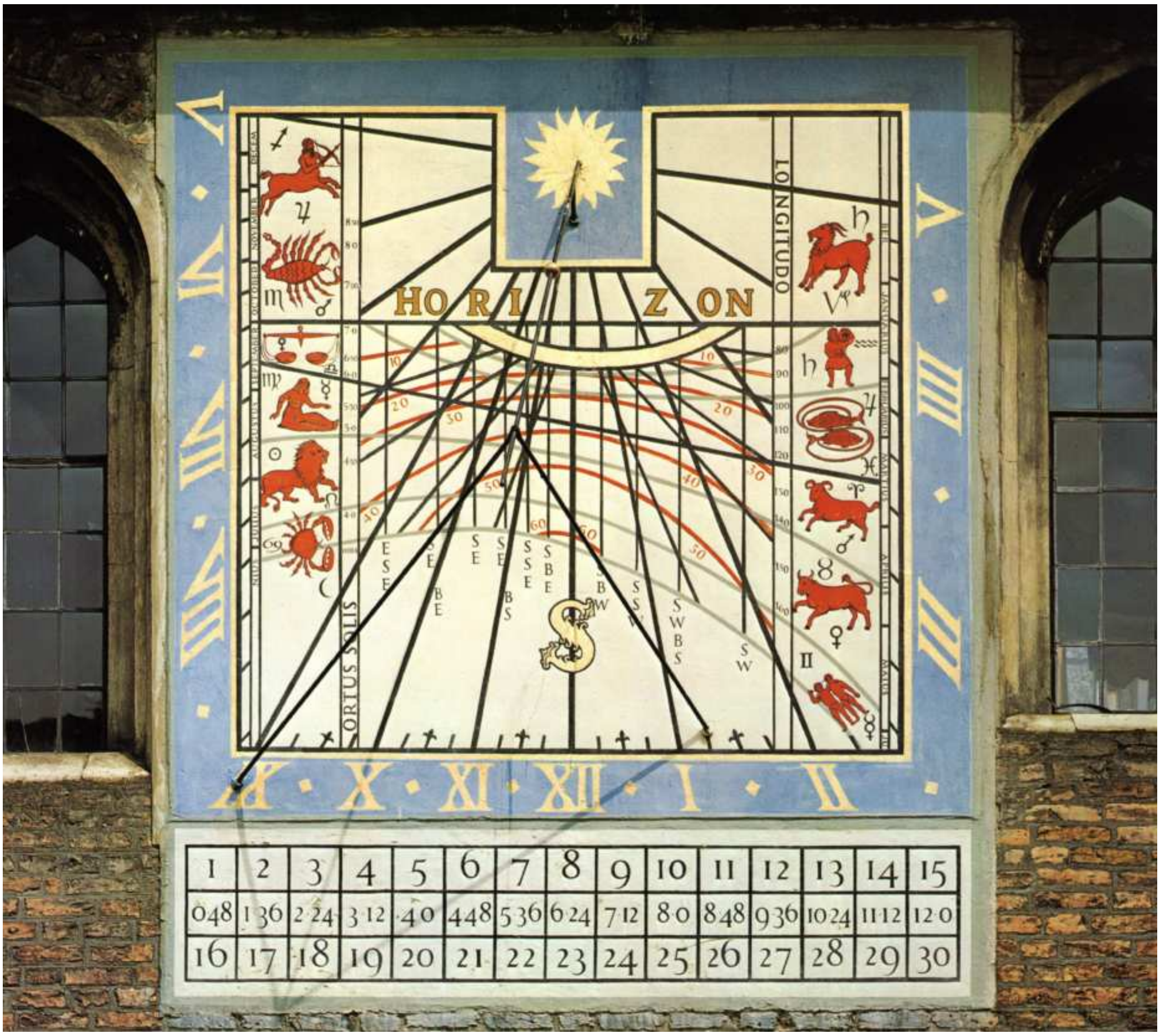
THE QUEENS' DIAL

REFURBISHMENT CONSIDERATIONS



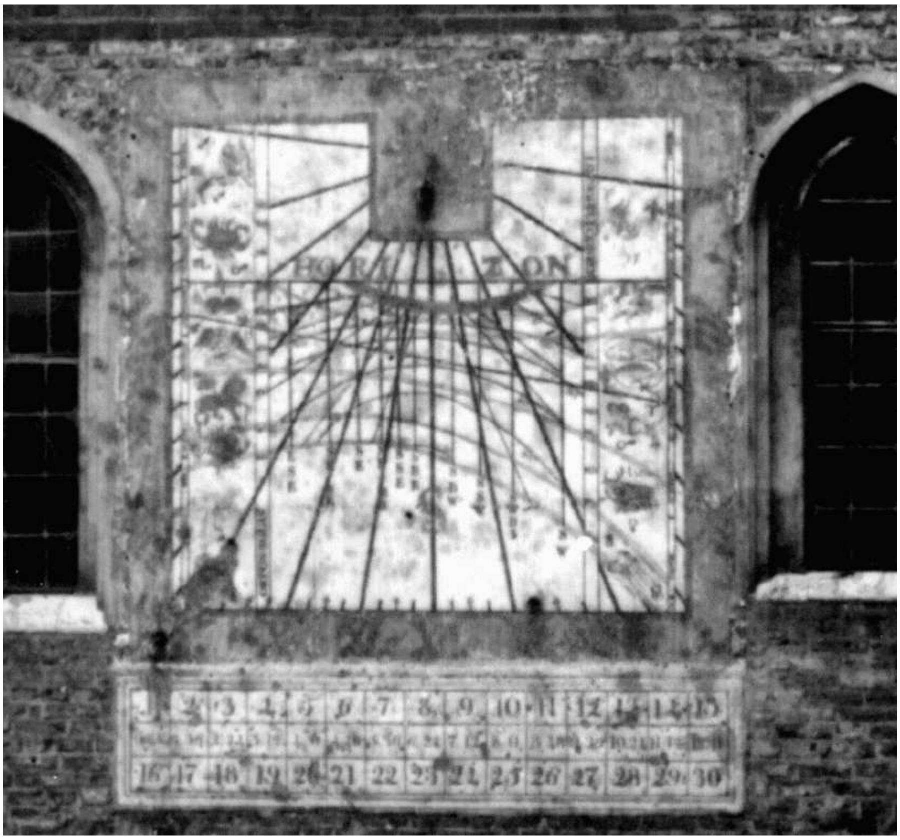
The 1971 Version of the Queen's Dial (Photograph: Cambridge University Press)

Frank H. King

August 2006

The Oldest Known Photograph

The image below is the oldest known photograph of the Queens' Dial. It is taken from a large photograph of Old Court and has been lightly processed by Robin Walker of Queens' who notes that it 'must have been taken before the 1864 re-painting'.



Photograph supplied by Robin Walker

Cover Photograph

A view of the 1971 version of the Queens' Dial which was the illustration for June in the Cambridge University Press Calendar for 1980. The photograph was taken about 10 o'clock (sun time) on a day close to the Summer Solstice, probably in 1979. The elliptical shadow of the nodus is just visible on the summer solstice curve close to where the 10 o'clock hour line intersects this curve.

Foreword

The Queens' Dial in Old Court dates from 1642 and is one of the best known sundials in the British Isles. It is rich in features and ornamentation.

It was most recently painted in 1971 and a new repainting is scheduled for 2006–07. This provides an opportunity to review the details of the dial and to correct errors.

Two Computer Models

At each repainting, the artist begins afresh with a blank area of rendered wall and builds up the dial in stages. Hundreds of decisions are made. For example, even when a line is well defined mathematically, one has to decide precisely what colour to use, how thick to paint the line, where the end-points are to be and whether it is to be painted over or under other lines that it crosses.

The figures in this document are from a computer model constructed *before* work begins on a repainting in 2006–07. It is hoped to prepare a second document describing a *post hoc* computer model which closely represents the result of the repainting.

The model described here has been constructed in advance of any survey. The orientation of the dial is not known to high precision and various measurements are known only approximately too. This draft is therefore based on estimates which can be refined later.

The model tidies up some gross errors but provisionally preserves some slight oddities noted in the 1971 version. Attention will be drawn to these matters and other limitations.

This document serves three purposes…

1. It serves as a guide to the artist showing the steps that might be used in setting out and painting the dial.
2. It serves as a discussion document so that interested parties can comment on the proposals before they are implemented.
3. With its *post hoc* companion, this document might be kept with the dial archives so those responsible for future repaintings will be able to see the kinds of issue that were considered in the 2006–07 repainting.

Outline Description

The cover photograph shows the dial a few years after its most recent painting in 1971. Diallists (those who study sundials) describe this as a vertical wall dial declining about 13° east of due south.

The sundial is equipped with a polar-oriented gnomon on which there is a small ball nodus. Local sun time is indicated by noting where the shadow of the gnomon falls within the hour lines and the half- and quarter-hour tick marks.

Numerous other features of gnomonic interest are indicated by noting where the shadow of the nodus falls on the dial. These will be described below. They are also well documented in:

`http://www.queens.cam.ac.uk/queens/Misc/Dial.html`

Step 1 — Preparation and the Root-of-Gnomon Origin

Fig. 1 shows a minimalist representation of the Queens' Dial at a 1:20 scale. Like all figures in this document, it should be viewed alongside the cover photograph.

The dashed line corresponds to the outer perimeter of the blue border which serves as the chapter ring (which carries the Roman numeral hour labels) and also runs across the top of the dial. This perimeter does not include the moon dial table. The dot near the top of the figure marks the root of the gnomon.

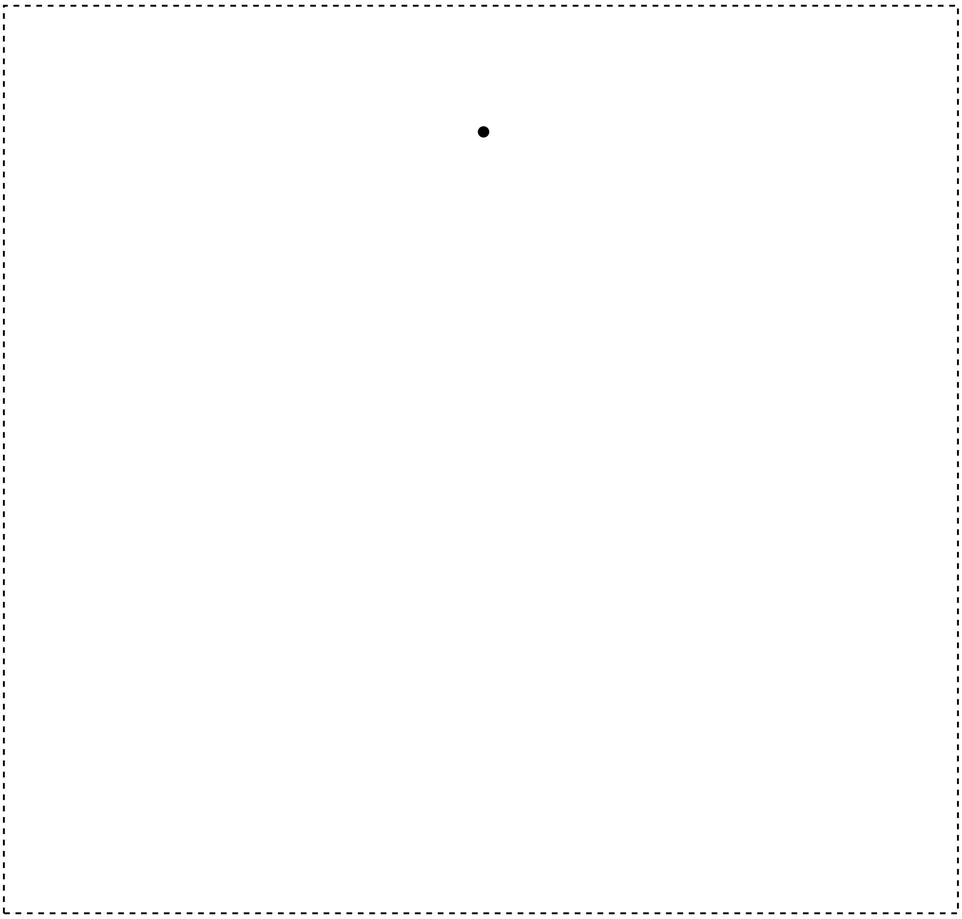


Fig. 1 — The Root-of-Gnomon Origin and the Dial Perimeter

Before this outline can be drawn as a pencil line, a good deal of preparation is required. The integrity of the rendering will need inspecting, the surface must be prepared for painting and the dial area will need careful surveying. Following the survey, proper measurements can be presented, mostly on spreadsheets, providing sufficient information for plotting out the mathematically defined features.

Such preparatory work is largely outside the scope of this document which is intentionally low in mathematical and numerical content. The computer model used in this document has been prepared in advance of a survey and the numerical values used are estimates only.

Dimensions

On the present dial, the perimeter runs a few millimetres inside the bevelled edges of the rendered area of wall. The extent of the rendering is itself limited by the stonework of the windows on either side of the dial.

Even if it were replaced, the rendering could not practically be any wider and it would not make sense to have it any narrower. The constraints at the top and bottom are almost as strong. There seems no good reason to change the extent of the rendered area. The perimeter therefore is essentially determined by the site and its precise dimensions must be established by survey.

The computer model that produced Fig. 1 used a rectangular system of coordinates with the root of the gnomon as the origin. The positive X-axis runs horizontally to the right and the positive Y-axis runs vertically upwards. If the wall has a detectable lean (and even modern walls lean!) then, strictly, the Y-axis will be a line of greatest slope rather than vertical. In the model:

- The abscissae of the left- and right-hand edges are $-1261.0\,\text{mm}$ and $+1247.1\,\text{mm}$.
- The ordinates of the top and bottom edges are $+334.2\,\text{mm}$ and $-2070.5\,\text{mm}$.

These figures are estimated from photographs and the implied precision is certainly not to be taken seriously. Better values will be available after the wall is surveyed.

The root of the gnomon is a little nearer the right-hand margin than the left-hand margin. Correcting this imbalance would require repositioning the gnomon but the asymmetry may be deliberate. There are more hour-lines on the left than on the right and this asymmetry gives them a little more space.

The Dial Surface

An early goal is to have a freshly prepared white surface which extends almost up to the perimeter. Many construction lines and survey points will be marked on this surface which, later, will later serve as the background for most of the dial furniture.

In principle, the perimeter can be marked out from the root of the gnomon using a steel tape and a spirit level but accurately setting out vertical lines 2.5 m long is astonishingly difficult. Spirit levels have a quoted accuracy of about 1 mm in 1 m. Over a distance of 2.5 m one may expect an error of the order of 2.5 mm.

Setting out is much easier if a grid has been first been marked out using a good surveying instrument but the grid has to be drawn on the white surface. This chicken-and-egg scenario can readily be resolved. The white surface is painted first, comfortably within the bevelled margins of the rendered surface. The grid is marked on this (see Step 2) and the precise perimeter can then be drawn using the grid.

The blue border is nearly 200 mm wide. The margin of the white surface can lie anywhere inside this border and therefore be quite ragged. The outer part of this area of white will be overpainted by the blue border later.

The Plan for Step 1

There are a couple of queries. Should the gnomon be repositioned? Should the rendering be replaced? After resolving these matters the white surface can be prepared.

Step 2 — The 500 mm Grid and the Sub-Nodus Point

Fig. 2 shows the grid. There are five vertical and five horizontal lines spaced at 500 mm intervals. The origin of this grid is *not* the root-of-gnomon origin and some explanation is merited...

The centre-line of the shadow of the gnomon is a line whose position indicates hours and sub-divisions of hours and nothing else. The centre of the shadow of the nodus is a point whose position indicates numerous other metrics. Accordingly, in dialling calculations, the nodus is a much more commonly used origin than the root of the gnomon.

