Stellarium software).

This tells us that the parapegma records months of the zodiac and weather indications beginning with the summer solstice. The first month is Cancer (the sun passes through Cancer in 31 days) and on the first day "*According to Kallippos, Cancer begins to rise: summer solstice, and a*

KALENDER.[34])

Zodiakalmonate und Witterungsanzeichen.

(Beginn mit der Sommerwende.)

I.

Krebs (31 Tage).

1. Anfang des Aufganges des Krebses nach Kallippos: Sommerwende, Witterungsanzeichen.
9. Südwind nach Eudoxos.
11. Vollständiger Frühaufgang des Orion nach Eudoxos.
13. Vollständiger (Früh)aufgang des Orion nach Euktemon.
16. Anfang des Frühunterganges der Krone nach Dositheos.

Fig. 2. A fragment from a German translation of the Geminos parapegma.

sign of the weather." The complete parapegma consists of 365 days divided between 12 months although the days without an observation are omitted. Table 1 indicates the length of each zodiacal month.

Cancer	31	Leo	31	Virgo	30
Libra	30	Scorpio	30	Sagittarius	29
Capricorn	29	Aquarius	30	Pisces	30
Aries	31	Taurus	32	Gemini	32

Table 1. The durations of zodiacal months in days.

This parapegma was about a quarter of a day shorter than the tropical year and, if used over an extended period, would have drifted from true seasons by one day every four years. However, it was recalibrated with every new observation and thus the shortfall never became apparent. With the aid of this parapegma, we can establish the position of the sun corresponding to each of the listed significant events throughout the tropical year. The dates of the risings and settings will slowly change because of the precession of the equinoxes but only at a rate of about one day every seventy-one years. Thus, the date of the observations is not critical within several centuries, but we shall use 200 BC for our calculations. The solar declination for each day of the year beginning with the summer solstice of 201 BC was established using the NASA JPL Horizons[4] software and was tabulated against the parapegma. Table 2 (opposite) is an extract from the annotated parapegma showing declinations around ±21.20°, ±14.35° and also the solstices and equinoxes as a check on the calculations. There is obviously some ambiguity because observation of the sun's declination cannot, except at the solstices, determine a unique date. However, this ambiguity is easily resolved by anyone who has been following the ascent and descent of the sun through the year.

The entries in bold type show a possible correlation between the declination arcs and events recorded in the parapegma. In particular:

- Declination +21.17° occurs 1 day after Sirius rising.
- Declination +14.20° occurs 3 days before Pleiades rising.
- Declination -14.44° occurs mid-way between two dates for Pleiades setting.
- Declination -21.16° occurs 3 days before Sirius sets.

We must now consider whether these events were significant enough to justify their inclusion on the dial.

Egyptian agriculture was heavily dependent on the River Nile to the extent that the year was customarily divided into three seasons which were known as inundation, growth and harvest.[5]

The pre-dawn rising of Sirius was believed to be the source of the scorching heat of midsummer[6] and an indicator of the imminent annual inundation of the Nile[7] when harvesting had to be complete.

Pleiades rising in late spring signified the time for reaping winter wheat while Pleiades setting just before sunrise signified the time to plough and plant seed.[8] This agricultural cycle had been recognised over the centuries and in c. 650 BC the poet Hesiod wrote:[9]

When the Pleïades, of Atlas born,
Before the sun's arise illume the morn,
Apply the sickle to the ripen'd corn;
And when, attendant on the sun's decline,
They in the ev'ning æther only shine,
Then is the season to begin to plow,
To yoke the oxen, and prepare to sow:

The author has been unable to discover any particular significance of the setting of Sirius for the ancient Egyptians.

It, therefore, appears that three of the declination arcs inscribed on the dial viz. +21.20°, +14.35° and -14.35° correspond to agriculturally significant events in the Egyptian year. Unfortunately, the middle arc on the winter face remains an enigma and might have been inscribed only to match the summer face.

What would be the advantage of including the declination arcs on a dial face? Although the parapegma observations allowed a fairly precise determination of the progress of the tropical year, they were cumbersome to implement. They required a learned observer with clear skies, with access to unobstructed east and west horizons and who was prepared to leave his bed an hour before dawn each day. If the equivalent information were displayed on a sundial, then it could be read at any time of the sunlit day by an untutored farmer and would allow him to anticipate the critical events in his year. Perhaps this explains the inclusion of these declination arcs on the dial.

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Day of the year	Zodiacal month	Day of the month	Solar declination	Parapegma
1	Cancer	1	**23.72**	Cancer begins to rise: **summer solstice (Kallippos)**
25	Cancer	25	21.69	**Sirius rises in the morning (Meton)**
26	Cancer	26	21.52	
27	Cancer	27	21.35	**Sirius rises in the morning (Euktemon and Eudoxos)**
28	Cancer	28	**21.17**	Aquila sets early; a storm occurs at sea (Euktemon)
53	Leo	22	14.86	Lyra sets in the morning (Eudoxos)
54	Leo	23	14.54	
55	Leo	24	**14.22**	
93	Libra	1	**0.02**	**Autumnal equinox (Euktemon and Kallippos)**
126	Scorpio	4	-12.73	**Pleiades set at sunrise (Demokritos)**
127	Scorpio	5	-13.08	Arcturus sets in the evening (Euktemon)
128	Scorpio	6	-13.43	
129	Scorpio	7	-13.77	
130	Scorpio	8	-14.11	Arcturus sets in the evening (Eudoxos)
131	Scorpio	9	**-14.44**	The head of Taurus sets (Kallippos)
132	Scorpio	10	-14.77	Lyra rises (Euktemon)
133	Scorpio	11	-15.10	
134	Scorpio	12	-15.42	Orion rises at sunset (Eudoxos)
135	Scorpio	13	-15.73	Lyra rises at dawn (Demokritos)
136	Scorpio	14	-16.05	Rainy weather (Eudoxos)
137	Scorpio	15	-16.35	**Pleiades set (Euktemon)**
156	Sagittarius	4	**-21.16**	
157	Sagittarius	5	-21.35	
158	Sagittarius	6	-21.53	
159	Sagittarius	7	-21.71	**Sirius sets (Euktemon)**; Sagittarius begins to rise (Kallippos)
182	Capricorn	1	**-23.72**	**Winter solstice (Euktemon)**
207	Capricorn	26	**-21.25**	
208	Capricorn	27	-21.06	Dolphin sets in evening (Euktemon); Cancer setting at an end (Kallippos)
233	Aquarius	23	**-14.43**	
234	Aquarius	24	-14.10	
235	Aquarius	25	-13.78	Pegasus sets in the evening (Euktemon)
271	Aries	1	0.38	**Spring equinox (Kallippos & Euktemon)**
272	Aries	2	**0.02**	
310	Taurus	9	13.89	Capella rises (Eudoxos)
311	Taurus	10	**14.20**	
312	Taurus	11	14.51	Scorpio begins to set in the morning (Eudoxos)
313	Taurus	12	14.82	
314	Taurus	13	15.13	**Pleiades rise; beginning of summer (Euktemon)**; Head of Taurus rises (Kallippos)
338	Gemini	5	21.06	Hyades rise (Eudoxos)
339	Gemini	6	21.24	
340	Gemini	7	**21.42**	Aquila rises in the evening (Eudoxos)

Table 2. Extracts from Geminos's Parapegma for specific solar declinations in 200 BC.

THE NOON SUNDIAL AT THE RESEARCH ESTABLISHMENT, FARNBOROUGH

DOUGLAS BATEMAN

This is a dial, SRN 3520, that I designed in 1996 to incorporate a noon line and an analemma. For most of year it will also tell the date. It is quite large – 1.8 metres tall – and is designed to be viewed from *inside a building*, where it was installed in the glass curtain wall of a new development, now part of a science park. Although I have described the dial at two BSS conferences and in an official report,[1] and it was seen by a group of our members in 1999,[2] it is high time I gave a summary in the *Bulletin*.

The idea to design the dial arose in two ways. The first was my background interest in noon dials, as per Greenwich and textbooks, and the second was the construction of new buildings at my place of work. Most of my career was spent in one of Europe's premier aviation research establishments, the Royal Aircraft Establishment, Farnborough. Following a national trend in the 1990s, it was being prepared for privatisation. The flying was moved to Boscombe Down, and after some name changes the RAE became the Defence Evaluation and Research Agency, and finally after privatisation in 2001 became QinetiQ. Further evolution has turned the area into a science park.

A major change was the sale of the airfield and the old site close to Farnborough, the historic centre of aviation research. Much rebuilding has taken place, although retaining some listed wind tunnels. The capital was to be invested in new buildings among pine trees to the north-west of the remaining site. The area houses the pressurised 5 metre wind tunnel and was the location of the former Concorde test building. The architects and the building programme managers were anxious to promote the new site, particularly because it meant that staff would be required to move from their former offices with pot plants, bookcases and cupboards to open-plan offices. A new and alien concept for many! To encourage the move, the architects provided a model of the site showing the new buildings together with displays of other material, such as the choice of honey-coloured bricks and artists' impressions. The proposed site layout is shown in Fig. 1.

From the plans I discovered that the main administrative building (at the start of the arrow in Fig. 1) was to have a two-floor-level glass curtain wall, and that the roof of the nearest building would not obstruct the sun in winter. As usual with such organisations and new builds, suggestions were invited. On 25 April 1994 I put forward a proposal

Fig. 1. The new Farnborough research site. The white building is the 5 metre wind tunnel. The direction of south is over the end of the runway.

with quite a bit of detail and rounding off with a bit of puff… "I commend the scheme because sundials can be non-intrusive, artistic, functional and informative, and befitting a scientific community." The idea was given a cautious welcome, no doubt helped by the fact that the head of the 'rationalisation programme' who was overseeing the rebuilding, both at Farnborough and at related establishments, was my former boss during my time at the Ministry of Defence in London.

The advice for any sundial project is to make a mock-up, and, having access to materials and other resources, I was able to construct, with a little help, the prototype as in Figs 2 and 3.

Note that the mock-up is on wheels so that the dial could be trundled out into the sun for assessment of the design and for one of the team of architects to give approval. For a time, it was perhaps the world's largest portable dial!

It may be noticed that there is something special about this design. It is intended that the dial be read from *inside* the building, with the shadow and spot of sunlight falling on frosted glass. The frosting was achieved by etching the outside-facing surface of the inner pane of the double glazing with hexafluorosilicic acid. Shot-blasting, through a mask, was then used to apply the detail to the frosted surface. Approval was given, and for the shape of the

Fig. 2. The full-size mock-up with the frame to the same dimensions as the planned window frames.

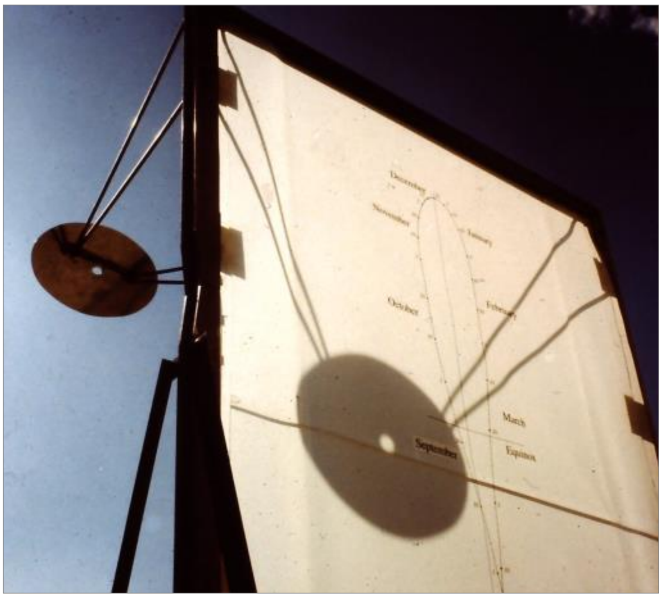


Fig. 3. The mock-up showing the struts for the aperture nodus, necessary because the transom of the window frame would not have been strong enough for a single strut.

analemma I obtained the formula for the equation of time from a booklet from the Greenwich Observatory (the nautical almanac section).[3] Averaging did not seem to make sense over an extended period for the maximum precision (averaged in most sundial books) and the informal advice was to "pick the best year". I picked 1999 (I cannot remember why, although I was due to retire in December 1999) and duly calculated the equation of time to minutes and seconds, and the solar declination. These gave the coordinates for plotting the curve. The computations also allowed for the longitude so as to give the mean-time noon.

The dial is 1.8 metres tall and the glass curtain wall declines 13.5° west of south. The dial is equipped with an aperture nodus, and the nodus height is 1057 mm. Another detail is that the nodus plate is quite large, and at the equinoxes the sun's rays are normal to it. In other words, it faces the sun as in other dials of this type. I also wished for the shadow of the plate and the spot of light to be circular on the dial at the equinoxes. This could be done quite easily if the nodus plate were fixed parallel to the dial's surface, but I wanted the plate to face the sun at the equinoxes, which required the plate to be elliptical and rotated slightly. This was a tricky bit of 3D geometry and a better mathematician than myself, none other than my close friend, Philip Woodward from the sister research establishment in Malvern, did the sums. Again, a mock-up was made at about half-scale, and it worked well. Decorating such a plate could be risky, and I wanted to avoid the 'sun's rays' metalwork common on a traditional aperture nodus. The only treatment was to score the sun-facing surface in a circular manner with coarse emery paper.

Another design feature is a series of dots to give the outline of the analemma. During the months March to September the sun's declination is changing by up to 0.4° per day, which is sufficient to show individual days on a dial of this size. In other words, the dial will show the date. There was a temptation to emphasise the date every seven days, but this makes it harder for the eye to count the days. I therefore chose every five days, in keeping with many scientific instruments – indeed, this dial is a scientific instrument in the general sense. In this case each day is marked by a dot with every fifth dot highlighted; days 10, 20 and 30 of each month are labelled. The final peer review was by a colleague in the Space Department, who was part of the Skynet satellite team. A critical part of a working satellite is the orientation of the solar panels, so the position of the sun, and hence the equation of time, was second nature to him.

Approval was given and the architects did two things. One was to allow the shuttered screening, which would descend in bright sunshine to prevent the glazed area from becoming too hot, to 'bracket' the planned position of the dial, and the other was to place a contract with a specialist glazing firm, T & W Ide, Glasshouse Fields, East London. Figs 4–8 show the progress of the construction of the etched and shot-blasted analemma. The gilding was suggested by the glass experts, even though when looking at an outline against the bright surface, any colour sense is lost in the strong contrast. They said: "what about the outside and its appearance on a rainy day?". They were quite right, and the gilding does indeed lift the appearance of the dial.