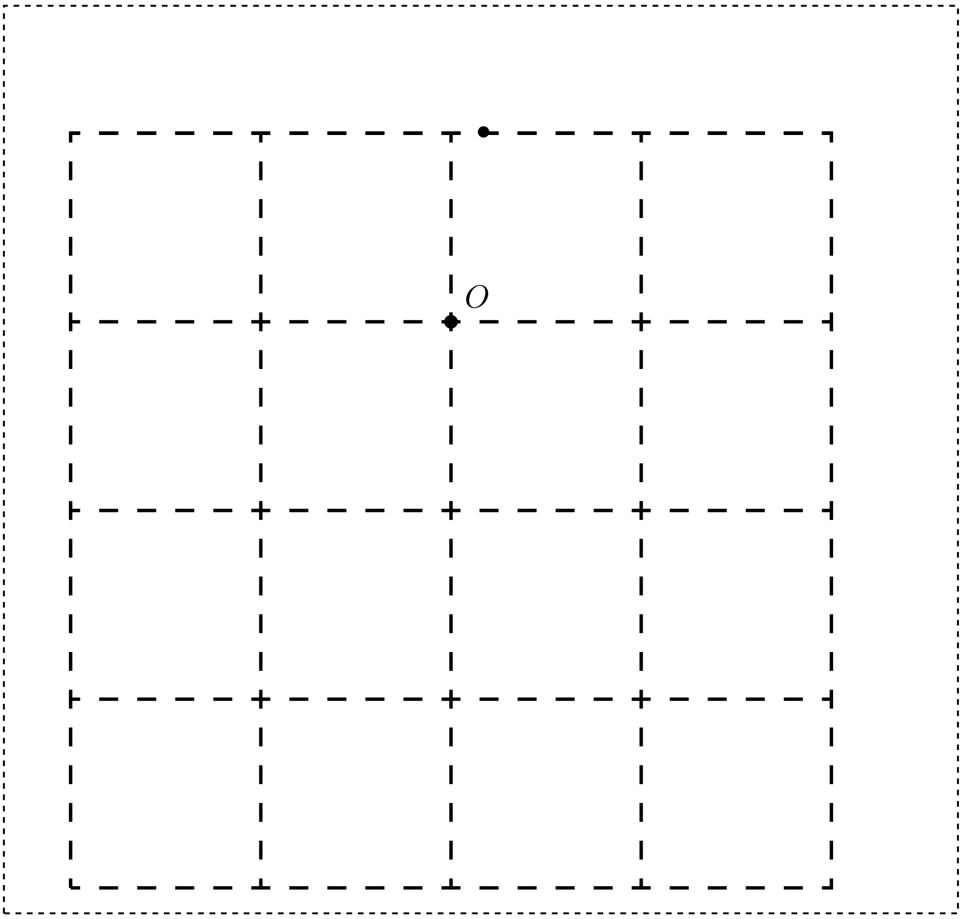


Fig. 2 — A Grid based on the Sub-Nodus Point

The nodus is not in the plane of the dial so a third coordinate, a Z-coordinate, is needed to specify the positions of points on the dial relative to the nodus. If the direction of the Z-axis is normal to the plane of the dial (and if the dial is a true plane) then the Z-coordinate of every point on the dial will be the same and can therefore be ignored.

The foot of the perpendicular from the nodus to the plane of the dial is known as the sub-nodus point. This is the origin used in the grid in Fig. 2 where it is marked by a large dot labelled O. Ignoring the Z-coordinate, the coordinates of O are $(0, 0)$.

What if the dial is not vertical?

If the plane of the dial is not vertical, diallists still use the foot of the perpendicular from the nodus to the dial as the origin but the origin on the dial and the centre of the nodus are no longer on the same horizontal level. This gives rise to a practical problem…

Surveyors, surveying instruments and surveying software all work in terms of horizontal levels. Surveyors see the grid in Fig. 2 in terms of an ideal 2m-square grid which is perfectly vertical and whose origin is the centre of the nodus itself. The 25 intersections in Fig. 2 are horizontal projections of the intersections of the perfect grid onto the dial.

Typically the projections will have different Z-coordinates, reflecting both the deviation of the dial from a true plane and the inclination of the dial to the vertical. Analysing these Z-coordinates will be an early task.

In practice, surveyors begin by establishing a horizontal line on the dial which is at the same horizontal level as the centre of the nodus. In dialling terminology, this is the horizon line. In the case of the Queens' Dial, it is clearly labelled as such in gold lettering!

The surveyors' origin on the dial is the foot of the perpendicular from the nodus to the horizon line which will not, in general, coincide with the foot of the perpendicular from the nodus to the plane. Horizontal gridlines are marked out at regular intervals above and (mostly) below the horizon line. Unless the dial is vertical (the assumption used in Fig. 2) diallists typically work from a grid which is slightly offset from what they would like!

There is a mathematical relationship between the root-of-gnomon origin and the sub-nodus point. In the present computer model, the root-of-gnomon origin is displaced about 86 mm to the right of the sub-nodus point and it is displaced about 503 mm higher. Inspection of Fig. 2 shows that the root-of-nodus origin is slightly above the 500 mm gridline.

Setting out the perimeter and, later, the chapter ring and dial outline are the few occasions that the root-of-gnomon origin is the more natural to use. One might be tempted to mark out a second grid but it is less confusing to work from the grid in Fig. 2 and add or subtract the displacements just noted as required.

Given these displacements, and the abscissae and ordinate values noted in Step 1, the specification of the perimeter now is:

- The abscissae of the left- and right-hand edges are −1175.0 mm and +1333.1 mm.
- The ordinates of the top and bottom edges are +837.2 mm and −1567.5 mm.

The Plan for Step 2

There are several queries. By how much does the dial surface deviate from the best-fit plane? What is the orientation of the best-fit plane? What is the ortho-style distance (the perpendicular distance from the centre of the nodus to the best-fit plane)?

Resolving these matters will include analysing the full survey data for the 25 intersections. The 500 mm grid can then be marked out on the freshly prepared white surface. The grid will be retained almost until the painting is complete. Most of the dial features will be plotted out via this grid.

It is accepted that the horizon line, the reference horizontal line in the grid, may not pass precisely through the sub-nodus point.

Step 3 — Integrity Check using a Test Hyperbola

One of the requirements of the survey is to determine the azimuth of the dial, the direction it faces relative to due north. The surveyors have been asked to determine the azimuth of the line of best fit through the five intersection points on the horizon line. This is about 77°, so the azimuth of the outward normal to the line of best fit is about 167°.

When the azimuth is known more accurately and the survey data for all 25 intersection points have been analysed, the ortho-style distance and the orientation of the best-fit plane will be known to high precision. One can then calculate where the shadow of the nodus should fall on the dial at different hour angles and declinations of the sun and compare the calculated positions with observed positions.

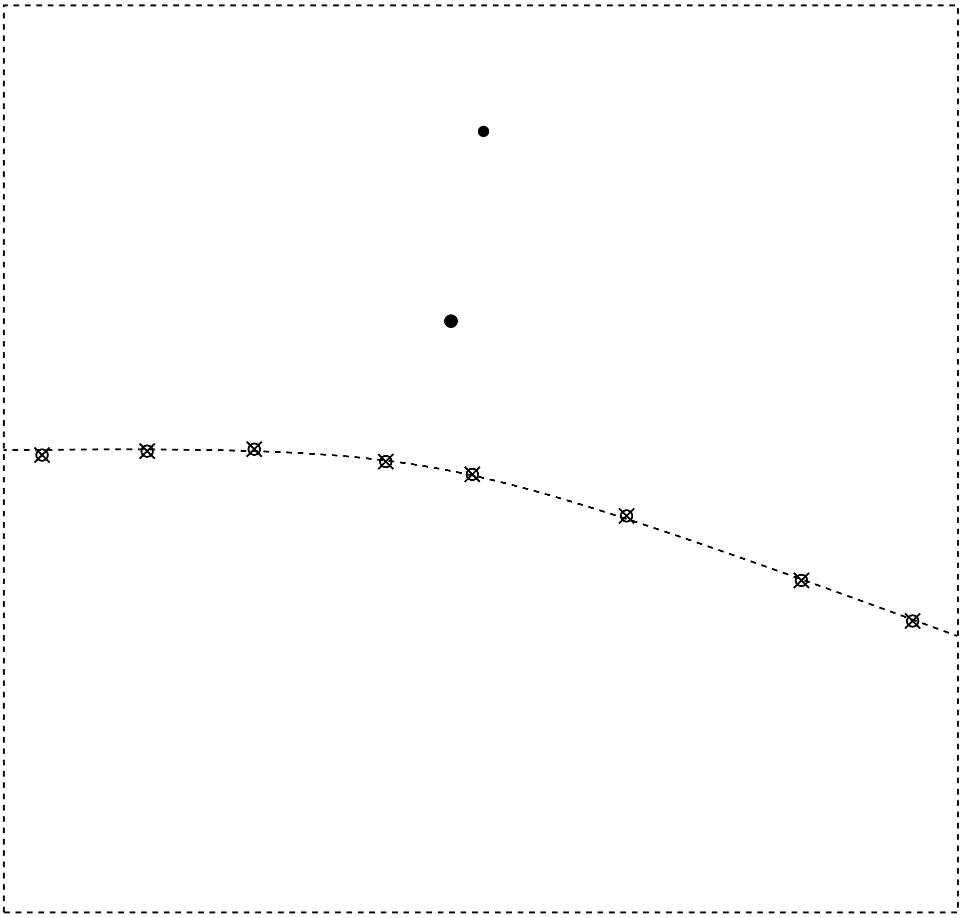


Fig. 3 — A Test Constant-Declination Curve

During the course of a day, the shadow of the nodus traces a path across the face of the dial. If the dial is a true plane, the shape of the path will (at Queens') always be a close approximation to a hyperbolic arc. The shape depends on the declination of the sun which is continually changing but the rate of change is so slow that one can usually assume that the declination is constant over a given day.

Integrity Check

Fig. 3 illustrates an experiment that serves as a check on the computer model which is set up after the survey data have been analysed. The model initially equates the dial to the best-fit plane and uses the ortho-style distance determined from the Z-coordinates.

The root-of-gnomon origin and the sub-nodus point are shown as isolated dots but, to avoid cluttering the figure, the grid has been omitted. The grid should, nevertheless, still be on the dial itself.

The eight circled crosses mark the positions of the shadow of the nodus *as observed* at intervals over a period of about eight hours. The coordinates of each mark are determined from the grid which is used as giant graph paper. The time of each observation has to be recorded too. Clearly a sunny day is required.

It is easier to make the observations via photography than by hand. One might also hope to make rather more than eight observations.

For each observation, the time and the observed coordinates are entered into a spreadsheet. The calculated coordinates are recorded on the spreadsheet too. The observed coordinates and the calculated coordinates are then plotted on a chart.

Fig. 3 is really more a partial presentation of the results than a depiction of the dial itself. The observed positions are shown. The calculated positions are not shown but the dashed line is a calculated trace showing the path that the shadow would follow if the dial were the best-fit plane and if the assumptions made about its orientation and about the ortho-style distance were correct.

The principal errors are likely to be observational. It is hard to estimate the position of the centre of the shadow of the nodus very accurately. These random errors result in the observed positions being displaced from where calculation suggests they ought to be.

More serious are systematic errors. If most or all the observed positions are below the dashed line then the model is incorrect. The ortho-style distance may be greater than estimated or, perhaps, the dial may be leaning forward more than the survey data suggests.

In the somewhat contrived example in Fig. 3, there is a hint that the observed positions near the beginning and end of the trace are systematically too low but elsewhere they are almost exactly on the calculated trace. This might be because the dial is slightly bowed.

The purpose of Step 3 is to gain confidence in the parameters of the best-fit plane and the estimate for the ortho-style distance. The parameters are the latitude (readily determined), the azimuth of the plane, and its inclination to the vertical. These latter two values and the ortho-style distance are determined from the survey data.

The Plan for Step 3

There are a couple of queries. What systematic errors are noted from observing the shadow of the nodus? What adjustments must be made to the computer model to correct them? Resolving these matters will probably require adjusting the model so that it takes account of the deviation of the dial from a true plane.

If the dial surface is badly bowed or otherwise irregular, the ortho-style distance has to be treated as a variable rather than as a constant. The computer model thereby becomes more complex but it can be used with more confidence.

Step 4 — Integrity Check using Selected Hour Lines

The gnomon has received little attention. Ideally, the gnomon should be parallel to the Earth's axis and thereby lie along the line that runs from the south celestial pole through the nodus to the north celestial pole.

The alignment of the gnomon is determined by the positions of the centre of the nodus and the root of the gnomon. Mathematically, it makes sense to start with the nodus and then specify where the polar-oriented line through the nodus intersects the dial. That point of intersection is where the root of the gnomon should be.

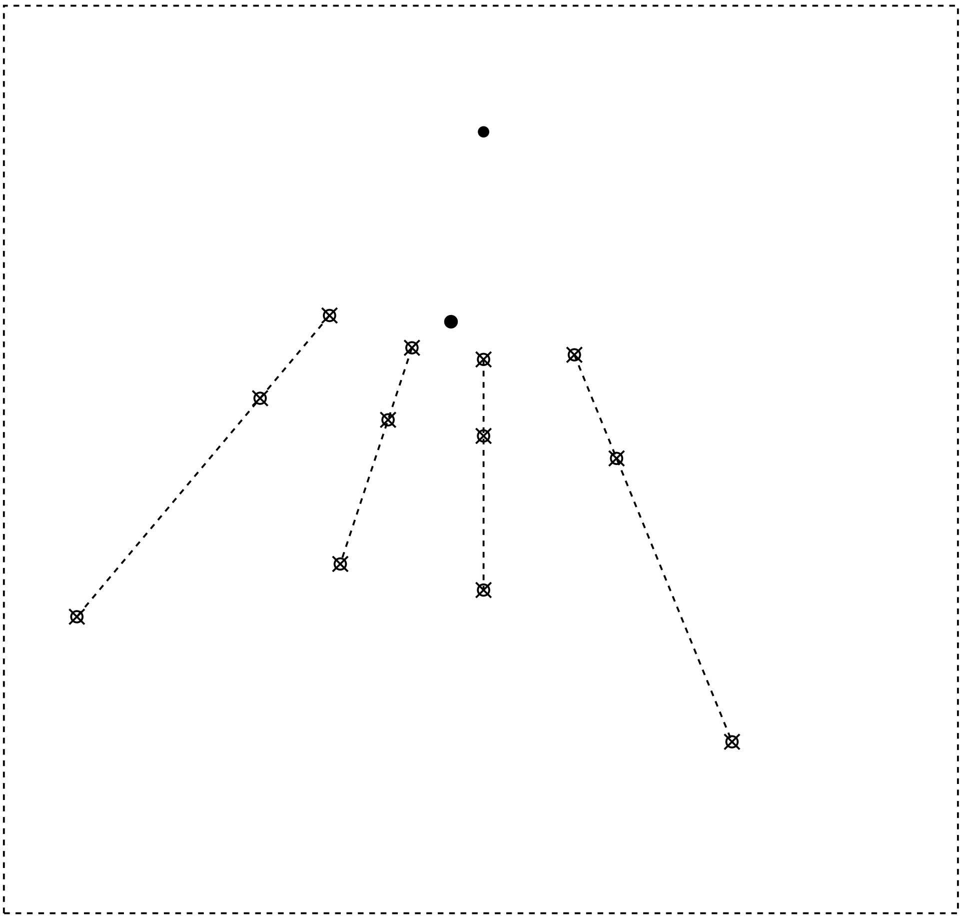


Fig. 4 — Four selected hour lines

In the simple model used in this document, the coordinates of the root of the gnomon relative to the sub-nodus point have already been given (using millimetres) as $(+86, +503)$.

Using the refined model based on the survey data, the coordinates can be specified more accurately and, using the grid, the calculated position can be noted on the dial. A first check is to verify just how close the actual root of the gnomon is to the calculated position.

Any error is likely to reflect an error in the alignment of the gnomon but it is possible that the model needs further refinement. A further check is merited…

Integrity Check

Fig. 4 illustrates an alternative way of checking the alignment of the gnomon. This figure again shows the root-of-gnomon origin and the sub-nodus point as isolated dots. As in Fig. 3 the grid has been omitted to avoid cluttering the figure.

There are also 12 circled crosses arranged as four triplets. The three circled crosses in each triplet mark the calculated positions of the shadow of the nodus at a given hour angle for three different declinations of the sun. Each circled cross is marked in its calculated position on the dial, again using the grid as giant graph paper.

The chosen declinations are those of the sun at the winter solstice, the equinoxes and the summer solstice. The chosen hour angles correspond to 8, 10, 12 and 2 o'clock sun time.

The dashed line through each triplet is a section of a line of constant hour angle and should (later) coincide with part of the corresponding hour line that radiates from the root-of-gnomon origin. If extended upwards, these dashed lines should meet at a common point and that point should be the true root-of-gnomon origin.

Mathematically, this common point is the position of the shadow of the nodus when the sun has a hypothetical declination of $-90°$. The sun would then be at the south celestial pole and the line from the sun through the nodus would run on to the north celestial pole, intersecting the dial at the root-of-gnomon origin on the way.

In practice, the dashed lines are likely to meet close to some common point which, in turn, is fairly close to the root of the gnomon. Any separation of the common point from the actual root of the gnomon is again likely to reflect an error in the alignment of the gnomon.

Note that the gnomon could almost be dispensed with. The time of day can readily be determined by drawing more sections of hour lines on the dial and then noting the position of the shadow of the nodus relative to the two nearest such line-sections.

Accommodating Alignment Error

If the alignment of the gnomon is seriously in error (and there is no reason to believe that it is) then it should be realigned. If the error is small then the gnomon can be left undisturbed. If the rendering is replaced, the gnomon will probably have to be set up from scratch anyway and a revised alignment error must be assessed.

Some approximating will necessarily be required when the hour lines are drawn. It looks bad if the shadow of the gnomon crosses an hour line. To avoid this, each hour line should radiate from the root of the gnomon even if it is not exactly at the correct angle.

Taking the 10 o'clock hour line as an example, one strategy would be to draw a line from the real root of the gnomon through the central circled cross on the dashed line for 10 o'clock. This new line may be slightly at an angle to the dashed line but it will, at least, radiate from the root of the gnomon. At the equinoxes, the correct time will be indicated. Small errors in indicated time have to be accepted at other times of year.

The Plan for Step 4

There is just one query. How well aligned is the gnomon? Resolving this matter may require the gnomon to be realigned. Whatever the outcome, alignment error will have to be taken into account when drawing the hour lines in Step 13.

Step 5 — The Chapter Ring and Heraldic Tinctures

This is an appropriate stage to start discussing the painting but the suggestions made here are not to be taken as too prescriptive. Whoever carries out the painting will have sound views on how to order the work.

Fig. 5 shows the blue border and this includes a roughly-square spur which carries a stylized sun centred on the root of the real gnomon. Two lines, one yellow and the other black, emphasize the inside of the border.

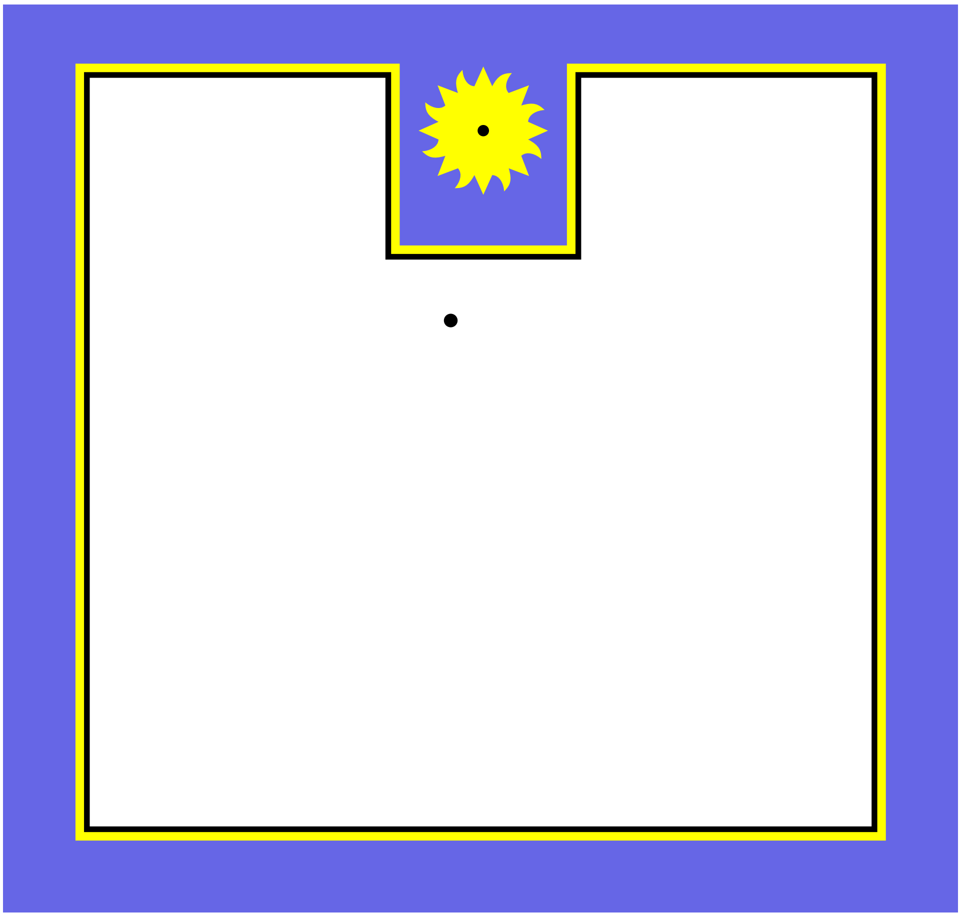


Fig. 5 — The Chapter Ring and the Stylized Sun

The grid is again omitted from the figure but is still on the dial. It will be used to draw the outlines of the blue border and the yellow and black edging. Some of the grid will be painted over by this border but the affected parts of the grid will by then have served their purpose.

The positioning of the perimeter has already been discussed. The width of the blue border can be taken from the 1971 dial. It is narrower at the top than at the sides and bottom. The widths of the yellow and black lines can likewise be taken from the 1971 dial.

Left–Right Symmetry

There are five considerations relating to the root-of-gnomon point:

1. This point is the centre of the root of the real gnomon.
2. The hour lines must radiate from this point.
3. The stylized sun should be centred on this point.
4. The point should be on the vertical centre-line of the roughly-square spur which carries the stylized sun.
5. The point should be on the vertical centre-line which lies mid-way between the vertical edges of the outer perimeter.

The first two are hard constraints. The third and fourth are clearly desirable aesthetically. The fifth consideration cannot readily be achieved but, fortunately, is not important.

It is clear from only the briefest study of the cover photograph that the 1971 version fails. The hour lines do not radiate from the root of the gnomon, the stylized sun is not centred on this point, and this point is not on the vertical centre-line of the roughly-square spur.

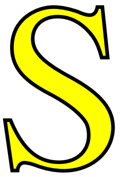
These well-known errors are easy to correct without any need to move the gnomon. The fifth consideration cannot be satisfied because the root of the gnomon is appreciably closer to the right-hand edge of the dial than it is to the left-hand edge.

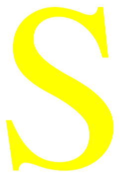
An Aside — Heraldic Tinctures

As is common on sundials and clocks, the Queens' dial uses heraldic colours. More strictly, the dial uses six heraldic tinctures. This generic term distinguishes metals (gold and silver, depicted as yellow and white in the figures) from colours (blue, red, black and green). Heraldic tinctures also include furs but none are used on the dial.

A general heraldic rule is 'never have a metal on a metal or a colour on a colour'. The gold (metal) sun on the blue (colour) background is correct. The gold lettering for HORIZON and the huge S for South on the silver (white) background is not heraldically correct. Interestingly, and perhaps intentionally, this violation of the rule echoes the Jerusalem quarter of the Queens' arms which has gold crosses potent on a silver field.

On the dial, there is a fudge. The gold lettering is edged in black so one has a metal on a colour on a metal. It is interesting to see what happens if the edging is removed:





Without the black edging the S doesn't show up. The gold is almost wasted; it is the edging that makes the letter stand out not the gold. By contrast, the stylized gold sun does not need black edging. It stands on a colour.

The Plan for Step 5

An expert should be consulted to resolve various queries about what kind of paint to use. The border can then be set out and painted and the stylized sun can be added.

Step 6 — The Butterfly-Shaped Region

Most of the mathematical lines on a sundial have indefinite extent. Each hyperbola runs from $-\infty$ to $+\infty$. Real sundials have practical limits. Lines are usually bounded by some kind of frame. On the Queens' Dial, the black edging which borders the central white area forms the major frame. No mathematical lines extend outside this frame.

A brief study of the cover photograph reveals that there are several minor frames to consider. In particular, there are four pairs of vertical tram-lines that confine different lines in different ways.

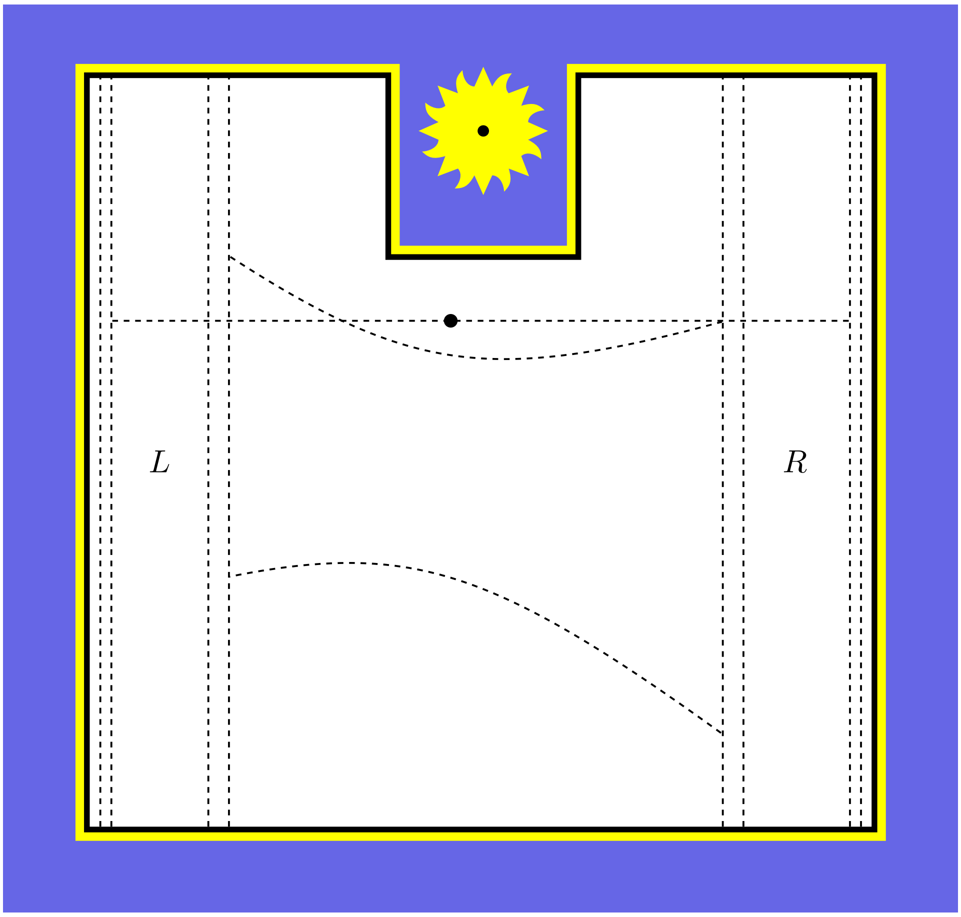


Fig. 6 — The Butterfly-Shaped Region

Fig. 6 shows the four pairs of tram-lines and a number of other framing lines including the horizon line. At this stage they are all drawn as pencil lines but most will be painted over as proper lines later.

Two sections of the winter and summer solstice curves have been pencilled in as framing lines. Diallists sometimes refer to a region framed by these curves and two vertical framing lines as a butterfly-shaped region. On the Queens' Dial the relevant vertical framing lines are the inner margins of the broad tram-lines.

Three Curiosities

The widths and the positions of the tram-lines and the thicknesses of the individual lines as they will later be painted can be determined from the 1971 version of the dial.

Most of the grid still remains and this should be used to mark out all the vertical lines.

The winter and summer solstice lines will be presented as sets of points on a spreadsheet. Using the grid, these points can be marked on the dial and then joined together using a length of bendy wood. These curves represent natural boundaries on various families of curves (for example the constant-altitude curves) because the declination of the sun is bounded by the declinations at the winter and summer solstices.

The horizon line is the principal horizontal line of the grid and has therefore already been drawn. This is another natural boundary. The sun cannot shine on the dial when it is below the horizon so the shadow of the nodus cannot fall anywhere above the horizon line. This boundary is taken as a weak constraint. Occasionally some features are shown above the horizon line, notably three Signs of the Zodiac on the Queens' Dial.

There are three curiosities:

1. The winter solstice curve does not feature in the 1971 dial except, perhaps, as a time of sunrise. Where the majority of the winter solstice curve should be there is, instead, a golden arc of no gnomonic significance. There will be further discussion about this.
2. The tiny lengths of horizontal lines in the narrow tram-lines are month separators which really ought to be tiny lengths of constant-declination curves. There will be further discussion about these lines too.
3. The placing of the broad tram-lines in Fig. 6 closely matches the placing on the 1971 dial. It is strange that the gap marked L between the narrow and broad tram-lines on the left is narrower than the gap marked R between the broad and narrow tram-lines on the right. This difference is especially strange given that there is less space on the right. The broad tram-lines on the right are therefore noticeably closer to the centre than are those on the left.

The gaps labelled L and R are where the Signs of the Zodiac will be painted and there seems no good reason for these gaps to have different widths. Moreover, shifting the broad tram-lines on the right slightly further to the right will make it easier to accommodate the LONGITUDO value 16 32 whose space (see Fig. 18) is otherwise intruded upon by the 2 o'clock hour line.

The Plan for Step 6

There are several queries. Where should the four pairs of tram-lines be placed? How wide should the gaps that they enclose be? Should the two gaps labelled L and R be the same width?

After resolving these matters, the tram-lines, the horizon line, and the winter and summer constant-declination curves can be marked on the dial. At this stage none is painted. Their purpose is to serve as guidelines.

Step 7 — The Constant-Altitude Curves

The family of constant-altitude curves is shown in Fig. 7. These are the first mathematical lines to be painted and if it is decided to postpone the painting of the chapter ring these curves will be the very first items to be painted.

The family consists of six red hyperbolic arcs which are bounded by the butterfly-shaped region. That is, they are bounded at top and bottom by the winter and summer solstice curves and at left and right by the inner margins of the broad tram-lines. By not being allowed to run above the winter solstice curve, the uppermost arc is broken into two parts.

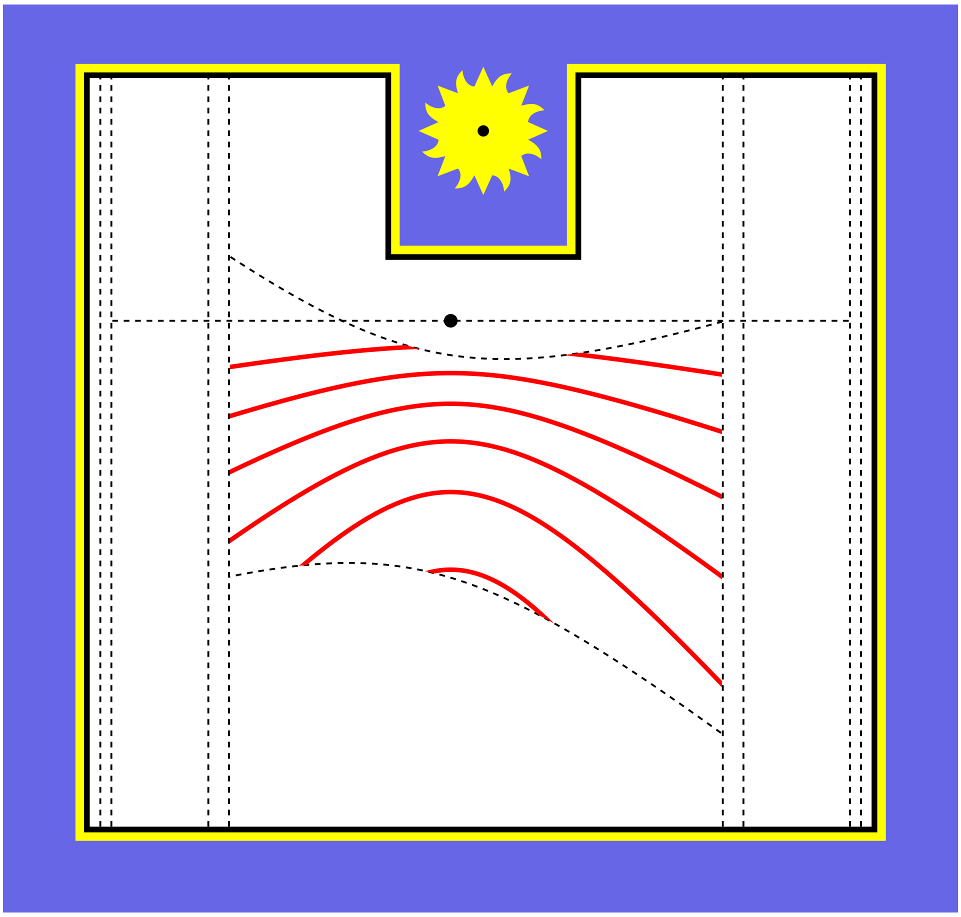


Fig. 7 — The Constant-Altitude Curves

There is an inconsistency in the treatment of the uppermost constant-altitude curve in the 1971 version of the dial. As can be seen in the cover photograph, the left-hand part of this arc runs only to the constant-declination curve labelled 80 in the LONGITUDO column whereas the right-hand part runs up to the golden arc.