Whilst the final artwork and glasswork was done elsewhere, I could have used in-house drawing and workshop facilities, with all the bureaucracy that that would have entailed, but it was quicker for me to make the nodus

Fig. 4. Initial layout. Photo courtesy of T & W Ide.

Figs 6–8. Applying the artwork (note the reversed lettering); gilding; the final stages of removing the excess gilding from the shot-blasted depressions. Photos courtesy of T & W Ide.

Fig. 5. Etching with hexafluorosilicic acid where the acid is retained by a ridge of tallow. Photo courtesy of T & W Ide.

Fig. 9. The dial framed by the solar-heat-limiting blinds that were raised in dull weather. The shiny nodus plate is just visible.

Fig. 10. A closer view of the dial where the effect of the gilded lettering can be seen. On approaching the dial from the outside the lettering is reversed although the analemma still functions properly!

plate and struts myself. I knew exactly what was required, ordered the stainless steel, and in a workshop at my disposal I did all the necessary metalwork. In addition, on a raised platform, I did the final installation on the mullions of the window frames.

The dial was installed in August 1996 and Figs 9 and 10 are views of the south face of the building showing how the protective shuttering was made to frame the dial. Fig. 11 is an internal view with the shuttering raised. Figs 12 and 13 show views of the dial from the balcony.

As stated earlier, the highlighted dates increase in steps of 5, with the alternate bold dots numbered 10, 20, 30, or for the longer months (with six small dots) to 31, the position of each dot being calculated for noon. The leap year day, 29 February, is not included (Figs 14–16).

Fig. 11. A view of the dial from the concourse. There is a balcony above right that can give a good view of the dial.

Fig. 12. A view from the balcony; a Buccaneer aircraft is in the background. It was a day of light cloud and the faint shadow of the nodus plate can just be seen.

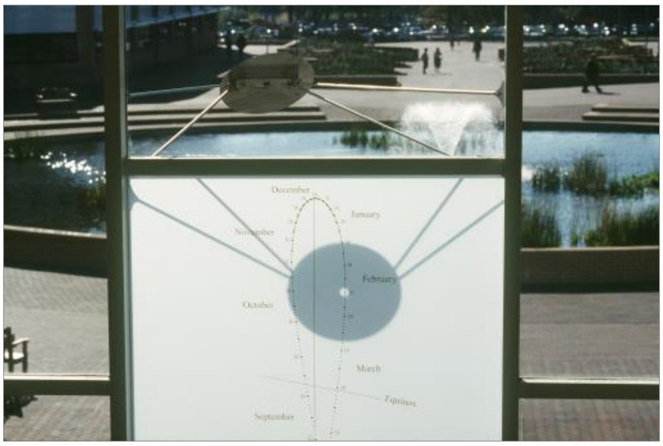
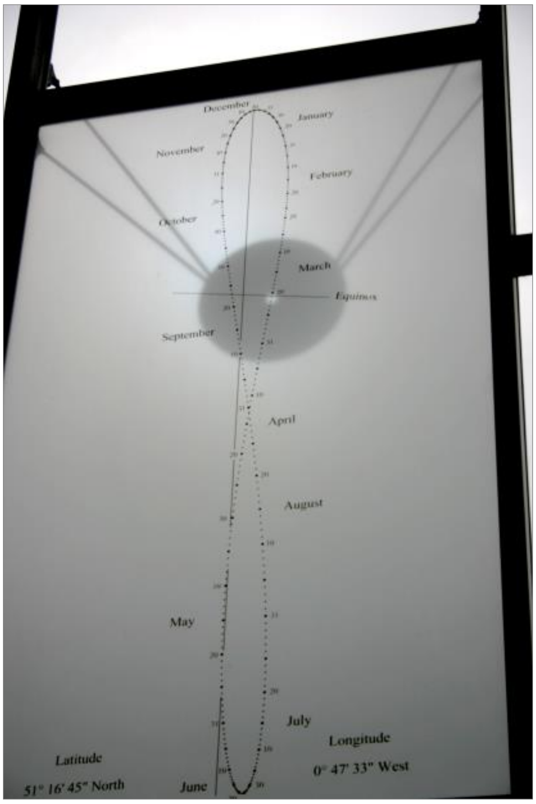
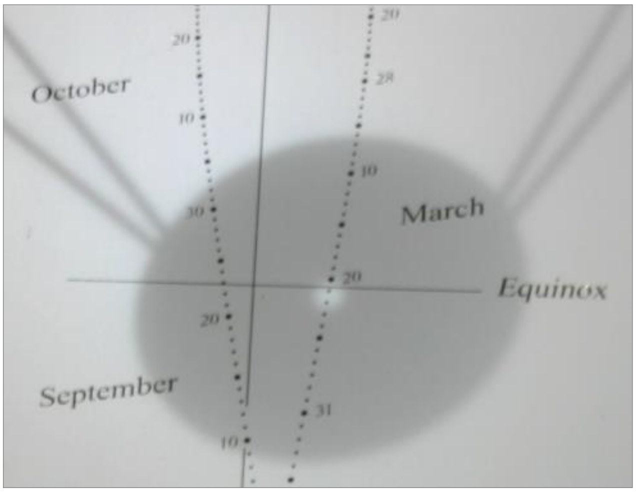


Fig. 13. Another balcony view on a sunny day.





Figs 14 and 15. The dial is remarkably effective on a hazy day. These photographs were taken on Monday 21 March 2005. The equinox was almost exactly 24 hours earlier, but access was not convenient at the weekend.

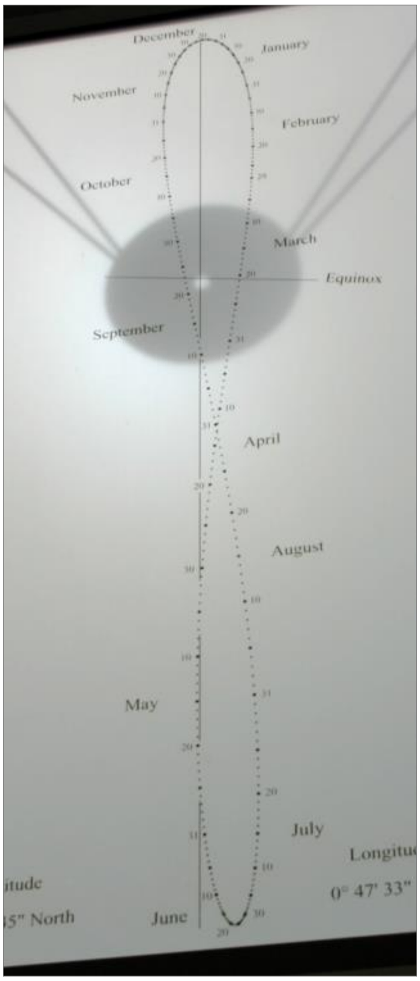


Fig. 16. The local apparent noon, seven minutes later.

Another detail is the displacement of the analemma for longitude. If one looks very closely at a similar dial at the Greenwich meridian (or the textbooks), the analemma is close to the vertical local noon line. On a similar dial in Oxford at 1° 15′ 46″ W (Fig. 17) there is a definite shift of the analemma to the left. The same effect can be seen in Fig. 16, but remember that we are looking at the dial from *behind* the surface. It is as if we are looking at the slate dial in Fig. 17 from the other side of the wall. If the dials were further west the analemma would become more and more displaced from the vertical local noon line.

One can record some satisfaction in that a group from the British Sundial Society came to see the dial on the autumn equinox on 23 September 1999. Just in case it was cloudy my colleagues made a short video recording with a radio ready for the 1 o'clock news, and the spot of light tracked across the centre of the line when the pips were going! Shortly after setting up the dial some young scientists came to see the transit, and on another occasion (the date the photographs were taken in Figs 14–16) the Senior Military Advisor, an RAF Air Commodore, appeared at noon. I asked if he was interested. He gave a brief reply: "Of course, I am a navigator". This phrase stresses the link between a sextant, chronometer and almanac to obtain the position of the sun while at sea, and hence solve the previously *unknown* location of the ship, contrasting with the sundial at its *known* location, to determine the time from the sun's position.

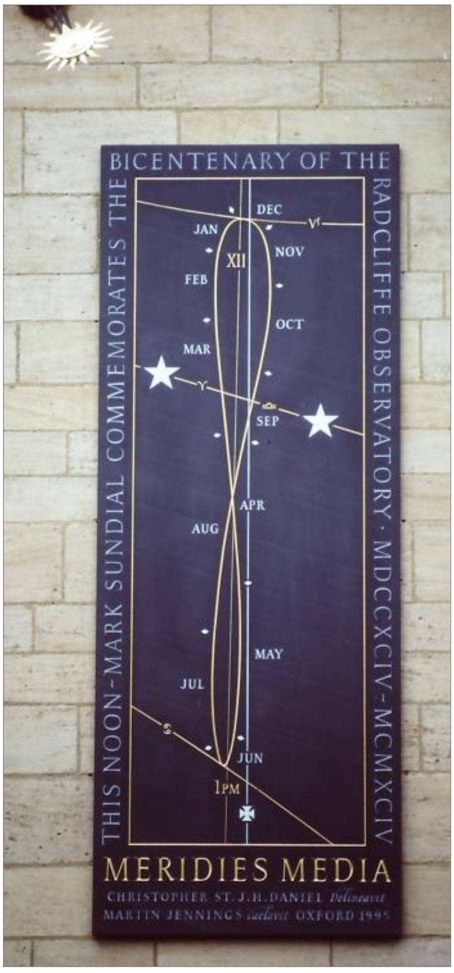
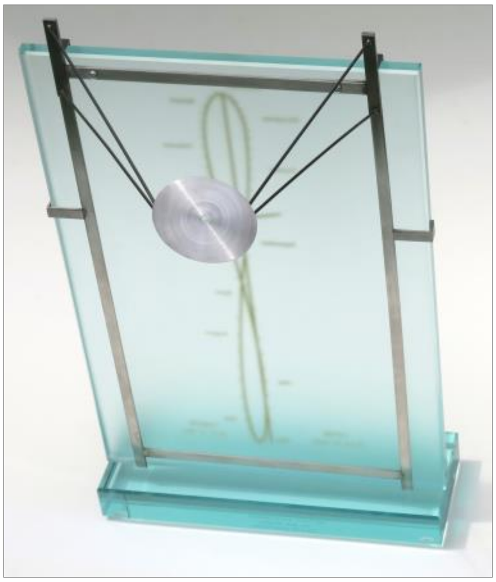


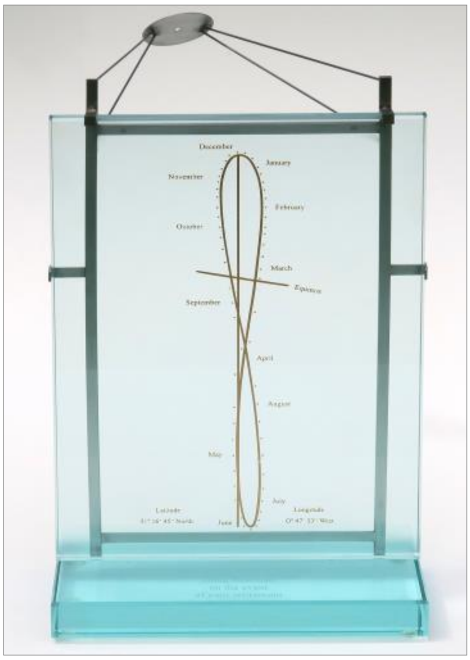
Fig. 17. The meridian noon dial at Green College, Oxford, SRN 2943.

In addition to the dial itself a plaque was installed nearby that gives a brief explanation with my name on it (at least when I left) and some leaflets that gave much more detail (Fig. 18).

Finally, at my retirement I was given an unexpected and handsome model of the noon dial on a vertical slab of glass, 30 cm × 21 cm, cemented to a glass base. On the slab a very close copy of the dial had been made with deeply cut outlines and gold painted. It did not have a nodus plate, which would have been a very tricky extra to add. I was delighted with the gift, and in my own workshop I made a

Fig. 18. Explanatory plaque and leaflets about the dial.





Figs 19 and 20. The model dial with the added steelwork.

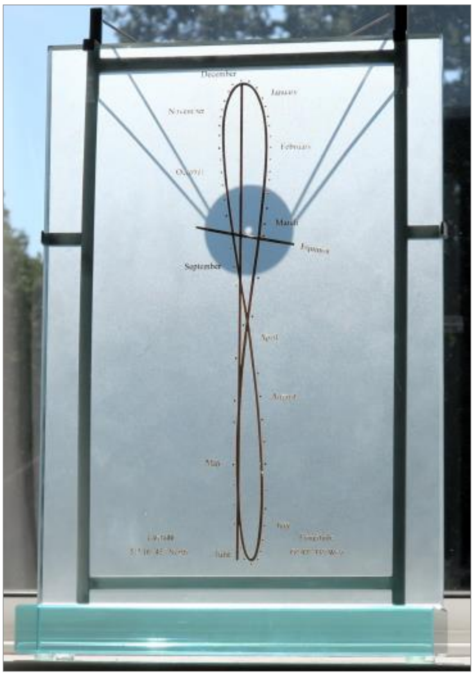


Fig. 21. The model dial on a windowsill. The scattering of light from the frosted surface is quite directional and is very bright when viewed in line with the sun.

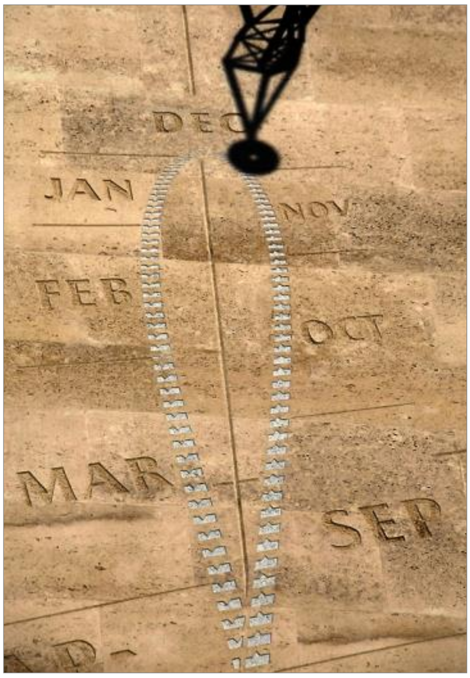


Fig. 22. Detail of the Stock Exchange noon dial. Photo: Estate of David Isaacs.

representative outline of the window and an accurate replica of the nodus plate (of course). To align the struts and nodus plate prior to brazing, I made a wooden jig that effectively filled the void between the plate, struts and attachment pieces. This did the job, even if the jig ended up a little charred in places by the blow torch. Figs 19–21 show the model.