At this stage it makes sense to let both parts run up to the winter solstice curve. If it is decided to keep the golden arc, that will simply override the winter solstice curve.

Notes on Painting the Constant-Altitude Curves

As described in Step 6, in the context of the winter and summer solstice curves, each constant-altitude curve will be presented as a set of points on a spreadsheet. These points are marked on the dial and then joined together.

As with the constant-declination curves, the lines will be presented as sets of points on a spreadsheet. The points are marked on the dial using the grid and then joined together.

In the current computer model, the red lines are 12 mm thick. This thickness has been estimated from the 1971 dial where the true thickness may be $\frac{1}{2}''$. Standard heraldic red should be used.

The constant-altitude curves have to be painted before any of the other mathematical lines because they represent a bottom layer. The cover photograph shows that all lines that intersect the constant-altitude curves are painted on top.

It is very difficult to end a line exactly at a prescribed boundary which is why painting the horizon line, the winter and summer solstice curves, and the tram-lines is probably best delayed. When these lines are eventually attended to they can be painted neatly over the ends of the curves which run up to them.

Each constant-altitude curve can be thought of as a trace showing the path that the shadow of the nodus would follow across the dial during the course of a day if the sun were (hypothetically) to maintain a constant altitude.

Note that the highest points of the constant-altitude curves fall on a vertical line which runs through the sub-nodus point. This is a useful check when setting out.

As can be seen in the cover photograph, each constant-altitude curve is labelled twice. The values 10 to 60 are altitudes in degrees. It is much too early to paint the labels. These should be placed in appropriate gaps in the dial furniture and the positions of these gaps will not be clear until the other lines have been painted.

It should also be noted that the horizon line is a member of the family of constant-altitude curves so it is worth considering whether it too should be red to reflect this membership.

The Plan for Step 7

There are several queries. How thick should the lines be? Should both parts of the uppermost constant-altitude curve run up to the winter solstice curve? Should (later) the horizon line be red too?

After resolving these matters, the constant-altitude curves can be set out and painted.

Step 8 — The Constant-Declination Curves

In Fig. 8 a start has been made on the family of constant-declination curves. Apart from the red constant-altitude curves, these are the only mathematical lines on the dial which are coloured. All the others are black.

The colour of these curves is rather faded in the cover photograph but on the dial itself it is clear that, with the exception of the equinoctial line, the curves are green. It is also clear, even from the cover photograph, that these curves cross over the red constant-declination curves but are, in turn, crossed by various black lines which will be added later.

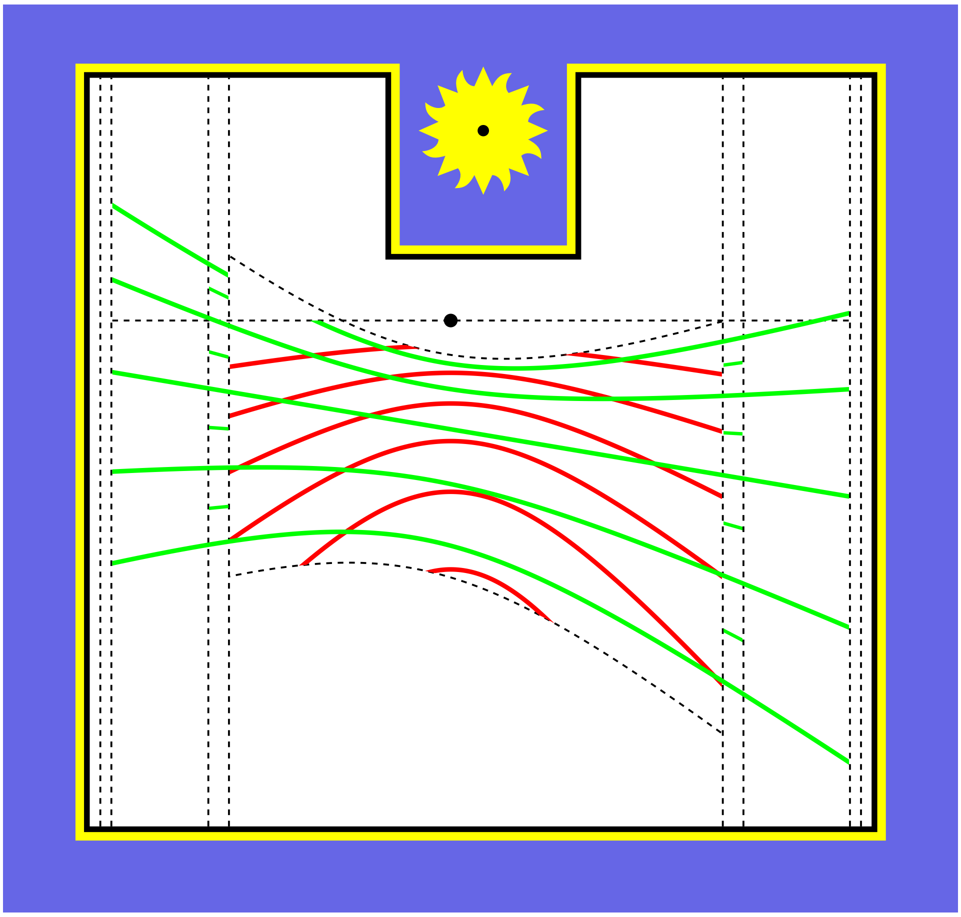


Fig. 8 — The Constant-Declination Curves

The summer and winter solstice curves are not painted at this stage. These curves mark the end points of many of the constant-azimuth lines and temporary-hour lines (which will be added later) and it makes sense to paint across the ends of those lines after they have been added.

The family of constant-declination curves presents several puzzles. Why, for example, is the straight equinoctial line black and not green? In Fig. 8 this line is shown as green to be consistent. Is there a good case for this line being black?

How big is the Family?

On sundials that feature constant-declination curves it is usual to have 1, 3, 7 or 13 lines. If there is just one, it will be the equinoctial line. Adding the winter and summer solstice curves brings the number up to three. It is very common to divide the period from one solstice to the next into six parts (equating to six Signs of the Zodiac) meriting seven lines.

The Queens' Dial is very suggestive of seven lines except that there is scant evidence of the winter solstice curve itself. In some sundials, the period for each Sign of the Zodiac is divided into two which requires six more lines, bringing the total up to 13.

Including the dashed winter and summer solstice curves, Fig. 8 illustrates the seven-line case. The five green curves are bounded at left and right by the inner margins of the narrow tram-lines. The constraint that no constant-declination curve may extend above the horizon line has not been imposed in Fig. 8 but a section of the highest green curve has been omitted where it would cross the first two letters of HORIZON.

There is scope for debate about whether or not all seven lines should be painted and, if so, whether they should all extend to the narrow tram-lines. A good reason for extending all seven lines across the two regions labelled L and R in Fig. 6 is that those regions would each be divided into six sub-regions. Subject to a little straddling across borders, the 12 Signs of the Zodiac could then be accommodated one to each of the 12 sub-regions.

Note that the seven declinations used in the current computer model correspond to equal intervals of solar longitude. At the winter solstice the solar longitude can be taken as -90° and this value is increased in six 30° increments to $+90^\circ$, the solar longitude at the summer solstice. These are the longitudes which traditionally separate the Signs of the Zodiac.

Fig. 8 also hints at the next level of sub-division (15° increments of solar longitude) which would bring the total number of constant-declination curves up to 13. Instead of adding six more lines, just four are added and they are shown only where they fall within the broad tram-lines. They are shown in the family colour, which should be standard heraldic green, but are thinner than the full-length curves, 9 mm rather than 12 mm. The thickness of the full-length curves is chosen to match the thickness of the constant-altitude curves.

The broad tram-lines are used for labelling the constant-declination curves. The labels in the broad tram-lines on the left indicate approximate times of sunrise and the labels in the broad tram-lines on the right probably indicate day-length.

In the 1971 dial, the four lowest-level sub-dividing lines are not shown explicitly but their labels are clear in the broad tram-lines close to the correct positions. There is a case for showing these four sub-dividing lines explicitly as in Fig. 8. There is insufficient space for the final two sub-dividing lines. They are omitted from Fig. 8 and the 1971 dial.

The Plan for Step 8

There are several queries. Should the equinoctial line be green? Should there be a winter solstice curve? Should any long lines be allowed to continue above the horizon line? Should the lowest-level sub-division lines be shown explicitly in the broad tram-lines? How thick should the lines be?

After resolving these matters, five of the seven long curves and all four pairs of short sub-dividing lines can be set out and painted.

Step 9 — The Month Separators

As can be seen in the cover photograph, the narrow tram-lines are destined to contain the names of the 12 months, most of them in Latin.

In the photograph, the narrow tram-lines also accommodate 12 black horizontal lines. These short lines are the month separators. They mark the divisions between adjacent months. For example, the third black line down from the top in the right-hand narrow tram-lines separates FEBRUARIUS from MARTIUS.

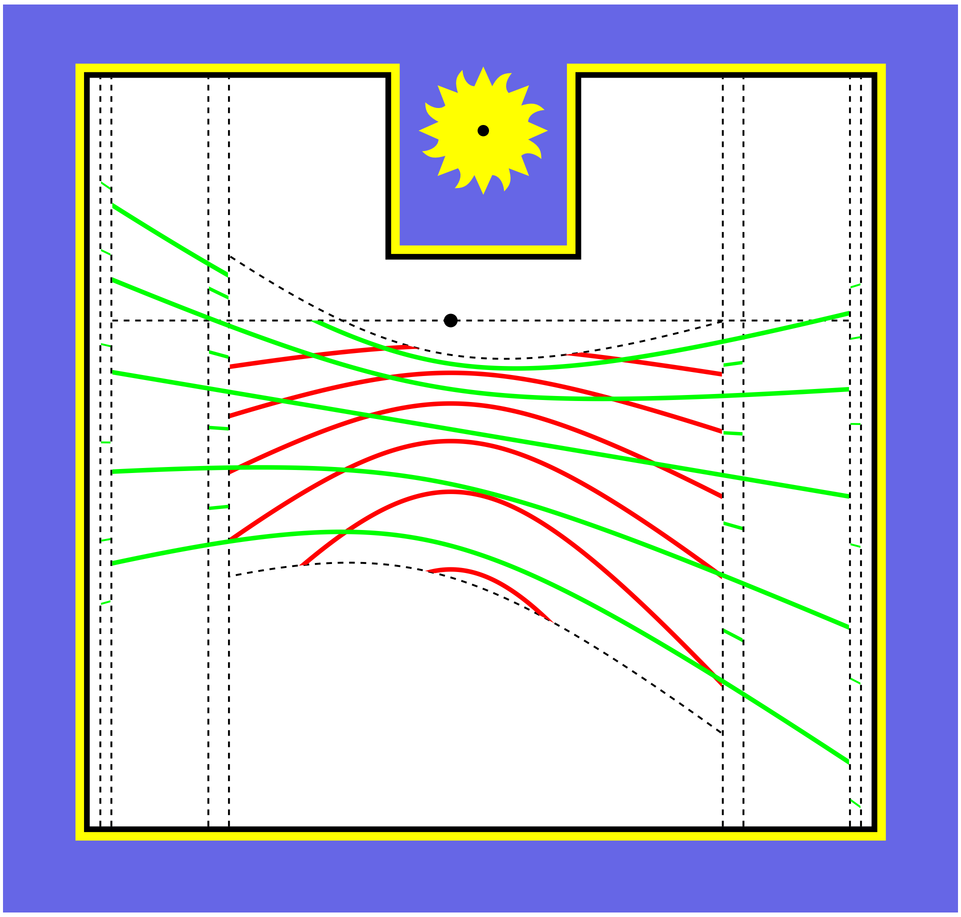


Fig. 9 — The Month Separators

An obvious thought is that each month separator should really be a very short length of constant-declination curve where the declination assumed is that which applies at the instant of midnight as one month turns into the next.

There has been considerable discussion about the month separators as they are marked on the 1971 dial, including the possibility that they originally related to the Julian Calendar. The Queens' Dial dates from at least 1642 and the Gregorian Calendar was not adopted in England for another 110 years so this is a reasonable conjecture.

Six Considerations

The proposed representation and positioning of the month separators implicit in Fig. 9 stem from a number of considerations of which six are the following:

1. It is assumed that the life expectancy of the proposed repainting is about 40 years.
2. It is assumed that the Gregorian Calendar should be used.
3. The month separators are assumed to be short lengths of constant-declination curves so they should be angled appropriately to fit in with other members of the family.
4. To emphasize that they are part of the family of constant-declination curves, the month separators should be green.
5. The month separators should be even thinner than the short lengths of constant-declination curves which are confined to the broad tram-lines. In the computer model they are 5 mm thick.
6. The declination at the instant of midnight at the beginning of any particular month varies from year to year, so some compromise value has to be chosen. This should be appropriate for the expected lifetime of the painting.

The last consideration needs amplifying. Take the FEBRUARIUS–MARTIUS month separator as an example. Over the period 2006 to 2050 the extreme values of the declination as February turns to March are $-7^\circ\,48'\,49''$ in 2007 and $-7^\circ\,24'\,37''$ in 2048. It clearly makes sense to use the average of these extremes, $-7^\circ\,36'\,43''$.

In simple terms, the value follows the four-year leap-year cycle on which is superimposed a slow drift. This will not be corrected until 2100 when 29 February is omitted.

When the time comes for another repainting in the mid-21st Century, and yet another towards the end of the Century, similar calculations can be undertaken which may again look 40 years or so ahead.

The seven-year run from 2097 to 2103 inclusive without a leap year will be a problem for a future diallist to ponder. Happily, the problem is not of great consequence. Many of the month separators on the 1971 dial are wildly out of place and the College receives few complaints. Not many people will notice an error of a quarter of a degree in the chosen value for a declination.

The Plan for Step 9

There are several queries. Should the month separators be green? Should each be a short length of constant-declination curve? How thick should the lines be? After resolving these matters, the month separators can be set out and painted.

Step 10 — The Constant-Azimuth Lines

Fig. 10 shows the dial augmented by the family of constant-azimuth lines. Each of these lines indicates a compass bearing of the sun.

If the dial is a vertical plane, as assumed in the current computer model, the lines will be straight, parallel and vertical. If the dial leans over slightly, the lines will converge on some point a long way above or below the dial.