The only comparable dial in the British Isles is the one by Frank King on the Stock Exchange, London (SRN 5355, Fig. 22).[4] It is very satisfying knowing that this rare form of noon dial is likely to be appreciated, hopefully, for many years to come.

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A SCAPHE DIAL POSTSCRIPT

FRANK H. KING

About 20 years ago, I saw a photograph of Mark Lennox-Boyd's Holker Hall scaphe dial and found it instantly captivating. I had never seen such a dial before, even in photographs. I immediately wondered how it was designed and fabricated.

At first sight, the delineation looked very challenging but it did not take long to discover that, provided the nodus is on the vertical axis of the enveloping sphere, the calculations are very straightforward. Alas, it seemed unlikely that I would ever find a client who wanted a scaphe dial and I soon turned my attention to other matters.

When Mark submitted the first draft of his article for this issue of the *Bulletin*,[1] my interest was quickly rekindled. He confirmed *inter alia* my speculation that he used a minor variation of polar coordinates for setting out points on the internal spherical surface.

The centre of his system of polar coordinates is the lowest point on the surface. This is equivalent to the south pole of a globe whose axis is vertical. The position of any point on the surface can then be specified by giving its *distance* from the pole along some line of longitude and the *azimuth* of that line of longitude. Mark denotes the distance by q and the azimuth by A but, since it is a standard convention to use A as the azimuth of the sun, I shall refer to the azimuth of the line of longitude as A'.

Mark's article is so full of interest that I wrote a short commentary. This included an alternative approach to the derivation of q and some additional thoughts. We agreed that I should turn my notes into a supplementary article, this postscript.

What is a Scaphe Dial?

Intriguingly, the Oxford English Dictionary[2] does not have an entry for 'scaphe' but it has an entry for 'scaph' which comes from the Greek *σκάφη* and has the general meaning of boat or tub, clearly suggesting some kind of concavity or hollow. The letter eta at the end strongly suggests that the English equivalent should end in 'e'. Indeed, the French word is *scaphé* where the acute accent emphasises that ending.

In the context of sundials, a general scaphe dial could be any kind of hollow with a suitably placed nodus. The hollows of real scaphe dials are usually simple geometric shapes. This reduces the complexity of the calculations and it is common to use the inside surface of an inverted spherical cap. It is also common for the rim of the cap to be horizontal. If the nodus is on the same horizontal level as the rim, then the rim can serve as the horizon line of the sundial and the shadow of the nodus will never fall outside the dial as can happen with an ordinary horizontal sundial.

A final simplifying constraint is to have the nodus on the vertical axis of the enveloping sphere, typically well below the centre of the sphere. With the rim on the same level, the dial will have the appearance of a fairly shallow dish. Changing the distance of the nodus from the centre changes the appearance of the dial markings. Choosing the distance is discussed later.

Delineation – A Modified Approach

When undertaking almost any exercise in geometry, the choice of coordinate system is very important. When delineating sundials one generally has to use at least two systems: one for specifying the position of the sun in the sky and the other for specifying the associated point on the dial itself.

For the position of the sun, diallists commonly use equatorial coordinates (hour angle h and declination δ and/or horizon coordinates (altitude a and azimuth A).

For specifying the positions of points on a flat dial plate, it is common to use polar coordinates (radial distance and angular offset) for hour lines and cartesian coordinates (X and Y) for plotting constant-declination curves.

When delineating a simple scaphe dial, whose nodus is on the vertical axis, horizon coordinates seem a natural choice for specifying the position of the sun and, for the position of the corresponding point on the internal surface, it seems natural to use Mark's modified polar coordinates. The azimuth A' of the point will be 180° greater than the

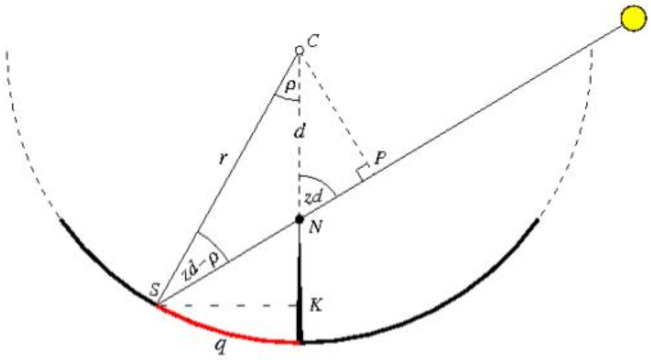

Fig. 1. A vertical cross-section of the scaphe dial in the plane of the sun.

azimuth A of the sun, and the distance of the point from the lower end of the vertical axis of the sphere directly relates to the solar altitude a.

Fig. 1 shows a modified version of the diagram in Fig. 3 of Mark's article. As in Mark's Fig. 3, this Fig. 1 also represents a vertical cross-section of the scaphe dial but it is *not* in the meridian plane. It is, instead, the vertical cross-section which includes both the nodus N and the sun (schematically represented by the yellow disc).

Point C is the centre of the enveloping sphere whose radius is r. The nodus is at the top of a vertical support which stands on the lowest point of the inside surface whose cross-section is indicated by the heavy black arc. The outer ends of this arc are two points on the rim of the dial; they are at the same horizontal level as the nodus. The nodus is a distance d below the centre C.

The continuation of the ray from the sun through the nodus N intersects the surface at S which marks the position of the shadow of the nodus. As noted above, the position of S can be specified by the distance q, the arc length (shown in red) from the lowest point to the shadow, and the azimuth A' of the appropriate line of longitude.

Viewed in plan, the shadow S is opposite the sun whose azimuth is A. Accordingly, the required azimuth A' is given by:

$$A' = A + 180° \qquad (1)$$

The expression for the distance q requires further analysis…

Viewed from the centre C, the angle shown as ρ in Fig. 1 is the angular offset of S from the lowest point. If ρ is measured in radians then arc length $q = r.\rho$.

The angle ρ is directly related to the altitude a of the sun but, instead of using a, its complement, the *zenith distance*, zd, is used: $zd = 90 - a$.

From the figure, and using the theorem about the exterior angle being the sum of the two interior opposite angles, we note that the angle at S is $zd - \rho$.

A construction line CP is also shown in Fig. 1; this is the perpendicular from C to the ray from the sun. Noting that the angle at P is a right angle, we can now write two expressions for the length of the perpendicular CP and equate one to the other:

$$r.\sin(zd - \rho) = d.\sin(zd) \qquad (2)$$

We now replace the two instances of zd with $90 - a$ to give:

$$r.\sin(90 - a - \rho) = d.\sin(90 - a)$$

This simplifies to:

$$r.\cos(a + \rho) = d.\cos(a)$$

So:

$$\rho = \arccos\left[\frac{d}{r}.\cos(a)\right] - a$$

It was noted above that if all angles are measured in radians $q = r.\rho$. Most readers will be more accustomed to working in degrees and, to convert degrees to radians, we further multiply by π/180 which leads to:

$$q = \frac{\pi.r\{\arccos[(d/r).\cos(a)] - a\}}{180} \qquad (3)$$

This is a simpler expression for q than that used in the Holker Hall article but, when setting out a sundial, it is usual to plot points corresponding to positions of the sun specified in equatorial coordinates h and δ rather than positions specified in horizon coordinates a and A.

Before we can use (1) and (3) to evaluate A and q from h and δ, it is necessary to convert equatorial coordinates into horizon coordinates. A standard way to do this is shown in the Appendix at the end of this article.

Mark uses a different approach to convert equatorial coordinates into horizon coordinates. This involves exploiting his 'y' formula which he quotes without proof in his article. This formula greatly intrigued me and is the subject of a later article in this issue.[3]

From the 'y' value, Mark derives both the azimuth A' (shown as A in his article) and a value z which he uses to calculate the zenith distance as:

$$zd = \arctan(z/N)$$

This term is used twice in his expression for q and readers wishing to verify Mark's expression for q should note that it almost directly follows from (2) above, though it is also necessary to note that he uses N for the nodus height and $r - N$ for the offset of the nodus from the centre, whereas I use d.

Mark mentions that he used QB64 BASIC to code his expressions. For preparing tables, giving the positions of points on the inside surface of the dial, one might also consider using a spreadsheet.

Choosing the Distance from the Centre

In writing of his model, Mark mentions that he chose the dimensions by eye. The radius of the enveloping sphere was 250 mm and the nodus was 50 mm above the lowest point. In terms of Fig. 1, radius $r = 250$ and the nodus height $r - d = 50$, so $d = 200$. If the sun were on the horizon, the shadow at S would be on the rim; the line NS would be horizontal and at right angles to NC. The triangle CNS then has a right angle at N and we know the lengths of the two sides $SC = 250$ and $NC = 200$. By Pythagoras, $SN = 150$ and, dividing each side by 50, we see that we have a 3–4–5 triangle.

Astonishingly, this triangle came about by serendipity; Mark had not appreciated that he had a 3–4–5 triangle at the heart of his design. He simply chose dimensions that made the pattern of hour lines and constant-declination curves look right.

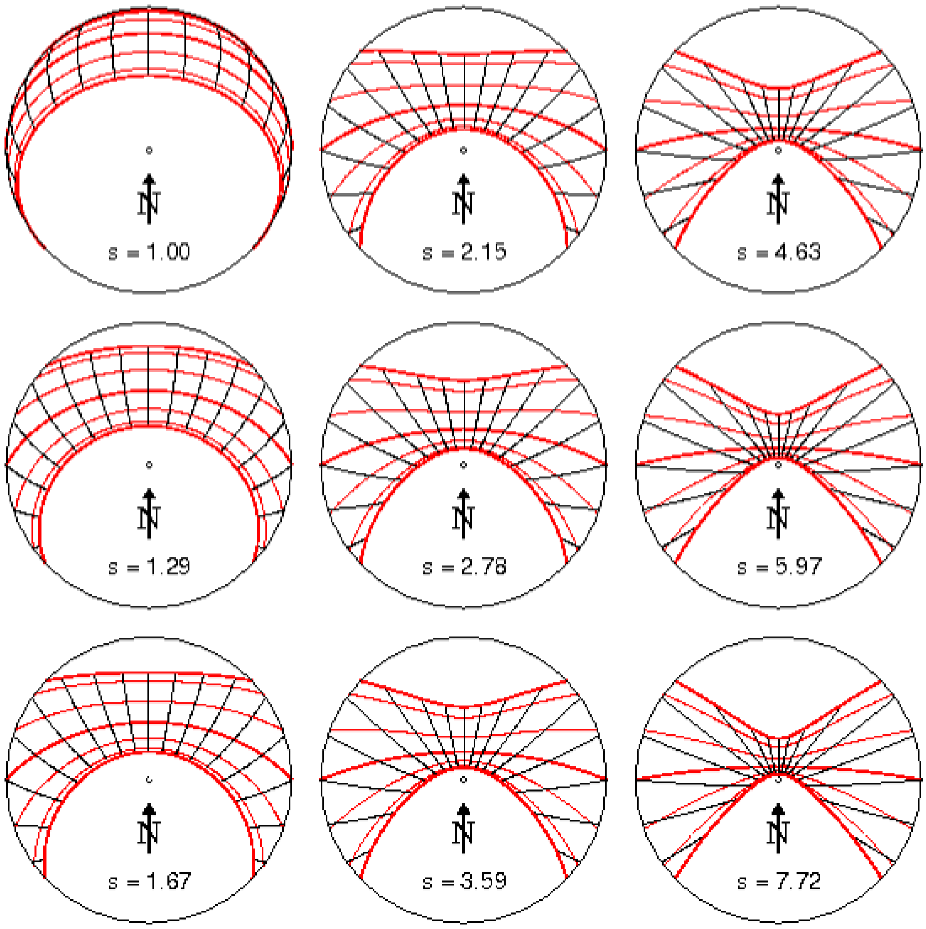

Fig. 2. Different outline designs of scaphe dial for Holker Hall (latitude 54°11′ 20″ N); all are shown strictly in plan. Each s-value is the ratio of the radius of the enveloping sphere to the radius of the internal rim of the dial. For the Holker Hall dial the ratio is 1.67, the example at bottom left. The appearance changes significantly with s-value but the ends of the constant-declination curves (which show the azimuths of sunrise and sunset) do not move.

It was noted earlier that changing the distance of the nodus from the centre of the enveloping sphere changes the appearance of the dial markings. I investigated how the appearance changes when the rim radius is held constant and the radius of the enveloping sphere is gradually increased. I used a scaling factor s defined as:

$$s = \text{radius of sphere} : \text{radius of rim}$$

If the dial is a hemisphere then $s = 1/1 = 1$. In the case of the Holker Hall dial, $s = 250/150 = 1⅔$. As s increases, the bowl becomes shallower and shallower and the markings become more and more like those of a normal, flat, horizontal dial.

Fig. 2 shows plan views of the dial markings for nine different values of s ranging from 1 to 7.72 and they are all calculated for the latitude of the Holker Hall dial. It is noticeable how the equinoctial line gradually becomes straighter as s increases. The markings for Mark's chosen value of 1.67 look the most pleasing. If s is increased much more, the winter solstice curve starts to develop a kink!

When $s = 1$, the dial markings appear squashed up on the north side. Although this appearance is decidedly less pleasing, the plan view in Fig. 2 is not entirely fair. At the rim, the surface of a hemispherical bowl is vertical and the same markings would look much better to someone studying the north side from a low viewpoint on the south side. The observer has to be careful not to stand in the way of the sun.

3D Engraving

Towards the end of his article, Mark refers to 3D engraving. A scaphe dial is clearly a three-dimensional object. Numerically controlled machine tools have been around for a long time and, in recent years, 3D printing has become popular in home workshops. It may not be long before people are 3D-printing scaphe dials!

In the analysis discussed earlier, the position of a point on the internal surface of a scaphe dial was specified using two values, a distance q and an azimuth A'. It is almost as straightforward to specify the position of the point in three dimensions using cartesian coordinates (X, Y, Z).

For the origin, I shall choose the centre of the enveloping sphere, point C in Fig. 1, with the positive Z-axis running vertically upwards through C. The positive X- and Y-axes are both horizontal and run due east and due north respectively.