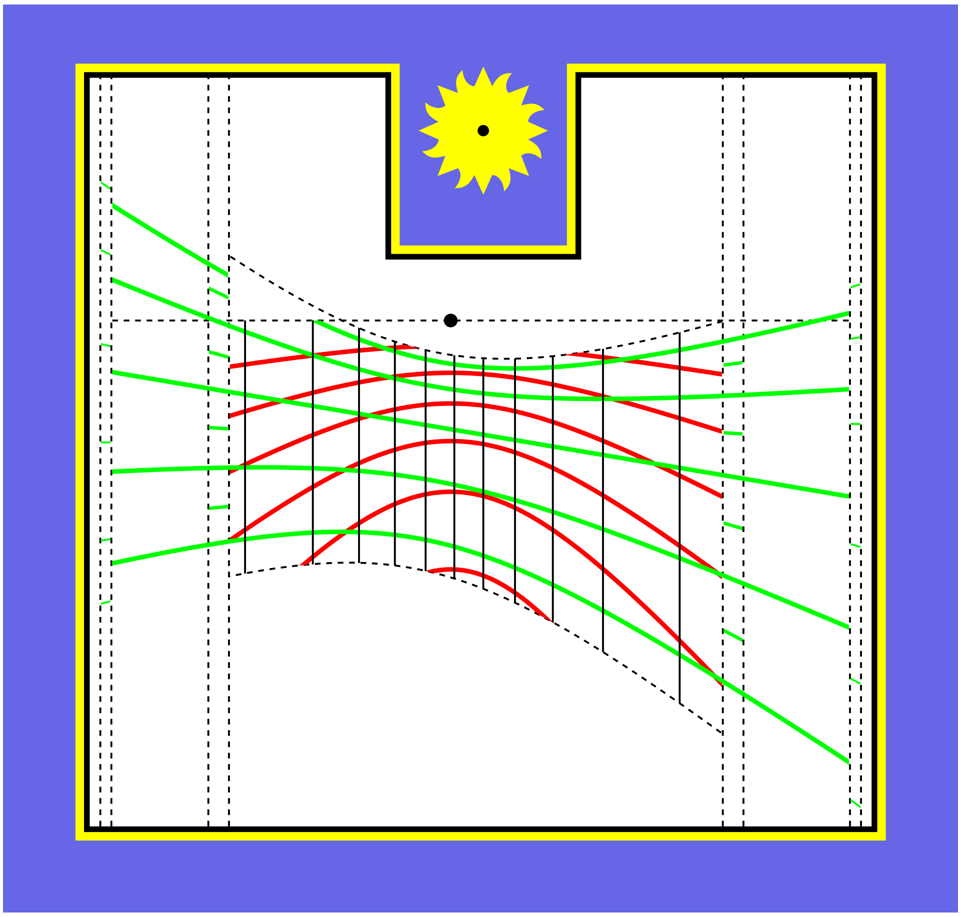


Fig. 10 — The Constant-Azimuth Lines

The family consists of 11 black lines, rather thinner than the lines drawn so far. In Fig. 10 the lower ends of the lines are bounded by the summer solstice curve and the upper ends are bounded by the winter solstice curve or the horizon line, whichever is lower.

In the photograph of the 1971 dial, the upper ends of all the lines run to the horizon line. This is inconsistent with the treatment of the constant-altitude curves. The extensions above the winter solstice curve do not have any aesthetic value and the shadow of the nodus can never fall on these extensions.

Notes on the Constant-Azimuth Lines

As with all the mathematical lines, each constant-azimuth line will be described by a set of coordinates. Since the lines are all straight, only the end points need be specified.

As can be seen in the cover photograph, the constant-azimuth lines are noticeably thinner than the principal lines already painted. In the current computer model, the constant-azimuth lines are 5 mm thick.

The ends of the lines will later be painted over by the winter or summer solstice curve or the horizon line. The painting of those lines has to wait.

Each constant-azimuth line can be thought of as a trace showing the path that the shadow of the nodus would follow during the course of a day if the sun were (hypothetically) to maintain a constant azimuth.

As can be seen in the cover photograph, each constant-azimuth curve is labelled. The labels are below the summer solstice curve and the label S for south is notable both for its size and its ornamentation. The constant-azimuth line corresponding to south will be pained over by the 12 noon hour line so it may be omitted at this stage.

It is much too early to paint the labels because they have to fit in with the hour lines which will be drawn later.

The Plan for Step 10

There are a couple of queries. Should the upper ends of the constant-azimuth lines be bounded by the winter solstice curve as in Fig. 10? How thick should the lines be? After resolving these matters, the constant-azimuth lines can be set out and painted.

Step 11 — The Temporary Hours Curves

Fig. 11 shows the dial augmented by the family of temporary hours curves. These curves divide the day into 12 equal intervals from sunrise to sunset. Mathematically, they are almost but not quite straight lines. They really are curves!

The question has been raised as to whether the corresponding lines on the 1971 dial really are temporary hours lines. There seems little doubt. If the equivalent lines in the cover photograph are compared with the lines in Fig. 11 the correspondence is very close. The only scope for quibble is that the first line on the left is missing from the 1971 dial.

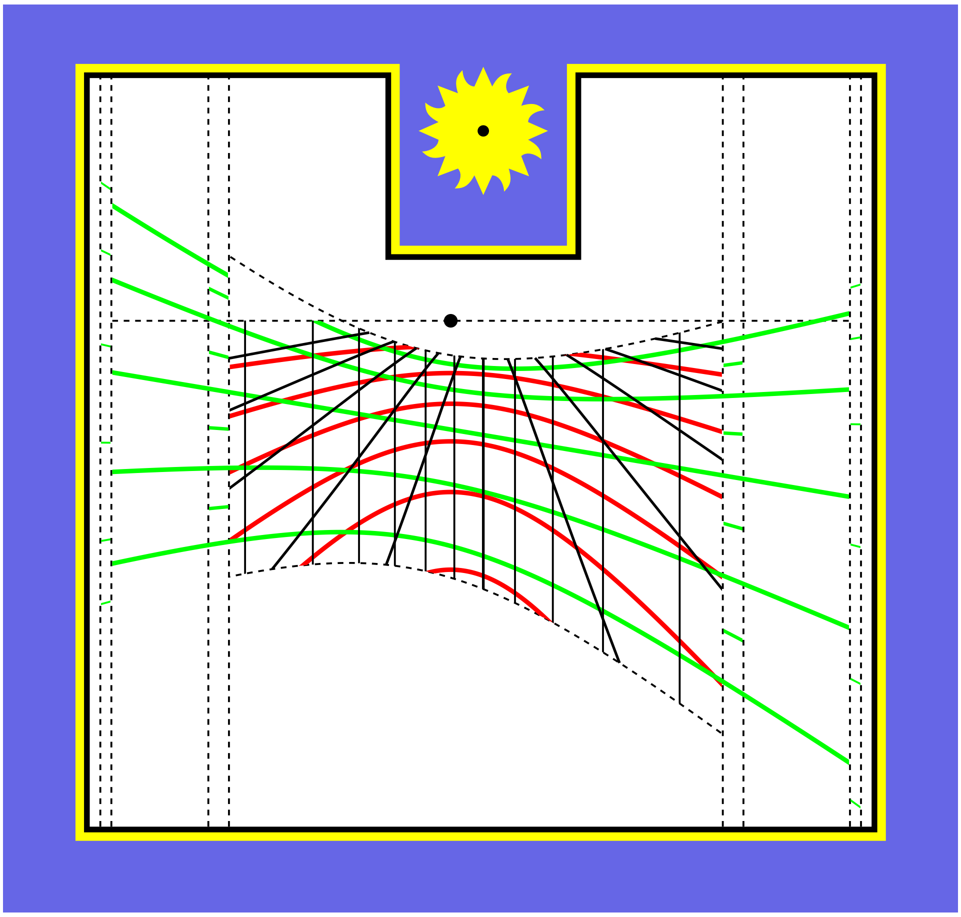


Fig. 11 — The Temporary Hours Curves

Strictly, the family consists of 13 curves which run from 6 a.m. to 6 p.m. The 6 a.m. and 6 p.m. lines are not drawn because they coincide with the horizon line. These special case lines *are* straight and, of course, refer to sunrise and sunset respectively. The central 12 noon line is also straight and coincides with the constant-azimuth line for south.

As in the cover photograph, the temporary hour lines are the same thickness as the constant-azimuth lines. The lines are bounded by the butterfly-shaped region.

Notes on the Temporary Hours Curves

The temporary hours curves will be described by sets of coordinates but they are so close to being straight lines that it will probably prove impossible, even at full scale, to distinguish them from true straight lines.

There is a hint in the cover photograph that the temporary hours curves are slightly thicker than the constant-azimuth lines. In the computer model the temporary hours lines are 7 mm thick.

The ends of the lines will later be painted over by the winter or summer solstice curve or a tram-line.

The mathematics of temporary hours is outside the scope of this document but it is worth noting that, if the temporary hour curves are extended upwards, they all converge on the point where the noon line intersects the horizon line. Clearly the lines in Fig. 11 would have to curve inwards to reach this point and most of the curvature is above the winter solstice curve.

As can be seen in the cover photograph, the temporary hours curves are not labelled. In 1642, users of the original dial might have been more familiar with temporary hours than today's users. Labelling the temporary hours curves was perhaps considered unnecessary.

Temporary hours long predate equal hours on sundials. The great *Horologium Solarium*, constructed in Rome 2000 years ago at the behest of Emperor Augustus, was marked out in temporary hours. This can be readily verified since fragments remain to this day.

The Plan for Step 11

There are several queries. Should the 7 a.m. line, missing from the 1971 dial, be included? Should the lines be straight? How thick should the lines be? After resolving these matters, the temporary hours curves can be set out and painted.

Step 12 — The Solstices and the Horizon Line

All the line-ends which are cut off by the winter or summer solstice curves or by the horizon line have now been painted so these lines can themselves be painted.

Fig. 12 shows the dial augmented by the winter and summer solstice curves and by the horizon line. The winter solstice curve, like the constant-declination curve immediately below it, is in two parts. Constant-declination curves are excluded from the region above the horizon line and between inner margins of the broad tram-lines.

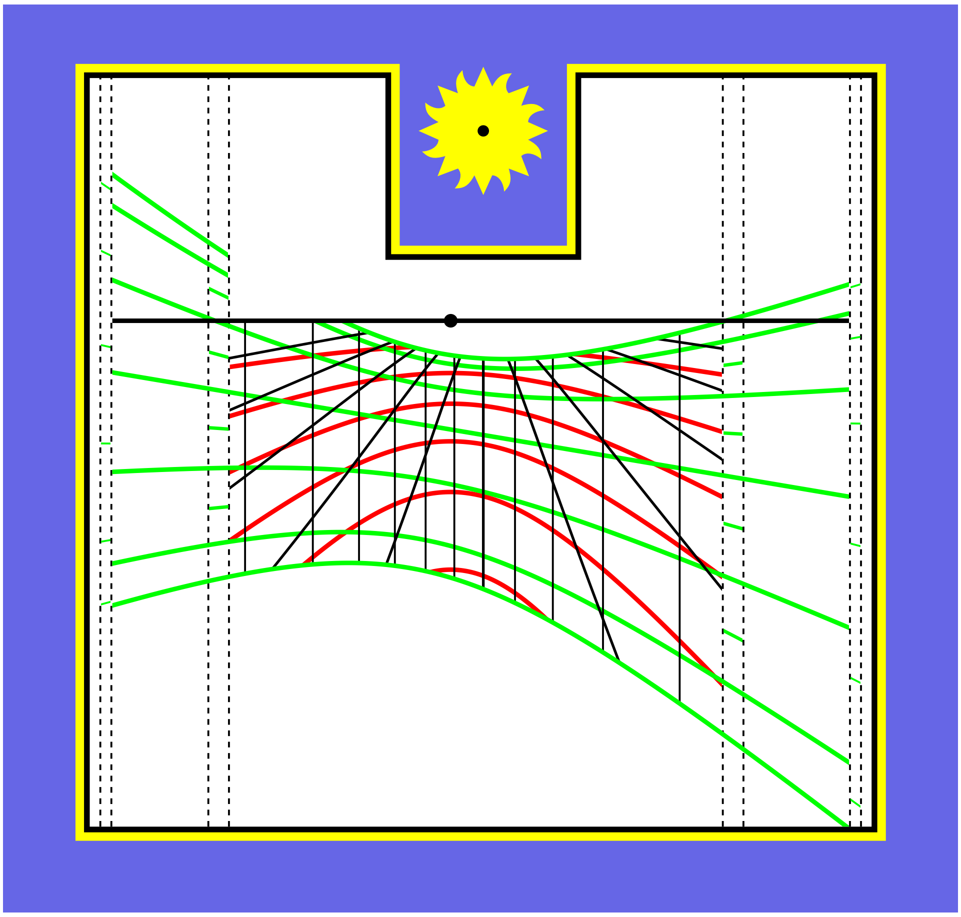


Fig. 12 — The Solstices and the Horizon Line

An implicit rule in the 1971 dial is that black lines run across the top of green lines. Accordingly, the winter solstice curve is painted before the horizon line.

The winter and summer solstice curves are painted over the extremities of the constant-altitude curves, the constant-azimuth lines and the temporary hours curves. The effect is that all these lines run up to but do not run into the winter and summer solstice curves. Likewise the winter solstice curve runs up to but not into the horizon line.

Notes on the Latest Additions to the Dial

The horizon line is bounded by the inner margins of the narrow tram-lines. This is how it is in the 1971 dial and also in the 1860s photograph (reproduced on the inside of the front cover). As such, it runs across two sub-regions where Signs of the Zodiac are to be painted, Libra on the left and Capricorn on the right. There is an argument for shortening it so that it is bounded by the inner margins of the broad tram-lines.

As can be seen in the cover photograph, none of the constant-declination curves in the 1971 dial is allowed to run above the horizon line, except to the extent that three times-of-sunrise labels appear to be aligned with the curves which they label. This seems also to be the case in the 1860s photograph.

If the upper constant-declination curves are to be allowed to run into the Signs-of-the-Zodiac column on the left then the argument for shortening the horizon line perhaps becomes stronger.

Each of the two uppermost constant-declination curves is in two parts. The missing sections are omitted deliberately. These sections are above the horizon line where the shadow of the nodus can never fall. The missing sections would serve no purpose here and the winter-solstice line in particular would interfere with the HO of HORIZON.

Where the lines pick up again they serve two purposes: they carry labels in the broad tram-lines and they separate signs of the Zodiac in the space between the narrow and broad tram-lines.

The missing section of the winter solstice curve itself has appeared in recent figures as part of the dashed line (representing a pencil mark) that formed the upper boundary of the butterfly-shaped region. This section of pencil line can now be cleaned off the dial. It is not shown in Fig. 12.

In the computer model, the horizon line is 12 mm thick, reflecting the fact that the horizon line is a member of the family of constant-altitude curves which are also 12 mm thick.

Note that the tram-lines cannot sensibly be painted yet. The tram-lines are used as bounds on certain hour lines and half- and quarter-hour tickmarks which are not painted until later.

The Plan for Step 12

There are several queries. Should any lengths of constant-declination line be shown above the horizon line? Should the horizon line be red? Should the horizon line be shortened? How thick should it be? After resolving these matters, the winter and summer solstice curves can be painted and then the horizon line can be painted.

Step 13 — The Hour Lines

Fig. 13 shows the dial augmented by the family of hour lines which radiate from the root of the gnomon. These, and the associated tickmarks, are the only lines which relate to the gnomon rather than the nodus.

The family consists of 13 lines which correspond to the hours from 5 a.m. to 5 p.m. Most of the hour lines are bounded at both ends by the black edging which runs round the periphery of the main (white) part of the dial. Accordingly, it may make sense to leave painting this black edging until after the hour lines are painted.

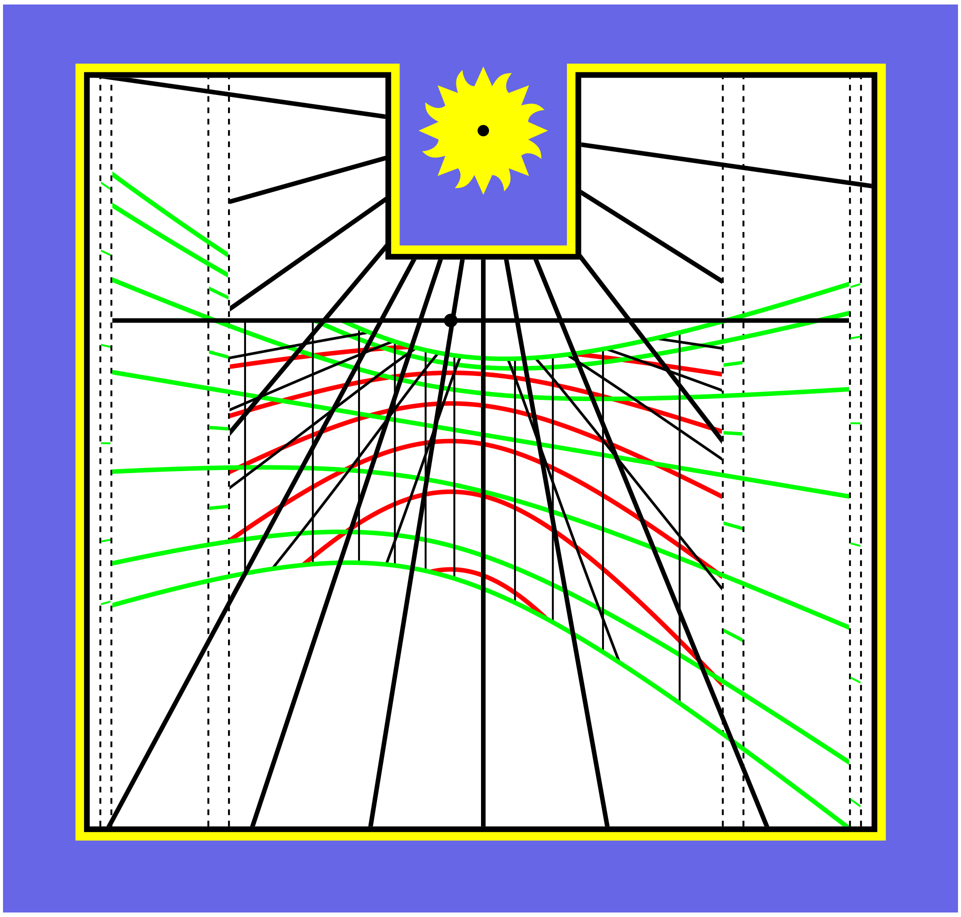


Fig. 13 — The Hour Lines

Some of the hour lines are relatively short. Their outer ends are bounded by the inner margins of the broad tram-lines which is why the tram-lines are not painted until later. By being cut short, these lines are prevented from cluttering the space allocated to the labels in the broad tram-lines or, worse, crossing the regions labelled L and R in Fig. 6 which are where the Signs of the Zodiac are to be painted.

The policy of not crossing the spaces allocated to the Signs of the Zodiac is clearly sound and, perhaps, adds to the argument for applying the same policy to the horizon line.

Notes on Painting the Hour Lines

In theory, each hour line could be set out by first marking the two end points using the coordinates supplied on a spreadsheet. It is assumed that sufficient of the grid is still visible to be used for setting out.

In practice, for all lines except 5 a.m., 6 a.m., 4 p.m. and 5 p.m., a third point should be marked. This is the point where the hour line crosses the equinoctial line. Moreover, these points are also where the temporary hours lines cross the equinoctial line. In Fig. 13, which is drawn with computer-regulated precision, six triple points can be seen on the equinoctial line.

One end of a length of string should be fastened to the root of the gnomon and, for each hour line, it should be drawn taught over the end points and the point on the equinoctial line to check alignment.

If the alignment is not quite exact, the point on the equinoctial line should take precedence. Not having the triple points nice and crisp draws attention to erroneous setting out. These points are certainly not nice and crisp on the 1971 dial.

If the alignment is seriously in error then something went wrong at Step 4 and wasn't put right.

In the computer model, the hour lines are 12 mm thick. It is possible they ought to be a little thicker than that but this would spoil the triple points which already suffer from one component (the temporary hours lines) being thinner than the other two. It is also possible that the 12 noon line should be thicker than the others.

The Plan for Step 13

There are several queries. How precise is the alignment of each line? How thick should the lines be? Should the 12 noon hour line be thicker than the others? After resolving these matters, the hour lines can be set out and painted.

Step 14 — The Tickmarks

In addition to the hour lines there are half-hour and quarter-hour tickmarks which are shown in Fig. 14 round the margin of the main white area of the dial.

As can be seen in the cover photograph, the half-hour tickmarks along the bottom edge of the main area of the dial are fleurs-de-lys, a very common symbol for half hours on old sundials and clocks. The computer model extends to these fleurs-de-lys too. Flanking each fleur-de-lys are two quarter-hour tickmarks.

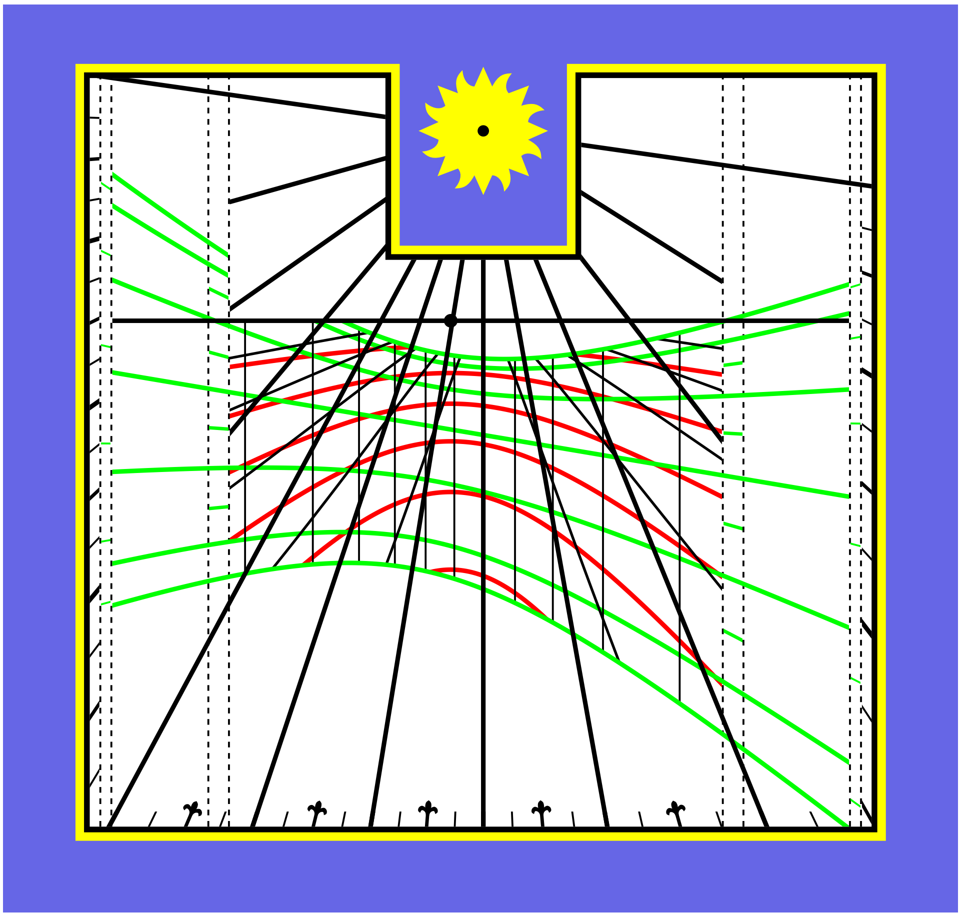


Fig. 14 — The Tickmarks

Along the vertical edges of the main area of the dial are further tickmarks. These lie in the narrow regions between the the vertical runs of black edging and the outer margins of the narrow tram-lines. This is why the tram-lines are not painted until later.

In these regions there are not only half-hour and quarter-hour tickmarks but, where appropriate, there are hour tickmarks too. These relate to the hours whose hour lines are cut short. On the 1971 dial, all the tickmarks have the same thickness which makes it hard to read. In the computer model the half-hour tickmarks are thinner than the hour tickmarks and the quarter-hour tickmarks are thinner still.

Notes on Painting the Tickmarks

As with the hour lines, the tickmarks can be set out by marking the two end points and, where they are in range, the relevant point on the equinoctial line. The length of string can again be used to check the alignment.

In the computer model, the hour tickmarks are 12 mm thick to match the thickness of the hour lines. The half-hour tickmarks (and the stalks of the fleurs-de-lys) are 9 mm thick and the quarter-hour tickmarks are 5 mm thick.

Both on the 1971 dial and the computer model, the tickmarks along the bottom are longer than the tickmarks along the vertical edges.

The Plan for Step 14

There are several queries. Should the thicknesses the half-hour and quarter-hour tickmarks be differentiated from the thicknesses of the hour lines? If so, what should the thicknesses be? How long should the tickmarks along the bottom margin be? After resolving these matters, the tickmarks can be set out and painted.

Step 15 — The Tram-lines

By painting the hour lines and tickmarks, all the line-ends which are cut off by the broad and narrow tram-lines are in place so the tram-lines themselves can now be painted.

Fig. 15 shows the dial augmented by the broad and narrow tram-lines which were set out at Step 6. As noted at that step, there is a wider gap for the Signs of the Zodiac on the right than there is on the left.

These tram-lines cross the ends of numerous other lines which were painted earlier. In particular, they cross the ends of lines which are confined to the tram-lines themselves, the in-between constant-declination curves and the month separators.

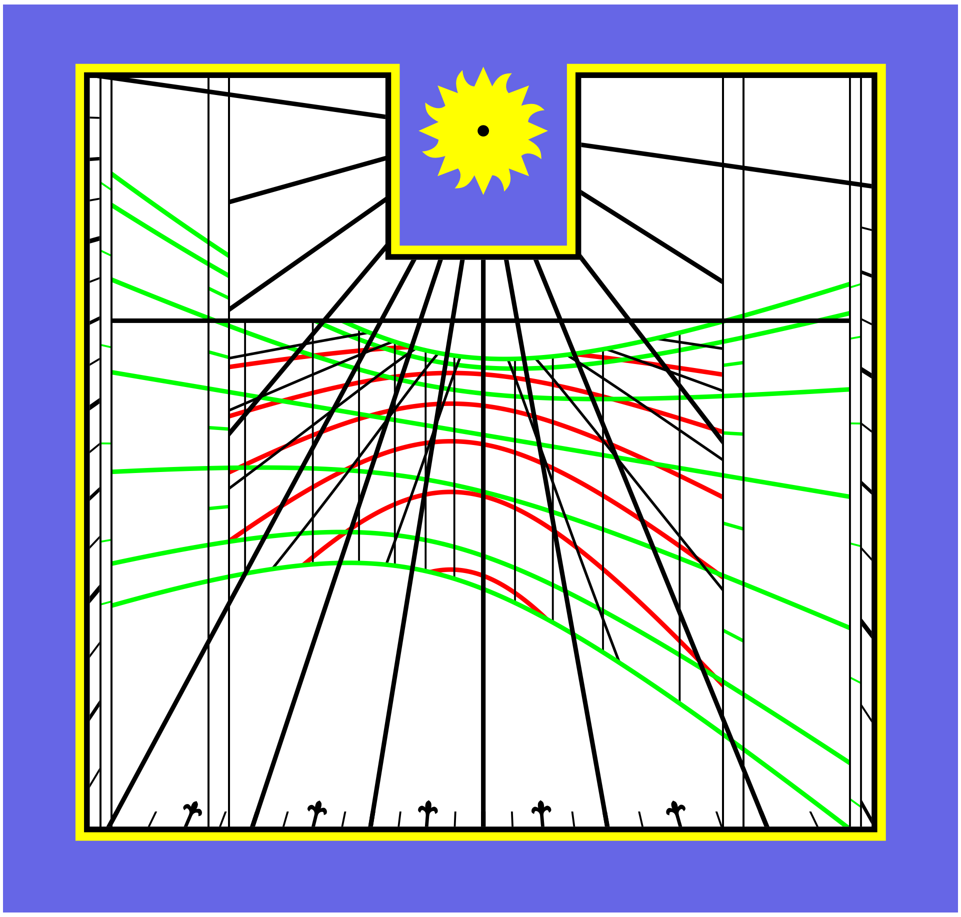


Fig. 15 — The Tram-lines

Painting the tickmarks concluded the painting of all the mathematical lines on the Queens' Dial and painting the tram-lines concludes the painting of all the framing lines on the dial.

There are numerous other items of dial furniture still to come and these items can be broadly classified into labelling and other ornamentation.

Notes on Painting the Tram-Lines and Tidying up

In the computer model, the tram-lines are 5 mm thick which matches the thickness of the constant-azimuth lines.

The grid, a few remnants of which still remain, can now be dispensed with. All the mathematical and framing lines which were set out with the aid of the grid are now in place.

This is the time to clean off the grid (and any indication of the sub-nodus point) and touch up any disturbed paint work. At the conclusion of this stage, the dial should look almost exactly like Fig. 15. Note that the sub-nodus point has gone.

The Plan for Step 15

There is just one query. How thick should the tram-lines be? After resolving this matter, the tram-lines can be painted, the dial can be cleared of all setting-out lines, and any imperfections can be touched up.

Step 16 — Outlining the Hour Labels

Most of the outstanding items, mainly lettering and artwork, can be copied without much change from the 1971 dial. This document will be largely silent on details but the placement of the hour labels in the chapter ring is considered now and the design of the individual Roman numerals is considered in Step 17.

In the 1971 design, each hour label is approximately centred on its associated hour line and the characters of a given numeral are slanted to reflect the angle of that hour line.

As can be seen in the cover photograph, the VII is too low, the IX is too far to the right and the slant angle of the III does not match the angle of three o'clock hour line.

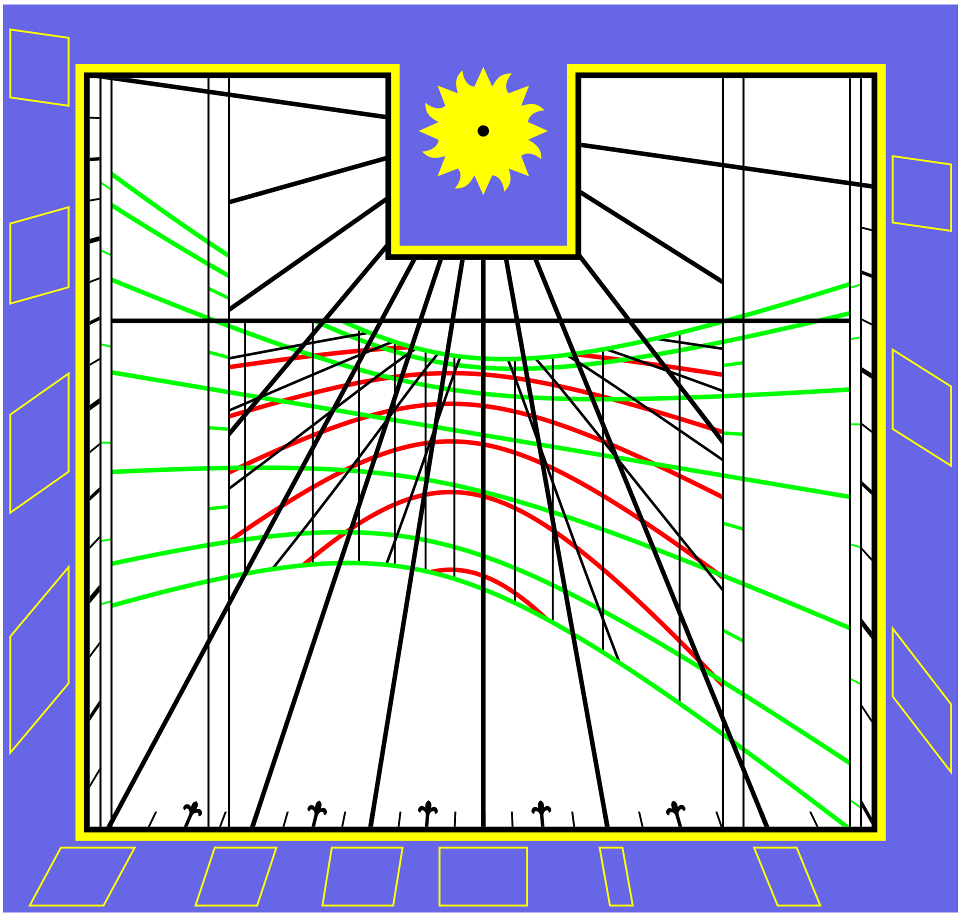


Fig. 16 — Outlines for the Roman Numeral Hour Labels

In Fig. 16, each Roman numeral is reduced to an outline parallelogram. A preliminary idea is to fit each Roman numeral into its outline as comfortably as possible.