A final line in Fig. 1 is SK which runs horizontally from the shadow point S to K on the vertical axis and we note, from the figure, that $CK = r.\cos(\rho)$. Subject to a minus-sign, CK is the Z-coordinate of S; since K is below the origin, the coordinate is $-r.\cos(\rho)$. SK is the radial distance of the shadow point S from the vertical axis and, from the figure,

$SK = r.\sin(\rho)$. In plan, SK has azimuth A' and so can be resolved into two directions east and north. The (X, Y, Z) coordinates of S can therefore be expressed as:

$$X = r.\sin(\rho).\sin(A')$$
$$Y = r.\sin(\rho).\cos(A')$$
$$Z = -r.\cos(\rho)$$

In these expressions, azimuth A' is measured clockwise round from true north. If you use south as the reference direction then the signs of X and Y will need changing.

In Fig. 2, I used just the X and Y values, thereby projecting the three-dimensional patterns onto the horizontal plane through the nodus. This results in plan views.

Appendix – Coordinate Conversion

The standard equations for converting from equatorial coordinates (h and δ) to horizon coordinates (a and A) given latitude ϕ are:

$$a = \arcsin\,[\sin(\delta).\sin(\phi) + \cos(h).\cos(\delta).\cos(\phi)]$$

$$A = \arctan\left[\frac{-\sin(h).\cos(\delta)}{\sin(\delta).\cos(\phi) - \cos(h).\cos(\delta).\sin(\phi)}\right]$$

In these expressions h is the hour angle of the sun. At noon $h = 0°$ and morning hours are negative, so at 6 am $h = -90°$. Azimuth A, also measured clockwise round from true north, can be anywhere in the range 0° to 360° but a simple inverse tangent function covers only half this range. Spreadsheets and many programming languages have built-in two-argument inverse tangent functions…

In particular, Excel has a two-argument function

ATAN2(denominator, numerator)

If the sub-expressions in the top and bottom lines of the expression for A are used, all 360° of azimuth will be correctly determined. The function fails only when the sun is directly overhead at noon (or is directly at the nadir at midnight) as can happen in the tropics. With the sun in the zenith or nadir, its azimuth has no meaning!

ACKNOWLEDGEMENT

I am most grateful to Mark Lennox-Boyd for rekindling my interest in scaphe dials and for his many helpful comments when I was preparing this article.

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A FINAL PHOTOGRAPH BEFORE LOCKDOWN

The noon mark on the new London Stock Exchange, in Paternoster Square opposite St Paul's Cathedral, was cut into the Portland Stone wall in the summer of 2003.

Subject to a bit of congestion at the two solstices, the giant analemma is divided into 366 strips, one for every possible date of the year including a special thin strip for 29 February. Just half the strips are cut into the wall. The alternate strips are left implicit as gaps between adjacent cut strips.

Each day, around GMT noon, the spot of light from the aperture nodus runs across the strip that indicates the day's date. In common years it misses out 29 February. It runs along the 28 February strip one day and the 1 March strip the next day. Only in leap years, on 29 February, will it run along the special thin strip.

I have visited the square every 29 February since inauguration. In 2004, I set off from Cambridge in a snow storm. I was with Annika Larsson, who had been in charge of the cutting operation. By a miracle, the clouds broke at exactly the right moment and the sun shone just long enough to convince both of us that the spot really was on the 29 February strip.

In 2008 there was heavy rain and in 2012 there was no rain but thick cloud. In 2016, it was so warm that office workers were walking about in shirt sleeves and, for the first time, I managed to take a photograph at 12 noon. I went again this year. When I arrived in Paternoster Square there was such

heavy rain that the square was under water but, amazingly, the clouds parted at the right moment and I captured the photograph above.

The spot of light is not centred on the strip. It is somewhat to the left of centre aligned with the triangular indentation. This marks 12 noon GMT. When the spot is centred on the strip it is 12 noon local mean time. Paternoster Square is significantly to the west of the Greenwich Meridian. When the spot of light crosses the almost vertical line it is 12 noon local sun time. The line is not quite vertical because the wall is slightly proclining. Little did I realise when I took this photograph that trips to London were about to be out of the question for some time.

FHK

AN AMATEUR'S ATTEMPT AT A VERTICAL DECLINING SUNDIAL

PETE CALDWELL

I got the idea from my uncle in the late 1980s, before even owning a house. My uncle is, amongst other things, a clockmaker; his house and garden are stuffed with clocks, orreries and sundials, and he has instilled in me from an early age a love of astronomy and clocks. So when my wife and I bought our house in Cambridge in the 2000s I always had a feeling that one day we would be putting a vertical sundial on our front wall, which faces south-west and has sun late into the evening during the summer. But it was not until the Covid-19 lockdown that I really decided to make it happen. And as that beautiful sunny spring of 2020 began to unfold, I started to get to work. Here is a description of that journey.

The Concept

The first stage was to create a concept design that we would be happy with. We considered a few 'design principles' to help steer the conceptual design. Modern or old-fashioned? Natural-looking or man-made? Metal or stone? Sparse or full of information? Angular or rounded? Copy of a famous dial or unique? We agonised over these choices and in the end decided on a fully customised, square piece of polished slate, with Roman numerals, the equation of time, coordinates of our location, and the year 2020. We wanted all the essential information about the dial, but in as uncluttered a way as possible.

The Mathematics

I had to calculate the angles and lengths myself as we had had a quote from a company promising to do the whole thing from start to finish, but it was way beyond our budget (20 times beyond, in fact). So I found my trusty, though dusty, Waugh's *Sundials*,[1] and with the help of Google Earth to determine the position and declination of our house, started to plot out the face of the dial. Initially I used the computational method and calculated the angles; however, as I was using Microsoft PowerPoint to draw it, it ended up being more accurate to use the graphical method as shown in Fig. 1.

I made some sketches on black corrugated card (Fig. 2), so we could see what it would look like. Our neighbours were amused to see me up the ladder sticking squares of card on the wall, but it gave us a good idea of how the sundial would look in various positions.

At about the time of making our initial sketches, I chanced to stumble across the British Sundial Society and Sue

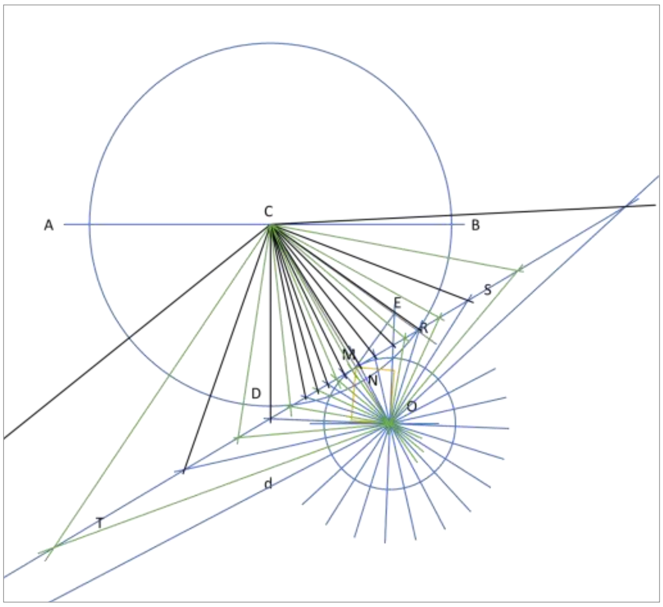

Fig. 1. Designing the dial face.

Manston of the BSS Help and Advice Service. Sue was kind enough to look over my sketches. She highlighted a number of points; in particular, she taught me about pulling the dial design apart by the width of the gnomon so that the sunlight falling onto either side of the gnomon would cast the shadow correctly onto the face. I had been worrying about that and was considering a knife-edge gnomon, but I

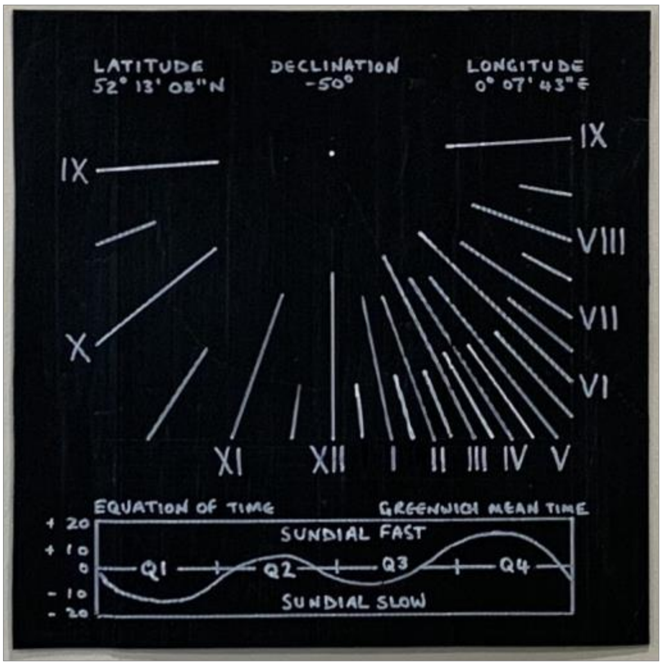

Fig. 2. Sketch on black corrugated card.

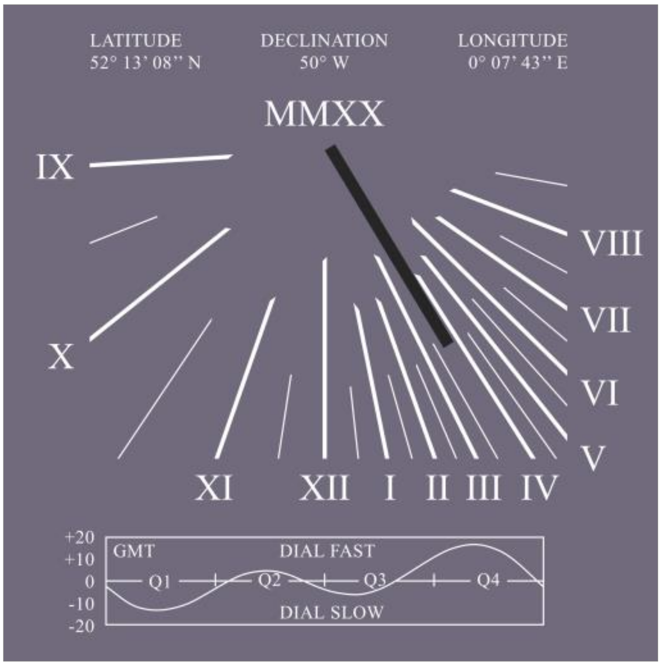

Fig. 3. The final drawing constructed in PowerPoint.

now understood I could go for a sturdy 8 mm thick square-cut piece of steel. I have to say the extremely detailed emails she sent me helped me not only in correcting my design, but also in spurring me on and believing that this project would actually make it past the drawing board.

The Final Design

Fig. 3 shows the final drawing constructed in PowerPoint. You can see the enlarged gap between 3:30 pm and 4:00 pm down the line of the gnomon. The two halves of the dial are 'pulled apart' perpendicular to the gnomon – the hour and half-hour lines left of the 3:45 line originate from the gnomon's top-left corner; the lines to the right of it originate from the gnomon's top right corner.

Fig. 4. The engraved slate dial.

In terms of the design of the face, the 9-, 12-, 3- and 6-hour lines are slightly longer than the others, and the half hour lines are shorter still. I did not want the lines to extend all the way upwards to the top of the gnomon, or to a perfect imaginary circle around it. I wanted the dial to look edgy and spiky, so I varied each line's length, and gave them each a jagged end.

I drew the equation of time by hand with PowerPoint's curve feature, using an example from the Internet as a guide. I wanted to keep the face as uncluttered as possible, so I decided not to use gridlines or month names; instead I used quarter-years. The gaps in the horizontal line – where Q1, Q2, etc. are written – are exactly a third of the quarter, i.e. one month, so it is quite easy to see where each month starts and ends. The gaps look smaller than the lines, but that is purely an optical illusion.

I decided to include the latitude, longitude and declination – they are not strictly necessary but at least if the sundial ever gets lost it can be posted back. I checked the spelling of 'latitude' about 100 times. And of course, I had to include the quasi-symmetrical year MMXX – after all, we will not have another such aesthetically pleasing Roman numeral year until 3000.

Building the Sundial

I sought two quotes for the slate dial and gave the contract to Lakeland Slate in Wales.[2] I emailed my PowerPoint drawing to the proprietor, Mike Charlton, who quoted under £200 for a piece of 450 mm square, 20 mm thick, dark blue Welsh slate, engraved, painted and polished, delivered from Wales to Cambridge. He was incredibly friendly and helpful, and I could not have been more pleased with the final outcome.

When the slate arrived (Fig. 4), we got down to designing the 150 mm long gnomon. Because we are musicians, we wanted the gnomon to suggest something musical. We tried many designs – a musical note, a clef, even something representing a piano lid. We could not make anything work exactly, so in the end we went for 'essence of treble clef' (Fig. 5).

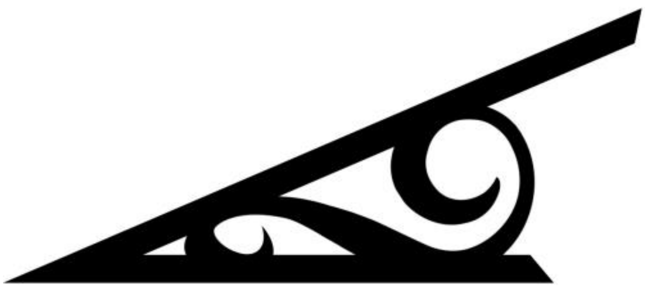
Fig. 5. The 'treble clef' gnomon design.

I designed the gnomon in PowerPoint and sent the drawing to Mike Overall,[3] a local Cambridgeshire blacksmith, who quoted under £100 to make it in 8 mm thick galvanised, coated steel.

Figs 6-9. The drilling and fixing of the rods and gnomon.

Completing the Sundial

At the time the gnomon was being made, our thoughts turned to how to fix the dial to the wall. I was introduced to David Brown from the British Sundial Society who gave me some fantastic advice on methods for doing this – four pages of advice in fact. We did not want any holes on the front face of the dial, so we decided to go for the blind-fixture method, whereby two 12 mm thick, 80 mm long threaded stainless-steel rods are epoxied into two holes that are drilled part-way into the back of the dial, one at each top corner, angled downwards by 15 degrees. This angle means that the dial will naturally lie snugly against the wall.

I 'socially-distanced'-visited Mike Overall's workshop with this information and he drilled and fitted the two threaded rods, using a couple of 15-degree wooden wedges as a guide. He also drilled and tapped the underside of the gnomon, threaded and bonded two 4 mm rods into the steel, drilled the dial, and epoxied the gnomon precisely in place. That gnomon is not going to move. See Figs 6–9.