Any ordinary upper-case letter can be enclosed in a rectangular box. The two horizontal edges are termed the base-line and the cap line. If the letter is given a slant, the horizontal edges stay horizontal but the vertical edges lean over. This gives rise to the parallelograms in the figure.

Notes on laying out the Roman Numerals

In the cover photograph, the XII is the only hour label whose naturally-enclosing box is a rectangle and this rectangle is shown in Fig. 16. The left- and right-hand sides are vertical because the 12 noon hour line is vertical.

As a second example from the cover photograph, the naturally-enclosing box for the II is clearly a parallelogram. The base-line and cap line are horizontal but the two Is slant at the same angle as the two o'clock hour line which, if extended, would run along the gap between the two Is of the II. Note that these two Is are parallel to one another just as are the left- and right-hand sides of the parallelogram for II in Fig. 16.

For the hour labels on the vertical sides of the chapter ring, the discussion is the same except that one's frame of reference is turned through ±90°.

Note, in particular, that the four Is of the IIII are parallel to one another. This accords with standard practice. On a stretch of chapter ring which has straight parallel sides, classic lettering rules require the four Is to look parallel to one another.

An alternative approach is to arrange for each of the four Is to align with the root of the gnomon. The enclosing box for the IIII would then be a trapezium rather than a rectangle. In Fig. 18, every enclosing box has been replaced by a trapezium. This is not standard practice but the suggestion introduces a subtlety...

There is a difference between *looking* parallel and *being* parallel. The IIII lies off the end of the four o'clock hour line which, in turn, is between the three and five o'clock hour lines. When the IIII is looked at in this wider context, the inner ends of the Is appear to splay outwards very slightly. There is then a case for deliberately slanting these ends inwards a little to compensate. This is somewhat akin to entasis. It is *not* a recommendation to adopt trapezoidal outlines, just a recommendation to correct an illusion.

In Fig. 16 the vertical edges of the 12 noon rectangle likewise appear slightly splayed but here there is no need for compensation. When the XII is viewed from ground level, natural perspective does the job. This rectangle should stay rectangular.

It has already been noted that the VII and the IX are poorly placed in the 1971 version of the dial. The positioning of the Roman numerals is important...

In Fig. 16, each parallelogram is arranged so the associated hour line runs through its geometric centre. As before, the eye doesn't necessarily see the truth. The parallelograms for the VIII and the III look much too low until a straight edge is offered up to the figure.

There is a case for slight repositioning to correct this illusion. One might further note that the optical centre (to use a David Kindersley term) of an asymmetric numeral such as VIII is not the same as the geometric centre of its associated parallelogram.

The Plan for Step 16

There are several queries. To what extent should the slants of the component letters of each Roman numeral be adjusted to make them appear parallel? How best should each Roman numeral be positioned so that it most satisfactorily fits with its associated hour line? Should gold leaf be used? Discussion with a skilled letterer should resolve these matters. The hour labels can then be designed and set out...

Step 17 — Designing the Hour Labels

Fig. 17 shows a first attempt at designing the hour labels for the chapter ring. The Roman numerals require an alphabet of just three letters: X, V and I. Two of these characters, the X and the I, appear unslanted in XII in the 1971 version of the dial and an early goal was to copy these two characters as closely as possible.

The figure on the facing page shows the evolution of the design process. The letter I at (a) is taken as the base character. Described geometrically, this letter consists of two parallel vertical lines which flair outwards in pairs of quarter-circles at the top and the bottom.

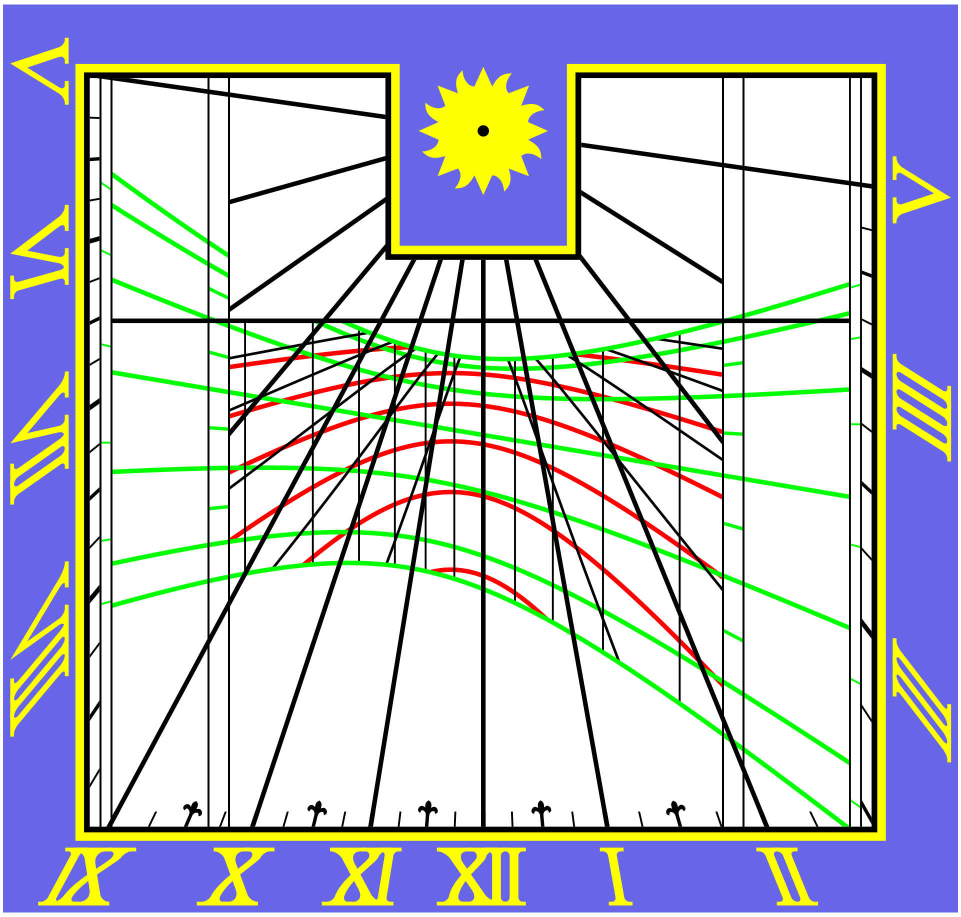


Fig. 17 — The Roman Numeral Hour Lables

This main part of the letter lies between two thin rectangular slabs. The lower one sits on the base-line and the upper one is capped by the cap line. On account of the slabs, this letter is sometimes called a slab-serif I.

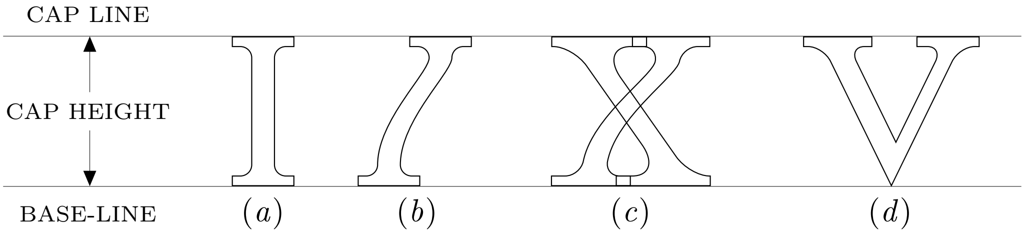
The cap height, the separation of the cap line and the base-line, is the single independent dimension. The width of the column, the dimensions of the rectangular slabs and the radius of the quarter-circles are all expressed as proportions of the cap height.

Modifying the Slab-Serif I

The slab-serif I can be distorted in several ways. The four serifs, the ends of the two slabs, can be specified independently as can the radii of the four quarter-circles.

A different distortion is illustrated at (b) where the top of the I (the upper slab together with its supporting flair) has been displaced to the right relative to the bottom of the I. The column in between is given a snake-like appearance.

A more significant distortion is to give a letter a slant. A forward slant is achieved by shifting each point in the letter to the right by an amount proportional to its height above the base-line. A rectangular element distorts to a parallelogram but the slanting procedure does not apply to the two slabs. They stay rectangular. Note that quarter-circles distort into non-circular arcs.



The letter X is formed by giving a backward slant to a simple I and superimposing a forward-slanting snake-I. Such an X is shown at (c).

The eight slab-ends, the eight radii, and the bend in the snake, have been adjusted so that the overall appearance of the X closely matches that in the XII in the 1971 version of the dial. Note that the bend is slightly over halfway up. The gap between the legs of the X is a little larger than the gap between the arms. This conforms with standard practice.

The V is also formed from two Is. Their tops are as at (a) but their bottoms are tapered to points. The four slab-ends, the four radii and the column widths are adjusted to match the Vs at 5 a.m. and 5 p.m. in the 1971 version of the dial. Such a V is shown at (d).

The forward-slanting stroke of an X or a V in Roman Numerals is usually very much thinner than the backward-slanting stroke. This convention has not really been followed in the 1971 version of the dial but it has been lightly applied in the lettering in Fig. 17.

Another convention is that a slanted letter has the same area as the unslanted original. This convention seems not to have been followed at all in the 1971 version of the dial but it *has* been followed in Fig. 17. The Is in VIII and III are noticeably longer and thinner than the Is in XII. In the 1971 version of the dial the Is in III are arguably much too stout.

The Plan for Step 17

There are several queries. To what extent should one attempt to make the lettering mathematically consistent? Should the components of the steeply-sloping hour labels be long and thin as in Fig. 17? Should the forward-strokes in the Xs and Vs be more noticeably thinner than the backward strokes? Again, resolving such matters requires discussion with a skilled letterer. The hour labels can then be painted.

Step 18 — The Half-Hour Lozenges and the Tram-line Labels

Fig. 18 shows the chapter ring augmented by gold diamond shapes, heraldic lozenges, which indicate half-hours. For interest, the parallelogram outlines of Fig. 16 have been replaced by trapezia. Using such outlines would be a radical change and is not proposed.

The lozenges in Fig. 18 are more elongated than in the 1971 dial and their long axes more accurately align with root of the gnomon.

Also new, is all the labelling that belongs to the four pairs of tram-lines. This labelling is entirely black on white and closely follows the labelling on the 1971 dial.

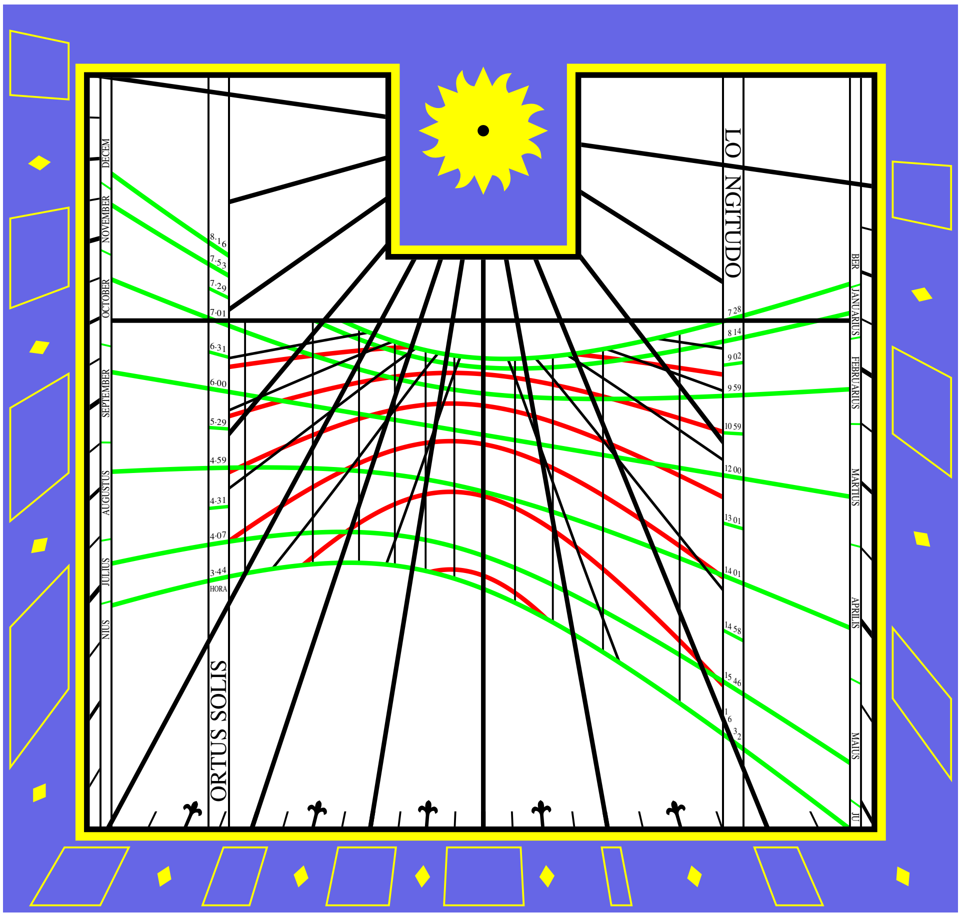


Fig. 18 — The Half-Hour Lozenges and the Tram-line Labels

The principal comment to make about the month labels is that they have to fit into really narrow tram-lines and the lettering has to be very precise if it is to be readable.

Apart from the link months of JU NUIS and DECEM BER which are split, all the month labels are centred between two month separators.

The tightest space is that for JANUARIUS and this label should definitely be painted first. The second-tightest fit is NOVEMBER.

Notes on the Labels in the ORTUS SOLIS and LONGITUDO Tram-Lines

In Fig. 18, the labelling in the ORTUS SOLIS and LONGITUDO tram-lines differs a little from the corresponding labelling on the 1971 dial. The words ORTUS SOLIS, LONGITUDO and HORA are as in 1971 but the labels for the in-between constant-declination curves are now on short lengths of hyperbolic arc rather than simply being suspended in space.

More significantly, the numerical values in these two pairs of tram-lines are different…

In the ORTUS SOLIS column of the 1971 dial, the times of sunrise are shown at half-hour intervals: 8.30, 8.0, 7.30, 7.0 and so on. One could take the view that these times are definitive and the selected declinations should be appropriate for these times.

There are two objections to this. First, the revised declinations would no longer correspond to equal intervals of solar longitude and, secondly, the sun never rises as late as 8.30 in Cambridge.

The constant-declination curves are used to delimit the spaces for the Signs of the Zodiac and these Signs divide the ecliptic into 12 equal intervals of solar longitude. Accordingly, it is more likely that the selected declinations correspond to equal intervals of solar longitude.

The times of sunrise shown in Fig. 18 are the times associated with the selected declinations and each time is correct to the nearest minute rather than to the nearest half-hour. The times are all are shown in the same format. In particular there is 6.00 rather than 6.0 to be consistent with all the other times.

Interestingly, seven of the times in Fig. 18 are no more that one minute different from the corresponding times in the 1971 dial. Perhaps the 1971 times are simply the correct times rounded to the nearest half hour.

The values given in the LONGITUDO column have sometimes been interpreted as unusual specifications of solar longitude but, very much more convincingly, Robin Walker of Queens' suggests that these values indicate the length of day, the time in hours and minutes between sunrise and sunset. In the 1971 dial these times are all given to the nearest hour with 160, for example, being interpreted as 16 hours 0 minutes.

In Fig. 18 the times are shown to the nearest minute so 160 is replaced by 15 46. Note that 15h 46m + 2 × (4h 7m) = 24h where 4h 7m is the time of sunrise shown as 4.07 in the ORTUS SOLIS column. Seven of the times in the LONGITUDO column are no more than two minutes different from the corresponding times in the 1971 dial.

In the 1971 dial the length-of-day times are not shown for the winter and summer solstices but the appropriate times have been included in Fig. 18.

The Plan for Step 18

There are several queries. Is the aspect ratio of the lozenges acceptable? Should the times of sunrise be as in Fig. 18 or should they be rounded to the nearest half hour as in the 1971 dial? Is 6.00 (rather than 6.0) acceptable? Should the hours of daylight be shown as 15 46 or as 15.46 or as 160? Should all four pairs of tram-lines be made a little wider to give more space for the labels? Should the portion of the 2 o'clock line that crosses the LONGITUDO column be omitted?