This was painstaking and nerve-wracking work. I can heartily recommend Mike; he is a true craftsman, and his wife Karen is a dab hand at steel coating.

Affixing the Sundial

It was now time to fix the dial onto the wall. I was slightly worried about fitting it permanently, because you never know when you might want to remove something from the house, and this was going to be permanently epoxied in. I had an idea – if we could epoxy some stainless-steel tubes into the wall, we could slide the dial into them via the threaded rods and slide it out again if we ever needed. So I

Fig. 10. The back of the dial.

Figs 11 & 12. Drilling holes and fixing the dial to the wall.

bought two 100 mm length tubes and placed them over the threaded rods. But I could not epoxy them into the wall on their own and simply hang the dial off them because the angles of the threaded rods and tubes would not necessarily be *perfect*. So we dry-fitted the tubes onto the rods, packed them with paper so that the rods were flush and tight against the bottoms of the tubes, sealed them off and masked the back of the dial so no epoxy could touch the rods or the dial itself (Fig. 10).

Then we drilled the holes in the wall using the same wooden angle wedges that Mike Overall had used, fitted a temporary batten to the wall to allow the dial to rest, level and stationary, while the epoxy set, then set the whole combination into the wall. Figs 11 and 12 show our friend Mark Pinder drilling the holes and fixing the dial to the wall. Now if ever we want to, we can slide the dial out of its tubes.

The Finished Sundial

A lick of paint on the wall completes the picture and the sundial looks stunning (Figs 13 and 14). We are so happy with it. The experience with the people from The British Sundial Society has been wonderful and enriching, and I have met other sundiallers along the journey. Of course I have become a member of the Society and I am looking forward to when the meet-ups take place to meet fellow members once this strange time has passed. In the meantime, I hope this is an interesting description of how it is possible even for an amateur to get a beautiful, useful and timeless ornament (or maybe 'ageless' would be a better term!) to grace a house for under £500 including installation.

Fig. 13. Painting the wall to show off the new dial.

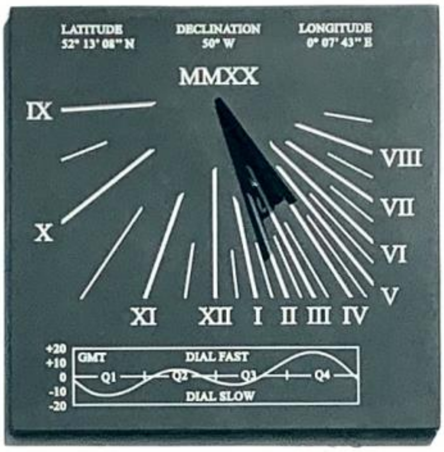

Fig. 14. The finished sundial.

ACKNOWLEDGEMENTS

Thanks to Mike Charlton, Mike Overall and Mark Pinder for their craftsmanship, and to Sue Manston and David Brown for their advice throughout the process.

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MICHAEL LOWNE 1929–2020

I am very sad to report that (C.M.) Michael Lowne died three weeks after his 91st birthday in May this year. He had been recovering slowly from major heart surgery in the autumn of 2018 but finally succumbed to bacterial pneumonia in Eastbourne Hospital.

He was born on the outskirts of Norwich and went to school at Fakenham Grammar. After his National Service, spent mostly looking out for Zeppelins (!) from Spurn Head, he joined the Royal Greenwich Observatory as a Scientific Assistant in 1950, a role for which he was much more suited than joining his father's bank. He became one of the last people working at the professional observatory at Greenwich, moving with them to Herstmonceux in the mid-1950s. It was here that he met and married Diana Damen in 1958, with the pair going on to have children David and Katie.

Michael continued to work at Herstmonceux until his retirement in 1989, mostly in the optical division of the RGO and spending many nights using the Isaac Newton Telescope until it was moved to the Canary Islands. He also made observations using the Equatorial Group telescopes.

In retirement, Michael joined the BSS as member 404. He had moved to Hailsham in East Sussex with the RGO where he was a few hundred yards from his friend and work colleague Gordon Taylor who probably kindled his interest in sundials. I first came across him via his article explaining the construction of the double horizontal dial, a type virtually unknown at that time. This article has the dubious honour of being the only item to have been published in two consecutive issues of the *Bulletin* after the printers made a nonsense of his equations on the first attempt. Most authors would have been apoplectic but, true to form, Michael was just mildly upset and pleased to have things corrected. Wanting to build my own DH dial, I contacted him and found a very patient and helpful enthusiast and we started a long collaboration with a monograph on DH dials as one product. In the early days, Michael would communicate by letter – I still have a thick folder of them – typed on an Olivetti typewriter with daisywheel printer, usually accompanied by a couple of neatly hand-drawn graphs and often including a cardboard model of something we were studying, with volvelles and other moving parts. Later, he moved online and, after sharing the user address diana.lowne for some time, had to accept being mike.lowne because his service provider already had a Michael. His early articles had been published under the rather formal authorship of C.M. Lowne but his stated preference was for Michael.

He was always happy to help other diallists with his knowledge, often putting much effort into explaining what had seemed a simple question. He liked solving puzzles, particularly optical ones, so among his articles were ones on reflecting and refracting sundials and the patterns caused by the reflections of sealed double-glazing units. He also produced a careful study of moon-dials, explaining why the traditional designs were such poor timekeepers and producing an optimised design with an example neatly made in plywood and MDF. One of his triumphs was to solve the operation of a unique universal altitude sundial in the form of a nomograph on brass. This had been made in the 17th century by John Marke and is now in the Science Museum but all previous descriptions had carefully avoided explaining what the unlabelled scales were for or how it worked. Other studies were into (true) planetary hours and the (faulty) declination lines on the 16th century sundials of Isaack Symmes, always providing a full but easy-to-understand analysis.

Michael was a tremendous foil as a collaborator and I learned to rely on his ability to root out my careless mistakes and silly misconceptions, gently correcting them without making me feel stupid. He was never a person to look for the limelight or even claim his due credit: even on projects where he had done the lion's share of the work he was reluctant to be the lead author. In the last decade of his life we ran out of double horizontal dials to study so we turned to medieval astrolabes where his knowledge of stars and the ability to work with several astronomical coordinate systems were a tremendous starting point in a new field. Although his failing eyesight and health limited him in the end, he was always keen to learn something new and make a contribution to scientific knowledge.

He will be much missed by his family (Diana, David and Katie) and by all diallists.

John Davis

DAVID O. LE CONTE 1940–2020

David Le Conte was born in Guernsey in 1940 and, sadly, died there in August this year. In between, he spent much of his distinguished working life in America, pursuing his passion for astronomy. Initially educated at Elizabeth College, St Peter Port, he studied physics at Edinburgh University, moving after graduation to the Royal Observatory there. After a spell at Aberystwyth researching and developing astronomical equipment, in 1964 he moved to the Smithsonian Institution's Astrophysical Observatory with NASA, tracking satellites with optics and lasers. This led to a move to Florida where he met a governess from Leicestershire called Dorothy. Following their marriage there came a move to Cambridge, Massachusetts, and then to Hawaii to become the manager of the SI's Maui, Hawaii astrophysical observing station. Here he took photographs of Apollo 7 and the significant photos of Apollo 8's trans-lunar injection in 1968. From Hawaii there was a move to Arizona where he tracked the progress of Apollo 11 which, of course, included the first moon landing. 1970 saw a move to Washington DC with the family now including their son Christopher and their daughter Sarah. In Washington he was the Executive Director of the Smithsonian Research Foundation and David also somehow found time to gain an MBA. His final appointment in the States was at AURA in Tucson, Arizona where he became the department manager at Kitt's Peak National Observatory and also contributed to a successful proposal to NASA to run the Hubble Space Telescope.

In 1978 the family moved home to Guernsey where David naturally became deeply involved in the Astronomy Section of La Société Guernesiaise, later being its Chairman. He was elected Fellow of the Royal Astronomical Society after being proposed by Sir Patrick Moore (who opened the Society's observatory's new building with a roll-off roof in 1993). David and Dorothy became important members of the island's social and intellectual life with David being elected one of four 'Jurats' of the Royal Court of Guernsey.

David helped to design the Guernsey flag adopted in 1985 and was also the key astronomical consultant in the development of the Guernsey Liberation monument (pictured at its opening). This consists of a tall obelisk which casts a shadow with its tip tracing the line between the seat and the back on 30 metres of curved stone seating around St Peter Port harbour at 7:00 am on the 9th of May (one day after VE Day) to celebrate the fiftieth anniversary of the liberation of the island in 1945. This was not a calculation that could be wrong.[1]

David and Dorothy were both very interested in sundials and collaborated on producing a booklet[2] cataloguing all the historical dials on the island – there had previously been none reported in the BSS Register. With his love of instruments, David had acquired a wartime Brunson sun compass (pictured) during his period in America and that was carefully described in a *Bulletin* article.[3] Another contribution was on the topic of the alignment of the churches resulting from his studies of the Neolithic passage graves and megaliths on the island.

This writer met him only a few times, but found him a delightful man, full of enthusiasm for all things astronomical and especially about his homeland. He is survived by his wife, Dorothy, his daughter, Sarah, and his sister, Margaret. His son Christopher predeceased him in 2015.

REFERENCES

1. 'The Guernsey Liberation Monument', *BSS Bull.* 97.3, 43-49. See also the video at *http://www.astronomy.org.gg/more/articles/liberation-monument*
2. David and Dorothy Le Conte: *Guernsey Sundials*, La Société Guernesiaise, St Peter Port, Guernsey (2004).
3. 'The Brunson Sun Compass', *BSS Bull*. 24(ii), 22-24 (June 2012).

John Davis

TIME FOR RECREATION AND RE-CREATION

Part 1: A Sundial on a Chimney

DAVID BROWN

I have always considered my interest in letter-cutting and sundial-making as recreation – an opportunity to get lost in the creative activity and escape from the challenges, responsibilities and toils of everyday life. Never more has this been apparent to me than during the seemingly never-ending succession of world-wide problems aggravated to a huge degree by the pandemic.

It was therefore a pleasant interlude when I was able to respond to a call from BSS member Harriet James, who has recently decided to curtail her sundialling activities for personal reasons, to help a client with the re-creation of a sundial that had been known to exist until around 1900 but had become lost as the building it was on had become derelict.

Fig. 1. The original sundial c.1900. Photo: owner's archive.

Fig. 2. Remnants of the original sundial. Note particularly the shortened V of the VII (top left). Photo: owner.

The building itself was a former thatched mill cottage in Wiltshire dating back to the 16th century, and in the process of restoration over the last twenty years the present owner had discovered a photograph clearly showing a vertical sundial on one of the chimney stacks (Fig. 1). In the rubble of the derelict building he also discovered remnants of the sundial (Fig. 2). With the full restoration of the whole building and the mill completed in early 2019, he wanted to add the finishing touch of the sundial on the chimney stack. I sent some sketches to the client with a number of options ranging from bare minimum (hour numerals only, as intimated by the client) to hour lines, hour numerals, seasonal date curves with a nodus on the style (Fig. 3). The happy medium was chosen, with Roman numerals, hour lines, no framing, no date curves. The gnomon was to match as closely as possible the shape and size of the original.

Quarantine and social distancing requirements meant that an on-site measurement of the chimney wall's declination had to be interwoven with availability of scaffolding and fair weather, but this was determined in June as 6.2° W. I was then able to refine the design and to include an interesting space-saving concatenation of the Roman numerals VII and VIII that had been noticed from the original fragments. The right-hand arm of the V had been omitted in each case and the left-hand arm moved to the right. A quotation was accepted for the waterjet cutting of the gnomon in 6 mm phosphor-bronze gnomon from Precision Waterjet (Lyme Regis).[1]

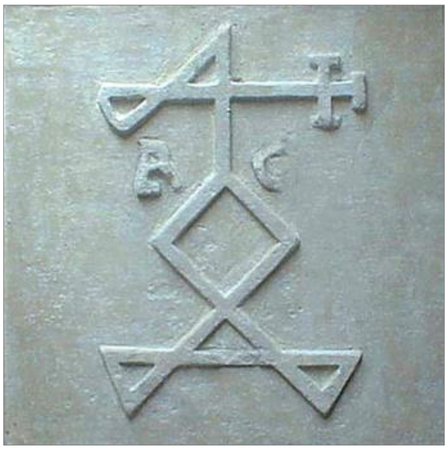

Fig. 4. The cloth seal (carved stone version) of the owner's ancestor. Image courtesy of Museum in the Park, Stroud.

The client's ancestors were cloth merchants. Leaden seals were attached to textiles from the late 14th to the early 19th century in England as part of a system of industrial regulation and taxation. The client warmed to the suggestion that his family seal (Fig. 4) be incorporated in the sundial but with his initials and this year's date.

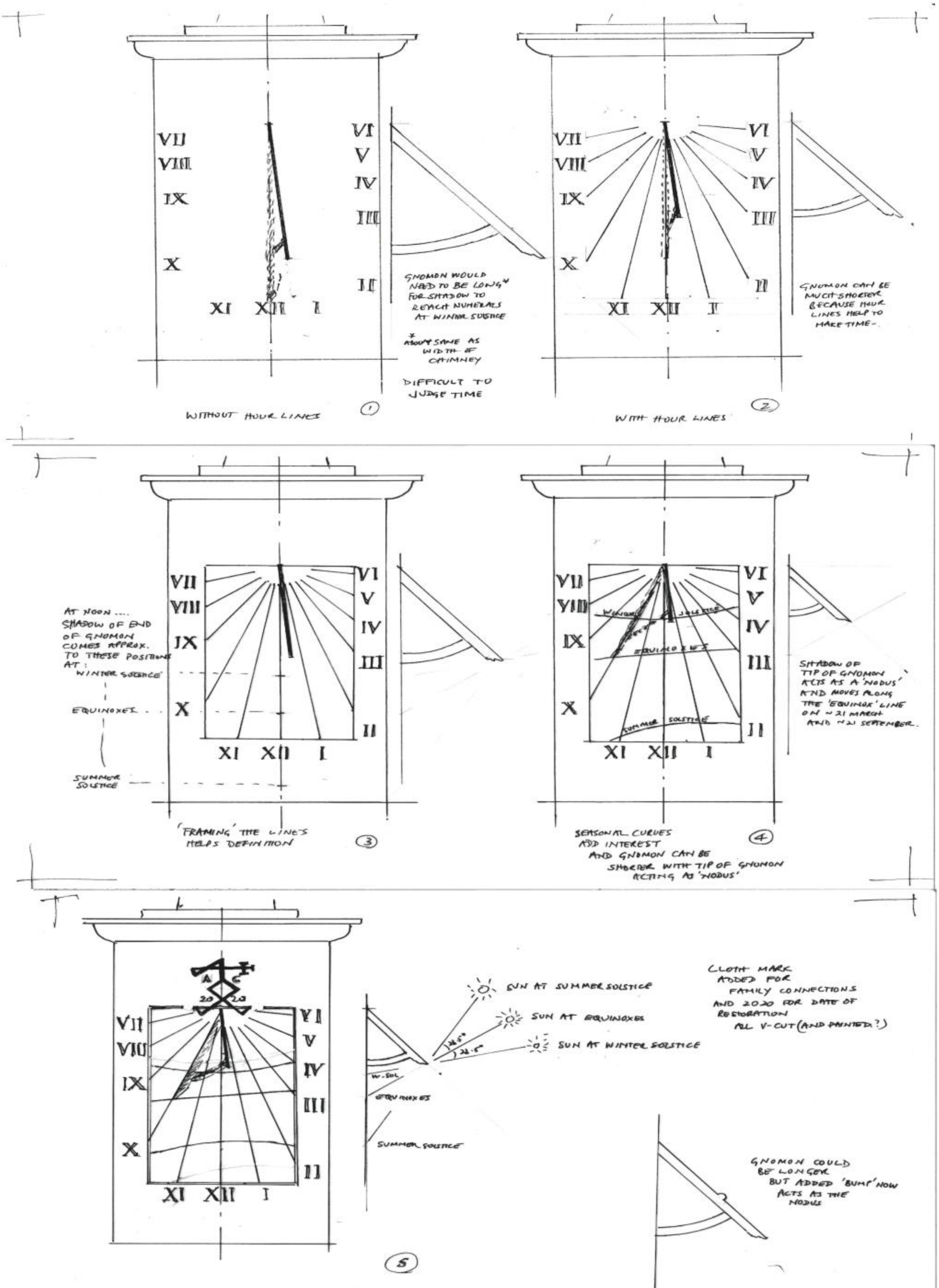

Fig. 3. Preliminary alternative drawings for the restored sundial.

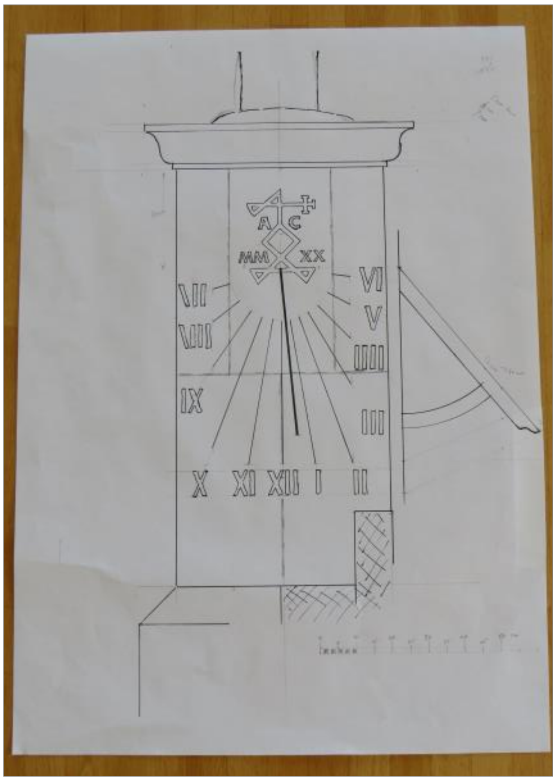

Fig. 5. Final drawing for the restored sundial, including cloth seal.

Fig. 6. Harry Jonas cutting the sundial in situ. Photo: owner.

Fig. 7. The completed sundial. Photo: owner.

The client had previously enlisted the services of Wiltshire stone-mason Harry Jonas[2] to do the incision work in the Chilmark stone of the chimney stack, so I provided him with a full-scale drawing of the sundial (Fig. 5) so that the layout on site would be straightforward (Fig. 6). Harry duly did the work and installed the gnomon (Fig. 7).

There is a good comparison between the dial as seen in the 1900 photo and the new one (Fig. 8), and the client was well pleased.

My suggestion that there should be an equation of time correction plate nearby was accepted. The restored mill is

Fig. 8. The new sundial in 2020. Compare with the original in Fig. 1.

Fig. 9. The equation of time plaque with mill-stone centre.

adjacent to and part of the cottage, so I incorporated a millstone image at the centre of a circular format of the EoT corrected for longitude (Fig. 9), and this slate disc is mounted on the wall below the sundial (Fig. 10).

ACKNOWLEDGEMENT

I am indebted to Tom Bell, a student in the Design School of Loughborough University who very willingly and at short notice converted my line drawing of the gnomon into .dxf format for use by Precision Waterjet.

Fig. 10. The finished ensemble of sundial and EoT plaque.

REFERENCES

1. Precision Waterjet Ltd., Lyme Regis, Dorset, DT7 3LS.
2. Harry Jonas Stonemasonry, Berwick St John, Shaftesbury, Dorset, SP7 0EX.

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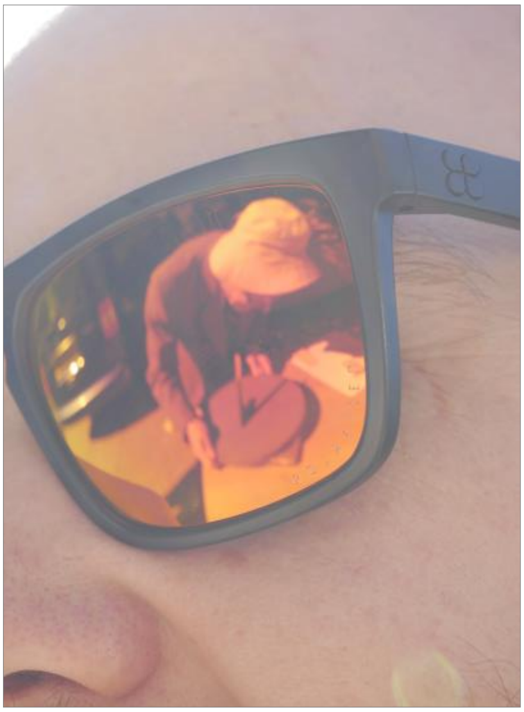
SUMMER REFLECTIONS

The unusual circumstances of this year did not significantly interrupt the work of the Cardozo Kindersley Workshop in Cambridge. Lida Cardozo Kindersley and her two sons along with a daughter-in-law count as a single household.

I interacted by e-mail and sent them my calculations for a simple horizontal sundial. I am a great believer in making and fitting the gnomon before marking out the design. Only with the gnomon firmly in place can you be sure where the hour lines should radiate from. Of course, you remove the gnomon when doing the cutting.

In late July I heard that the gnomon was ready, and the hour lines had been marked out. Could I come along to check?



I like to make my checks on a sunny day. I start by making sure that the hour lines are at the correct angles to the two noon lines and I then rotate the sundial, in the sun, to ensure that the shadow really does fall along each hour line. The shadow may have an impossible length at certain hours but that is not important, only the direction matters.

A camera seems always to be on hand when I visit and a stray click caught one of the lenses of Hallam Kindersley's sunglasses providing the unusual and unexpected view shown above.

Am I checking a morning time or an afternoon time?

FHK

A BEGINNER'S GUIDE TO DELINEATION
Deriving the Lennox-Boyd 'y' Formula
Using a Gnomonic Tetrahedron

FRANK H. KING

In his article on the Holker Hall scaphe dial,[1] Mark Lennox-Boyd referred to his 'y' formula, a useful expression which he derived and proved many years ago.

An explanation of the 'y' formula turns out to be an elegant way of introducing novice diallists to some of the tools used for delineating sundials. In outline, this article introduces 'y' and shows how to construct four important triangles which are the four faces of a tetrahedron. They can readily be drawn with a ruler and a protractor and lead to 'y'.

What is 'y'?

Fig. 1 shows the first triangle. Informally, we see a rudimentary horizontal sundial with the sun due south at 12 noon on the day of an equinox.

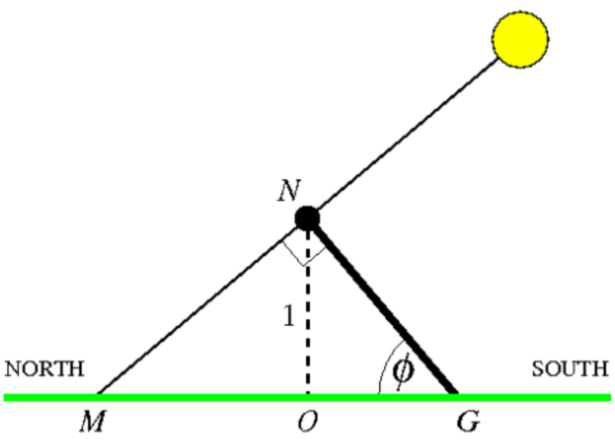


Fig. 1. A horizontal sundial when the local solar time is 12:00 and the solar declination is 0°.

The triangle and the sun (the yellow circle) are in a vertical north–south plane, the *meridian plane*. The horizontal green line represents the surface of a well-manicured lawn. The thick black line is the polar-oriented gnomon which runs into the ground at *G* and has a ball nodus mounted on its outer end at *N*.

At an equinox, the solar declination is 0° and the sun is in the *equatorial plane*. The ray from the sun to the nodus will be perpendicular to the polar-oriented gnomon. In the figure, the continuation of the ray through the nodus strikes the ground at *M* which is where the shadow of the nodus falls. The shadow of the gnomon runs due north from *G* to *M*. The length of the side *GM* of the triangle is the Lennox-Boyd 'y' value (at equinoctial noon). The 'y' value is simply the length of the shadow of the gnomon; it changes continuously with the time of day and it also depends on the solar declination. At 12 noon, the shadow will be shorter in summer when the sun is higher and longer in winter when the sun is lower.

The angle shown as ϕ is the slope of the gnomon which matches the local latitude. In this example, the latitude is taken as 50°, appropriate for the Lizard peninsula in Cornwall. The height of the perpendicular from the nodus to the ground at *O* is the *nodus height* and is taken as unit length, 1. All consequential lengths such as the lengths of the three sides of the triangle are derived from it. If required, all lengths can be scaled up or down. The position of the sun is purely schematic and the distance to the sun will not be used.

When the solar declination is 0°, the line from the sun through the nodus to the ground is one line in the equatorial plane, the plane which is perpendicular to the polar-oriented gnomon and perpendicular to the plane of the figure, the meridian plane.

Another plane which is perpendicular to the meridian plane is the *horizontal plane*, the plane of the lawn. This plane intersects the meridian plane along the green line *GM*.

For completeness, there is a third plane that is perpendicular to the meridian plane and this is the *six o'clock shadow plane*, which intersects the meridian plane along the line of the gnomon *GN*.

Try visualising a packet of Toblerone™ standing on one of its triangular ends. The three long faces of the packet give a rough idea of the three planes just described.

All four planes intersect each other in pairs and, in particular, the equatorial plane and the horizontal plane intersect along a line which is perpendicular to the meridian plane and runs through point *M*.

A Perspective View

Fig. 2 is a perspective view of the lawn with a pair of *X*, *Y* axes marked on it. The triangle *GNM* stands upright on the *Y*-axis which runs due south–north. The unmarked origin is

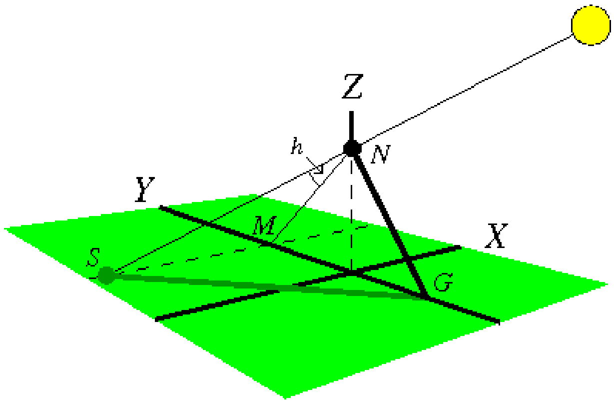


Fig. 2. Mid-morning when the solar declination is 0°.

vertically below the nodus and the positive *Z*-axis runs upwards from the origin through the nodus *N*. The broken line running along the lawn through *M* and parallel to the *X*-axis is the line of intersection of the horizontal plane and the equatorial plane. If we were to mark constant-declination curves on the lawn, the broken line would be the equinoctial line.

In Fig. 2, the sun is shown a few hours before 12 noon but is still assumed to have declination 0°. The ray from the sun through the nodus now strikes the ground at *S* which marks the shadow of the nodus. The shadow of the gnomon on the lawn is the line *GS* and the length of *GS* is the 'y' value now.

If we could freeze the solar declination at 0°, the sun would stay in the equatorial plane and, as the day progressed, the shadow point *S* would move along the equinoctial line and reach point *M* at 12 noon. The angle marked *h* is the *hour angle* of the sun (unusually, positive *h* is taken here as a time *before* noon). In this example *h* = 45° indicating that the time is three hours before 12:00.

Note that the triangle *SNM* is in the equatorial plane and that, because the gnomon *GN* is perpendicular to the equatorial plane, the line *GN* is perpendicular to line *NS* and to line *NM*. In like manner, because the line *SM* is perpendicular to the meridian plane, the line *SM* is perpendicular to line *MG* and to line *MN*.

We now list the four promised triangles. Together, these form the four faces of a tetrahedron. Note that each face is a right-angled triangle which simplifies mathematical analysis:

1. Triangle *GNM* is the original triangle in the meridian plane depicted in Fig. 1. The angle at *N* is a right angle.
2. Triangle *SMN* is in the equatorial plane and the angle at *M* is a right angle.
3. Triangle *SMG* is in the horizontal plane and the angle at *M* is a right angle.
4. Triangle *SNG* is in a *shadow plane* and the angle at *N* is a right angle. At any time of day, the shadow plane is the plane defined by the gnomon and the sun; in practice, only a small part of the plane on the down-sun side of the gnomon is actually in shadow. In the illustration in Fig. 2, only points inside the triangle *SNG* are in the shadow of the gnomon. An insect flying into this triangle would find itself momentarily in shadow as it passed through.

As the sun moves, the shadow of the nodus at *S* moves along the equinoctial line dragging the shadow *GS* with it; the triangle *SNG* continuously changes shape and coincides with the original triangle *MNG* at 12 noon. At 6 o'clock in the morning, the angle *h* = 90° and the point *S* will be at infinity. The shadow plane will then be perpendicular to the meridian plane.

The Run-up to Noon

Fig. 2 shows the triangle *SNG* at latitude 50° N when the solar declination is 0° and the time is 9 o'clock in the morning. Fig. 3 shows the triangle *SNG* at 10, 11 and 12 o'clock when the angle *h* is 30°, 15° and 0°.

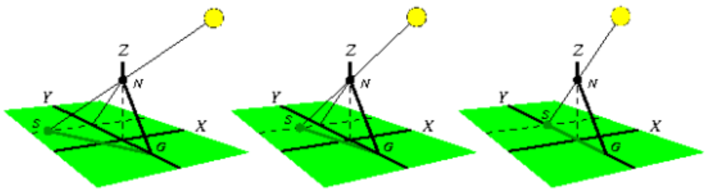


Fig. 3. The point S two hours before noon, one hour before noon, and at noon itself.

The three instances of *GS* show where the hour lines for 10, 11 and 12 o'clock might be marked if we were marking out a sundial on the lawn. Notice that the 'y' value *GS* gradually gets shorter in the run-up to noon and it will then gradually get longer again in the afternoon.

The Equinoctial Shadow Triangle

Fig. 4 is almost identical to Fig. 2 but the triangle *SNG* is now shaded in. The sun is again three hours before noon and the solar declination is again 0°.

The crucial property of a polar-oriented gnomon is that the direction of its shadow at a given time of day is independent of the time of year. At 9 o'clock in the morning, the sun will always be in the same plane as the

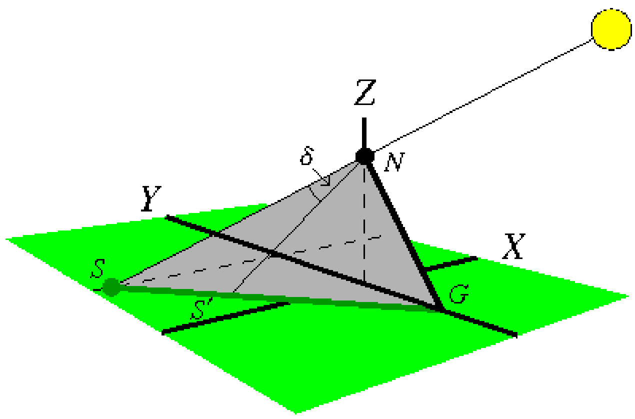


Fig. 4. The equinoctial shadow triangle at 9 o'clock.

triangle *SNG* but it may be higher or lower in the sky. In Fig. 4, point *S* is the outer extremity of the 9 o'clock shadow of the gnomon when the declination is 0°. When the declination is higher, the shadow of the gnomon will be shorter but its outer end will necessarily fall somewhere on the line *GS*. When the declination is negative, the shadow of the gnomon will be longer and its outer end will fall somewhere on the line *GS* produced.

Point *S′* shows a possible position of the shadow of the nodus at 9 o'clock in summer; the length of *GS′* would be the associated 'y' value. The relevant shadow triangle would be *S′NG* which is in the same plane as *SNG* but is smaller. Triangle *SNG* is special: not only is vertex *S* on the equinoctial line but it is also a right-angled triangle. Let us call *SNG* the *equinoctial shadow triangle*.

Recall that a polar-oriented gnomon is perpendicular to the equatorial plane, so *GN* is perpendicular to all lines in the equatorial plane; note also that all shadow planes are perpendicular to the equatorial plane. The angle *SNS′* is not only the angle that *NS′* makes to the line *NS* but it is also the angle the line *NS′* makes to the equatorial plane. This is the solar declination δ; it is zero at an equinox, positive in summer and negative in winter.

The General Shadow Triangle

Fig. 5 is a diagram to illustrate the general case when the latitude is ϕ, the hour angle is h and the solar declination is δ. Any real diagram has to illustrate a particular case and here $\delta = 50°$, $h = 45°$ and $\phi = 20°$.

The principal difference between Fig. 5 and Fig. 4 is that the sun has been moved so that the line from the sun through the nodus now falls on the ground at *S′*. Additionally, the shaded region is now confined to the triangle *S′NG*. Let us call *S′NG* the *general shadow triangle*.

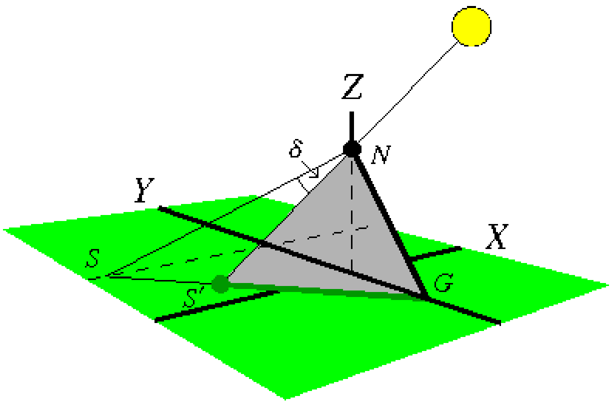


Fig. 5. The general shadow triangle.

Importantly, point *S* has been retained. The goal of determining the position of *S′* is best achieved by first determining the position of *S*. It is then simple to draw or calculate triangle *SNG*, after which the position of *S′* can be determined by drawing a line from *N* at an angle δ to *NS* and noting where the line intersects *GS*. The intersection point is *S′*.

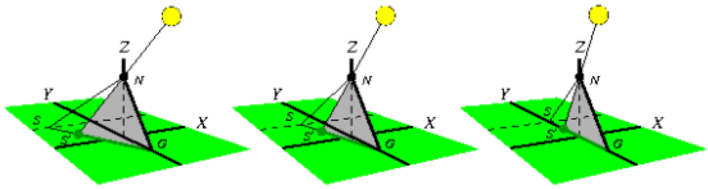


Fig. 6. The point S′ two hours before noon, one hour before noon, and at noon itself.

Fig. 6 is analogous to Fig. 3 but shows the abbreviated shadow triangle *S′NG* at one-hour intervals in the run-up to noon. Point *S* continues to run along the equinoctial line but the actual shadow of the nodus at point *S′* runs along a hyperbolic arc. This is the constant-declination line appropriate (in this case) for $\delta = 20°$.

The Associated Net

Fig. 7 shows the tetrahedron opened up into a flat net. At the heart of the figure is the original triangle *GNM* in Fig. 1. Attached to the edges of this triangle are the three other faces of the tetrahedron. One can imagine cutting this out and folding it up to form the tetrahedron itself. The three outer points S_1, S_2 and S_3 will then coincide; they meet at one vertex of the tetrahedron.

If you ignore all the internal details, this figure is a simple, but unusual, way of depicting a horizontal sundial at latitude ϕ on the day of an equinox when the solar declination δ is zero and the solar hour angle is h. In the figure $\phi = 50°$ and $h = 60°$ (indicating four hours before

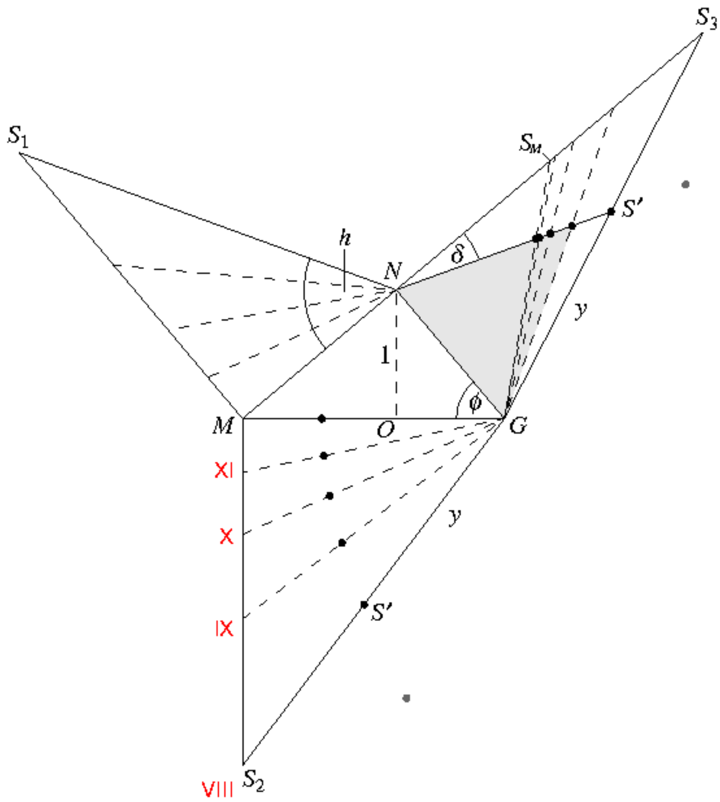


Fig. 7. The tetrahedron opened up into a net.

noon so one hour earlier than the tetrahedron in Figs 4 and 5).

Given that we have four right-angled triangles, this figure is easy to draw. Begin with the triangle *GNM* which is in the meridian plane. Draw the vertical line *NO* (perhaps one inch long) standing on a horizontal line and, from *N*, draw two lines to *M* and *G*. Note that the line *NM* makes an angle ϕ (here 50°) to *NO* and the line *NG* makes an angle $90 - \phi$ (40°) to *NO*.

Next draw triangle S_1MN which is in the equatorial plane. Draw a line at right angles to *MN* and draw another line at angle *h* (here 60°) to *NM*. These two lines intersect at point S_1 whose position is thereby determined.

We now know the lengths of MS_1 and NS_1 which are the lengths of MS_2 and NS_3 respectively. You can immediately draw line MS_2 at right angles to *MG* and of length MS_1 and line NS_3 at right angles to *NG* and of length NS_1. Now that you have points S_2 and S_3 you can complete the outline figure by drawing the final lines GS_2 and GS_3. Check that these final lines have the same length!

Triangle S_2MG is the horizontal face of the tetrahedron. This is the embryonic dial plate marked out on the well-manicured lawn. S_2 is the position of shadow of the nodus *N* at 8 o'clock and the line GS_2 is the 8 o'clock hour line which has been labelled with Roman numeral VIII in red. The length GS_2 is the distance from *G* to the shadow of the nodus; this is the Lennox-Boyd 'y' value at 8 o'clock when the declination is 0°.

One hour later $h = 45°$ which is the new angle MNS_1. The line NS_1 is now the uppermost broken line running to the left from *N*. The new length of MS_1 provides the new length for MS_2 and hour label IX marks the new hour line. Notice that the line MS_2 is the equinoctial line. If the solar declination is frozen at $\delta = 0°$ the shadow of the nodus at S_2 runs along this line and is at *M* at noon.

The broken lines in S_1MN and S_2MG are the hypotenuses of the two triangles at hourly intervals. The triangles shrink to nothing when $h = 0°$ (12 o'clock) at which time both point S_1 and point S_2 coincide with point *M*. *GM* is the 12 o'clock hour line (though it is left unlabelled).

Plotting Constant-Declination Curves

Triangle S_3NG is in the shadow plane. This triangle can be ignored if all you are interested in is delineating the hour lines but the triangle is required for delineating constant-declination curves other than the equinoctial line.

As the morning progresses, points S_1 and S_2 move towards *M*. Point S_3 moves towards *N* but it does not reach *N* which suggests that it cannot possibly reach the more distant point *M*. At noon, S_3 is at the point marked S_M, the outer end of the shortest broken line in triangle S_3NG. The length GS_M equates to the length of GS_2 at noon which is *GM*. In Figs 3 and 6 we see, in three dimensions, that face S_MNG folds flat against face *MNG* when S_M really does coincide with *M*.

When $\delta = 0°$ the shadow of the nodus runs along the straight equinoctial line S_2M which is at right angles to *GM*. At other declinations the shadow follows a curve (a hyperbolic arc) and, to set this out, we make use of the straight line *NS′* (in triangle S_3NG) which is offset at an angle δ to the line NS_3. In the figure, $\delta = 20°$ and point *S′* is where the line intersects GS_3. Note that *GS′* is the Lennox-Boyd 'y' value at 8 o'clock when $\delta = 20°$. At an equinox, when $\delta = 0°$, point *S′* coincides with S_3 and the 'y' value is GS_3.

Now GS_3 and GS_2 are the same line, the same edge of the tetrahedron, and in Fig. 7 point *S′* is marked on both GS_3 and GS_2 by a black spot. This is the shadow of the nodus at 8 o'clock when $\delta = 20°$.

As time moves towards noon, *S′* moves towards *N* and the changing values of $y = GS'$ give rise to the succession of black spots on the hour lines in triangle S_2MG. In this way the constant-declination arc for $\delta = 20°$ can be plotted on the dial plate.

In Fig. 5 the shadow triangle for $\delta = 20°$ at 9 o'clock is shown shaded and this same triangle is shown shaded in Fig. 7.

Fig. 7 can readily be adapted for other times of day. At 7 o'clock angle $h = 75°$ (five hours before noon) and although the associated triangles are not shown, the shadow of the nodus (when $\delta = 20°$) is drawn as a lone grey spot near the bottom of the figure. The corresponding position of this spot is shown off the end of the line *NS′* near the right-hand margin of the figure.

At 6 o'clock angle $h = 90°$ and all three points S_1, S_2 and S_3 are at infinity. The three outer triangles turn into long thin rectangles. The tetrahedron turns into an indefinitely long Toblerone™ packet! It is not possible to mark the shadow of the nodus at 6 o'clock when $\delta = 0°$ but it *is* possible to plot the shadow of the nodus at 6 o'clock when $\delta = 20°$. The relevant spots will not fit on the diagram but we know that one spot will be vertically below *G* (on the lawn this spot is due west of *G*) and the line from *G* to this spot is one half of the 6–6 line which is a characteristic feature of horizontal sundials. In the shadow plane the same spot will be where *NS′* intersects the line through *G* which is parallel to NS_3.

Notice that in the shadow plane, the spots are collinear; they all fall on the line *NS′*. In the horizontal plane, the spots are on a hyperbola. This is an interesting illustration of mapping a straight line into a conic.

Analysing the Triangles

A major advantage of opening the tetrahedron up into a net is that all the angles and linear dimensions that are needed when delineating the dial plate can be determined without any need to think in three dimensions.

We have seen how to draw the triangles; let us now look at the associated trigonometry, noting that right-angled triangles are particularly easy to analyse.

Triangle GNM in the meridian plane

Fig. 8 shows the triangle in Fig. 1 and we recall that the only known linear dimension is the nodus height which is assumed to be 1. The only angle given is ϕ, the local geographical latitude. From these two values, the red expressions for the lengths of the three sides of the triangle can easily be determined…

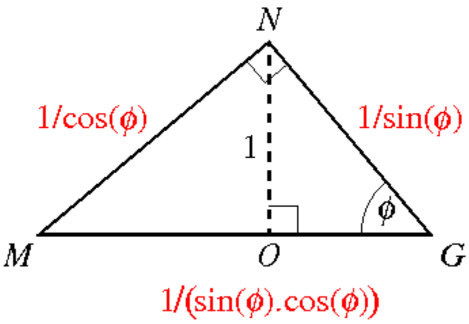


Fig. 8. Triangle GNM in the meridian plane.

First, note that $\sin(\phi) = NO/GN$ and since $NO = 1$ we have $GN = 1/\sin(\phi)$.

Also, $\cos(\phi) = GN/GM$ and since $GN = 1/\sin(\phi)$ we have $GM = 1/(\sin(\phi).\cos(\phi))$.

Further, $\sin(\phi) = MN/GM$ and since $GM = 1/(\sin(\phi).\cos(\phi))$ we have $MN = 1/\cos(\phi)$.

Triangle SMN in the equatorial plane

Fig. 9 shows *SMN*. We know side *MN* from the previous triangle and we know that angle *SMN* is a right angle. We also know that the angle $MNS = h$ where h is the hour angle which, in this analysis, corresponds to the time (in degrees) before noon. The lengths of the sides *SM* and *SN* can easily be determined…

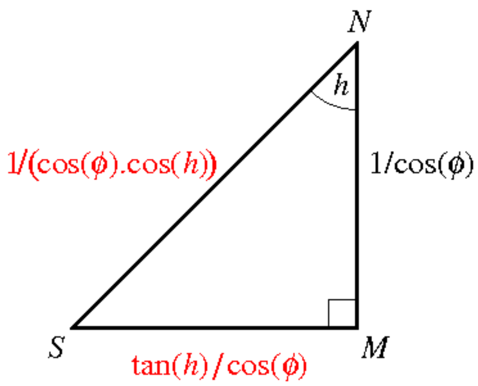


Fig. 9. Triangle SNM in the equatorial plane.

First, note that $\tan(h) = SM/MN$ and since $MN = 1/\cos(\phi)$ we have $SM = \tan(h)/\cos(\phi)$.

Also, $\cos(h) = MN/SN$ and since $MN = 1/\cos(\phi)$ we have $SN = 1/(\cos(\phi).\cos(h))$.

Triangle SMG in the horizontal plane – the dial plate

Fig. 10 shows *SMG* which is the embryonic dial plate. Point *S* marks the shadow of the nodus at hour angle h when the declination is 0°. The position of *S* could be specified in cartesian coordinates relative to some origin (perhaps the sub-nodus point) but it can also be specified in polar coordinates. Polar coordinates are conventionally specified as (r, θ) where r is some distance and θ is an angle relative to some reference direction. If the root of the gnomon *G* is taken as the origin and due north (the direction of *GM*) is taken as the reference direction then the polar coordinates of *S* are (GS, θ).

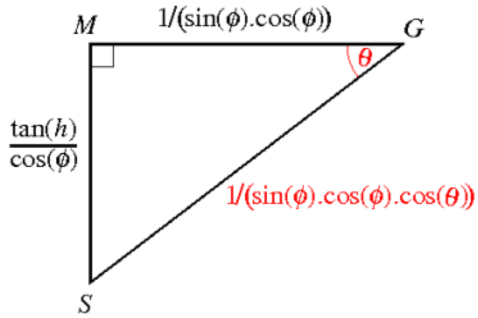


Fig. 10. Triangle SMG in the horizontal plane.

The angle θ (marked in the figure) is the *hour line angle*. In both the *BSS Glossary* and the Holker Hall article it is given as *X* but, to avoid confusion with the *X* coordinate used earlier, θ is used for the hour line angle here.

Now $\tan(\theta) = SM/GM = (\tan(h)/\cos(\phi)) / (1/(\sin(\phi).\cos(\phi)))$
$= \tan(h).\sin(\phi)$

Accordingly:

$$\theta = \arctan(\tan(h).\sin(\phi)) \qquad (1)$$

This is one of the best-known formulae in dialling.

With θ now known we can determine the equinoctial 'y' value, *GS*…

Note that $\cos(\theta) = GM/GS$ so:

$$GS = (1/(\sin(\phi).\cos(\phi))) / \cos(\theta) = 1/(\sin(\phi).\cos(\phi).\cos(\theta))$$

This expression is written against *GS* in Fig. 10.

At noon, the angle h reduces to zero so, from (1), θ also reduces to zero. In consequence, $\cos(\theta) = 1$ and the expression for *GS* reduces to the expression for *GM*, the equinoctial 'y' value at noon.

Triangle SNG in the shadow plane

Fig. 11 shows *SNG*; this is the triangle which incorporates the line *NS′* which is offset from line *NS* by the angle δ, the solar declination. In general, the shadow triangle is *S′NG* but when $\delta = 0°$ the entire triangle *SNG* is in shadow.

All three sides of *SNG* are known from analysing the three other triangles. The goal is to determine the general 'y' value which is the length *GS′*. To facilitate the analysis, the angle at *S* has been labelled k and the angle at *S′* has been labelled $\delta+k$ (this is the sum of the two opposite angles in the triangle $S'SN$). Also, a perpendicular from G to NS' has been introduced and it is noted that this makes an angle δ to GN.

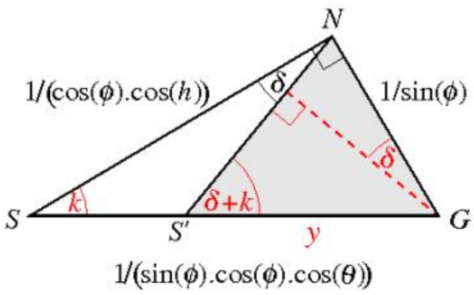


Fig. 11. Triangle SNG in the shadow plane.

Before determining the general 'y' value, it is useful to determine sin(k) and cos(k). From Fig. 11:

$$\sin(k) = NG/SG = \cos(\phi).\cos(\theta)$$

$$\cos(k) = \sqrt{1-\sin^2(k)} = \sqrt{1-\cos^2(\phi).\cos^2(\theta)}$$

Next we note that the length of the perpendicular from G can be expressed in two ways which equate one to the other:

$$y.\sin(\delta + k) = \frac{1}{\sin(\phi)}.\cos(\delta)$$

Rearranging and expanding sin($\delta + k$):

$$y = \frac{\cos(\delta)}{\sin(\phi).[\sin(\delta).\cos(k) + \cos(\delta).\sin(k)]}$$

$$= \frac{1}{\sin(\phi).[\tan(\delta).\cos(k) + \sin(k)]}$$

Now replace sin(k) and cos(k) with the expressions derived above:

$$y = \frac{1}{\sin(\phi).[\tan(\delta).\sqrt{1-\cos^2(\phi).\cos^2(\theta)} + \cos(\phi).\cos(\theta)]} \quad (2)$$

This is the general Lennox-Boyd 'y' formula exactly as used in the Holker Hall article but with ϕ, δ and θ used where Mark uses f, d and X and assigns the nodus height a more general value which replaces the '1' on the top line.

Exploiting the 'y' value

Fig. 12 is adapted from Fig. 5 and shows a plan view of the horizontal sundial. Point G is the root of the gnomon and, due north of G, point O is the sub-nodus point. Either may be used as the origin of a system of cartesian coordinates or polar coordinates. The points serve as gnomonic centres and it is important to know their separation, GO. From Fig. 1, tan(ϕ) = NO/GO so:

$$GO = NO/\tan(\phi) = 1/\tan(\phi) \quad (3)$$

In Fig. 12, X, Y axes are drawn through O, and X', Y' axes have been drawn through G. The Y and Y' axes coincide; these axes run due north through points G, O and M. The equinoctial line through M is drawn as a broken line. The line GS' is the shadow of the gnomon.

The length of the shadow GS' is the Lennox-Boyd 'y' value and the hour line angle is θ. Using G as the origin and GO as the reference direction (due north), point S' may be specified in polar coordinates as:

$$S' = (y,\theta)$$

To set out hour lines, simply calculate θ for different values of hour angle h using expression (1).

To set out the constant-declination curve for some declination δ, determine a sequence of points S'. Each point stems from a given hour angle h and, for each h, the hour line angle is calculated from (1) and the 'y' value from (2).

Point S′ can alternatively be specified in X', Y' coordinates as:

$$S' = (-y.\sin(\theta), y.\cos(\theta))$$

By shifting the origin from O to G, point S' can also be specified in X, Y coordinates as:

$$S' = (-y.\sin(\theta), y.\cos(\theta) - GO)$$
$$= (-y.\sin(\theta), y.\cos(\theta) - 1/\tan(\phi))$$

The X coordinate is the same as the X' coordinate but the Y coordinate is not the same as the Y' coordinate; to reflect the shift of origin from G to O, the separation GO, shown in (3), has to be subtracted from the Y coordinate.

Finally, using G as the origin and north as the reference direction, point S' may be specified in alternative polar coordinates as:

$$S' = (z,A)$$

where distance z and angle A are marked in red in Fig. 12 as is a new point P, the foot of the perpendicular from S' to the Y axis.

From the figure, $z^2 = S'P^2 + OP^2$ where $S'P = y.\sin(\theta)$ and $OP = y.\cos(\theta) - GO$. GO is given in (3) so:

$$z = \sqrt{y^2.\sin^2(\theta) + (y.\cos(\theta) - 1/\tan(\phi))^2}$$

Also from the figure, tan(A) = $S'P/OP$ so:

$$A = \arctan\left[\frac{y.\sin(\theta)}{(y.\cos(\theta) - 1/\tan(\phi))}\right]$$

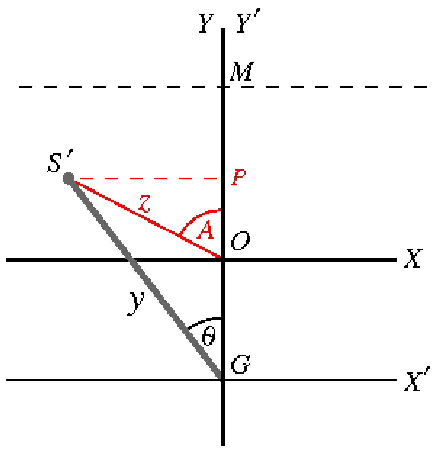


Fig. 12. The embryonic dial plate. GS′ is the shadow of the gnomon.

Both these expressions are given in the Holker Hall article and z might, perhaps, be called the Lennox-Boyd 'z' value.

A is well known to diallists as the Azimuth. If the nodus were on a vertical support, then A would be the azimuth (or bearing) of the shadow of the support.

Throughout this article, the nodus height has been assumed to be 1. A bug resting at S' would note that the nodus was directly in its line of sight to the sun. If a is the altitude of the sun then $\tan(a) = 1/z$ so:

$$a = \arctan(1/z)$$

The altitude a is not used in the Holker Hall article, instead the zenith distance ($90° - a$) is used:

$$\text{zenith distance} = \arctan(z)$$

Summary

There are several ways of deriving the altitude and azimuth of the sun and there are several ways of plotting constant-declination curves and this article demonstrates that exploiting the Lennox-Boyd 'y' value can be added to the list of possibilities.

ACKNOWLEDGEMENT

Using the distance of the shadow of a gnomon-mounted nodus from the root of the gnomon was a novel approach to me. I am most grateful to Mark Lennox-Boyd for introducing me to it since explaining its derivation allows for much pedagogic interest such as the gnomonic tetrahedron. I am also grateful to Mark for his helpful comments on the first draft of this article.

REFERENCE

1. Mark Lennox-Boyd: 'A Scaphe Dial for Holker Hall', *BSS Bulletin*, 32(iv), 16-18 (December 2020).

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READERS' LETTERS (2)

Thomas Stringer

I thought I would try to find out about Thomas Stringer, the dedicatee of the book referred to inside the front cover of the September 2020 *Bulletin*. He was steward to the late and not much lamented first Earl of Shaftesbury. He was involved in a complicated dispute over a portrait of John Locke, possibly the one now in the National Portrait Gallery. His family archives are in the Hampshire Archives, although I think he himself died in Alderbury in 1702 as there is a memorial to him in St Mary's Church there. He and his wife moved to Ivychurch House (formerly Ivychurch Priory), Alderbury, near Salisbury in the 1680s. I have not been able to find his coat of arms, but I bet John Davis is right that those are his.

I think the classification of dyalls on that page is interesting. It looks very much like a 'mind map' which Tony Buzan seems to claim to have invented.

Since writing the above, I have also found some recent research on Thomas. His father was Sir Thomas Stringer who, according to the History of Parliament, adopted the arms of the Stringer family of Yorkshire, although it seems unclear if he was actually entitled to them. The only Stringers in Yorkshire with arms that I have found had eagles, not ravens, so that does not help.

However, I have now found a great deal about our Thomas. He was very well connected and had been very wealthy at one time. He helped Shaftesbury (the third earl, sometime Lord Chancellor) manage his extensive estates in America and, although an employee, was treated by Shaftesbury as a friend. I note that Blome's works include his 1687 *Isles and Territories of America*, for Shaftesbury.

But I have seen much of Blome's book and it is clear that each section was dedicated to some other benefactor. For instance, the Cosmography and Astrology section was dedicated to the Earl of Castlemaine. So I do not think Stringer was anything but a sponsor.

On the subject of sundials he seems to have been very well informed. He goes on to state:

> "In respect of the *Hours, Dyals* are divided in *Astronomical, Italick, Babilonick, Antient,* or *Judaick.*
>
> "The *Astronomical* declares equal *Hours* from *Noon* to *Midnight,* and from *Midnight* to *Noon* again; and it is principally in use all over *Europe.*
>
> "The *Italick* reckons equal *Hours* from *West* to *West,* so as that of the 24 *Hours,* that is said to be the first *Hour* which is the *Hour* of the *Suns setting,* and this would be called by us the *6th, 7th,* or *8th Evening Hour,* whereas on the contrary the *Babilonick* reckons equal *Hours* from *Sun-rising* to *Sun-rising.*
>
> "The *Ancient,* or *Babilonick* reckons unequal *Hours,* as *Chronology* shews more at large. I shall therefore treat only of *Astronomical Dyals,* because the knowledge of them leads to all the rest."

This is followed by many pages of instructions for creating sundials. I found the text at:

https://quod.lib.umich.edu/e/eebo/A28396.0001.001/1:29?rgn=div1;view=fulltext

The whole book is at:

https://quod.lib.umich.edu/e/eebo/A28396.0001.001?view=toc

Chris Lusby Taylor