After resolving these matters, the half-hour lozenges and the tram-line labelling can be incorporated into the dial.

Step 19 — The Azimuth Labels and Horizon Label

Fig. 19 shows the dial augmented by labels for the constant-azimuth lines and by the label HORIZON on the horizon line. The hour labels are back to being the Roman numerals introduced in Fig. 17.

The constant-azimuth labels are all below the summer solstice curve and include the huge golden S, with its black edging, for south. Apart from the S, each label is a two-, three- or four-letter abbreviation for a compass direction on a 32-point compass.

By default, the letters of each abbreviation align with an implicit extension of the relevant constant-azimuth line. In two cases, SEBE and SEBS, the abbreviation has to straddle an hour line. In these cases the abbreviation is split into two in a rather uncomfortable way.

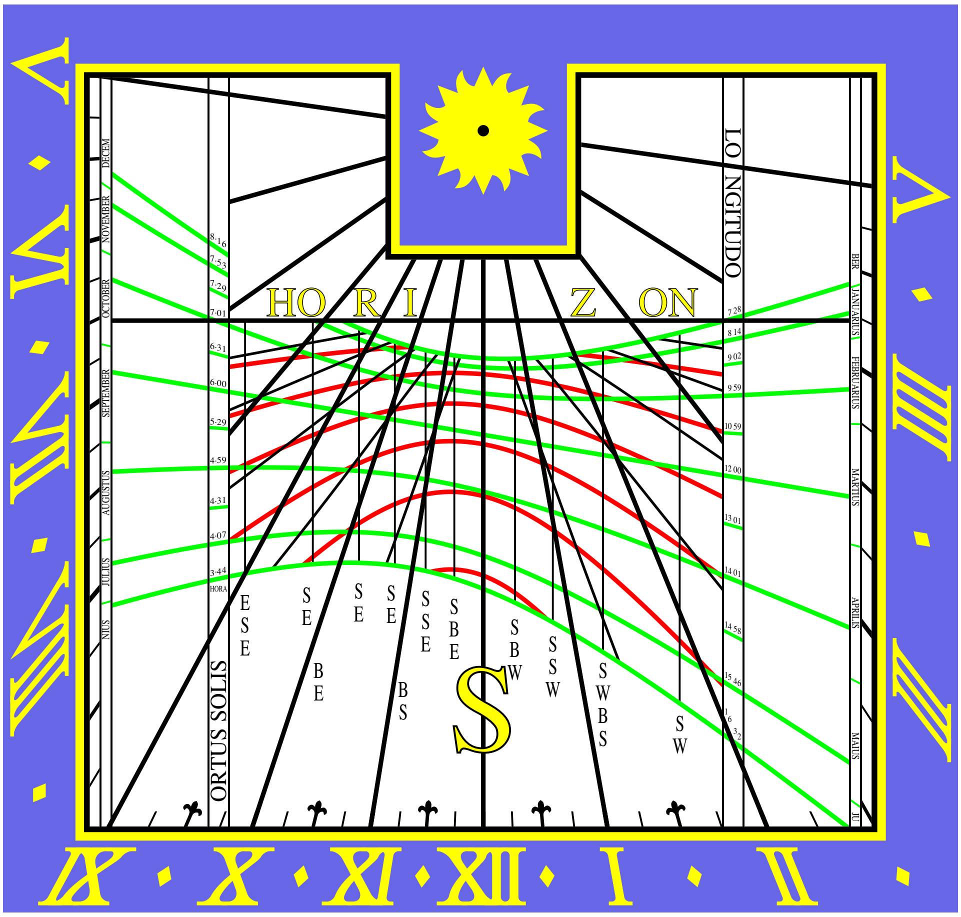


Fig. 19 — The Azimuth Labels and Horizon Label

In Fig. 19, the large S is a feeble representation of the ornate version which is displayed on the 1971 dial. An almost identical ornate S can be seen in the 1860s photograph. In the proposed repainting, the flamboyant S should undoubtedly be retained.

Like the S, the letters of the word HORIZON are of gold and have black edging. These letters are placed in the gaps between hour lines as on the 1971 dial.

Outstanding Labelling and other Ornamentation

Fig. 19 is almost as far as the computer model is going to be taken. When compared alongside the cover photograph, the principal outstanding items are:

1. The Moon Dial table
2. The red labels of the constant-altitude curves
3. The Signs of the Zodiac
4. The pairs of symbols associated with each Sign of the Zodiac
5. The golden arc

The Moon Dial table is separate from the main dial and is largely outside the scope of this document. The present table was painted quite recently and is in good condition.

The red labels for the constant-altitude curves have to fit into little gaps. Given that the dial parameters are not yet known very accurately, the relevant gaps might be different in an adjusted computer model. Accordingly, these labels are best left until the last minute.

The Signs of the Zodiac can be very much as in the 1971 version though a few will need slight repositioning. There are 12 explicit sub-regions for these Signs but there is very little space for Sagittarius and Capricorn in particular. Both of these Signs can straddle the winter solstice curve in the way that Cancer straddles the summer solstice curve but there is a side-effect: Scorpio may have to be painted slightly smaller if it is to fit into the space allocated to it and not spill over into Sagittarius.

The pairs of symbols associated with each Sign of the Zodiac can mostly be accommodated as in the 1971 version. The symbols fit round the associated Sign in convenient gaps.

The golden arc seems to serve no gnomonic purpose. It hides the winter solstice curve and has doubtful ornamental value. Nevertheless it is a distinguishing feature of the Queens' Dial and its absence would not go unnoticed! Two possible ways of alluding to this arc are presented in Steps 20 and 21.

The Plan for Step 19

There are a couple of queries. Should the labelling of the constant-azimuth lines and the horizon line closely copy the 1971 version? Should gold leaf be used for the huge S and the letters of HORIZON?

After resolving these matters, the labelling can be can be incorporated into the dial.

Items 2, 3 and 4 above can then be attended to. Finally, the golden arc or some variant of it must be considered...

Step 20 — The Modified Golden Arc

Fig. 20 shows the previous figure with the central portion of the winter solstice curve depicted in gold and drawn wider than the other constant-declination curves. Such a treatment is hinted at in the 1860s photograph.

An alternative approach would be to fill the central space between the two uppermost constant-declination curves with gold and then redraw these curves so that they run along the top of the central hour lines. They would both be green rather than black.

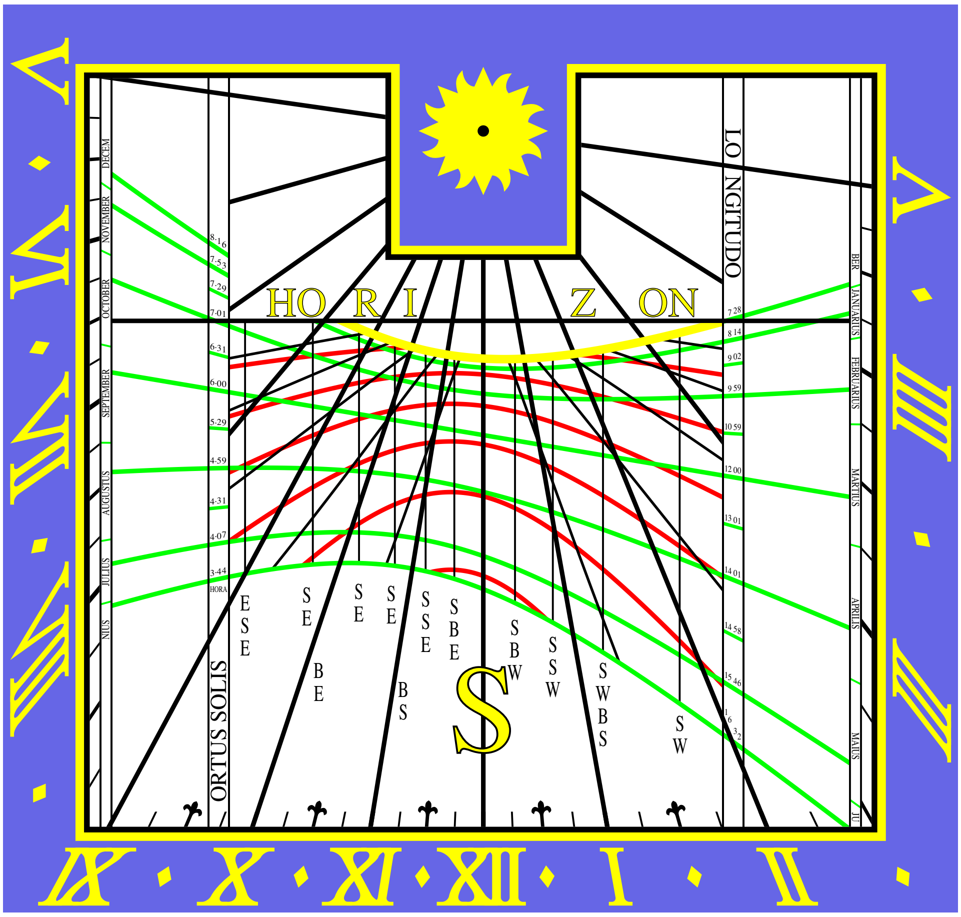


Fig. 20 — The Modified Golden Arc

It is a matter for debate whether either form of modified golden arc is acceptable. In each case the result is an arc which lies in very much the same place as the golden arc in the 1971 dial but without hiding the winter solstice curve. The principal demerit is that the arc is very thin. There is insufficient space for it to be made wider or to be given black edging. It is hardly the dominant feature that one sees in the 1971 dial.

Step 21 — The Golden Wing

Fig. 21 shows an alternative to the golden arc. This fills the space between the winter solstice curve and the horizon line.

The winter solstice curve is then redrawn so as to run along the top of the central hour lines. It retains its green colour.

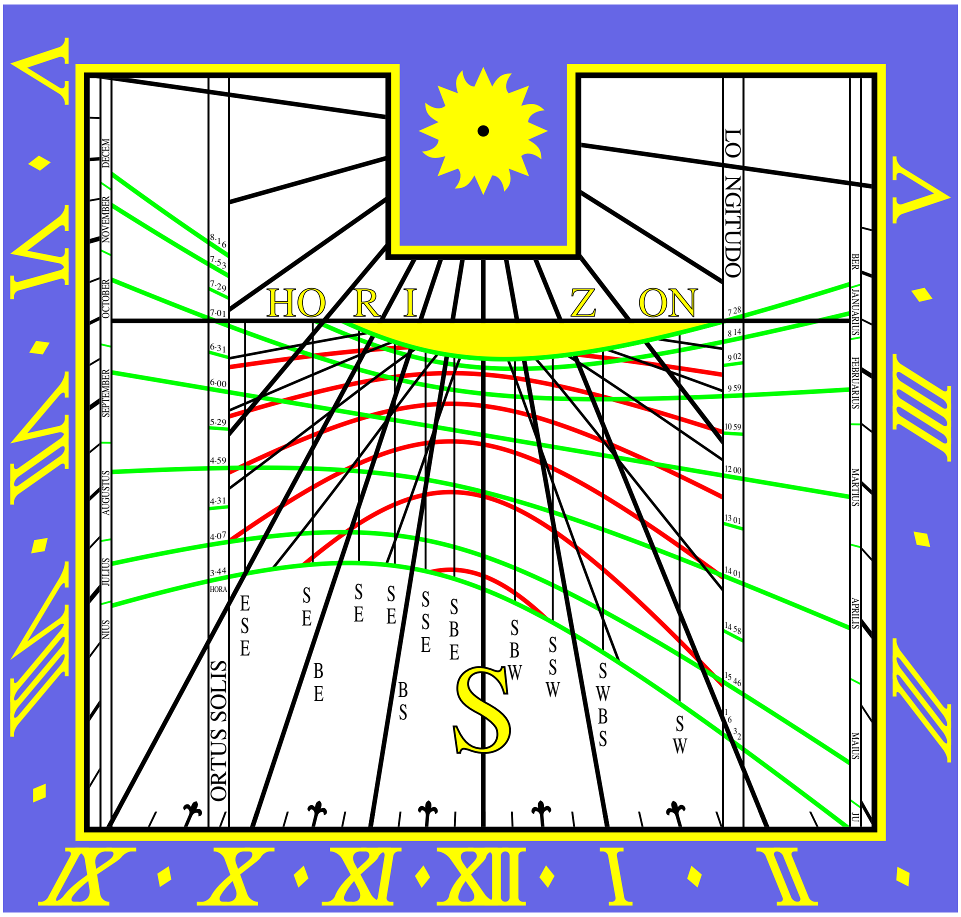


Fig. 21 — The Golden Wing

It is again a matter for debate whether this version, referred to here as a golden wing, is acceptable. It has the merit of being about as dominant a feature as the golden arc in the 1971 version. As with the version in Fig. 20, it leaves the two uppermost constant-declination curves unhidden. The principal demerit is that the arc shape is lost.

Appendix A — Numerical Notes

The computer model that produced the figures in this document incorporates numerous angles and linear dimensions. Their values are listed in the tables below. Many of the values will change when more accurate survey data are available.

Four Dial Parameters

Three angles describe the orientation of the best-fit plane to which the surface of the dial approximates. The perpendicular distance from the nodus to that plane serves as a scaling factor.

$$\begin{aligned} \text{latitude} &= 52^\circ\,12'\,8.7'' \\ \text{azimuth of outward normal} &= 167^\circ\,20' \\ \text{forward lean} &= 0^\circ \\ \text{ortho-style distance} &= 380.75\,\text{mm} \end{aligned}$$

The Constant-Declination Curves

The following table shows, for each of 13 solar longitudes, SL, the associated declination, δ, the time of sunrise, SR, (as local sun time) and day-length, DL. Note that:

$$\sin\delta = \sin SL\,\sin\epsilon_0 \qquad \text{and} \qquad \tan H_{SR} = \frac{\sin\phi\,\sin\delta}{\sqrt{0.5\,(\cos 2\phi + \cos 2\delta)}}$$

where ϵ_0 is the mean obliquity of the ecliptic (which is taken as $23^\circ\,26'\,18''$), ϕ is the latitude of the Queens' Dial, and H_{SR} is the hour angle of sunrise.

The columns headed OS and LO show values copied from the 1971 version of the dial: the times of sunrise copied from the ORTUS SOLIS column and day-lengths copied from the LONGITUDO column. Values missing from the dial are shown as blank entries.

SL	δ	SR	OS	DL	LO
−90°	−23.4383°	8.16	8.30	7 28	
−75°	−22.5946°	8.10		7 40	
−60°	−20.1496°	7.53	8.0	8 14	80
−45°	−16.3354°	7.29	7.30	9 02	90
−30°	−11.4715°	7.01	7.0	9 59	100
−15°	−5.9090°	6.31	6.30	10 59	110
0°	0.0000°	6.00	6.0	12 00	120
15°	5.9090°	5.29	5.30	13 01	130
30°	11.4715°	4.59	5.0	14 01	140
45°	16.3354°	4.31	4.30	14 58	150
60°	20.1496°	4.07	4.0	15 46	160
75°	22.5946°	3.50		16 20	
90°	23.4383°	3.44		16 32	

The seven declinations which correspond to solar longitudes that are multiples of 30° give rise to the long constant-declination curves. The others (except for ±22.5946°) give rise to the short lengths in the broad tram-lines.

Declinations of the Month Separators

These are the values built into the computer model. Each declination is the average of the maximum and minimum values at UTC midnight at the start of the relevant month over the years 2004 to 2050. In most cases the extreme values are in the years 2007 and 2048. In the cases of JANUARIUS and FEBRUARIUS the extremes are in the years 2008 and 2049.

month	*declination*
JANUARIUS	−23.0233°
FEBRUARIUS	−17.1857°
MARTIUS	−7.6119°
APRILIS	4.5149°
MAIUS	15.0574°
JUNIUS	22.0435°
JULIUS	23.1071°
AUGUSTUS	18.0256°
SEPTEMBER	8.2988°
OCTOBER	−3.1601°
NOVEMBER	−14.4015°
DECEMBER	−21.7846°

The raw data from which these values were calculated came from:

`http://www.gcstudio.com/suncalc.html`

Using this web site the declinations at midnight at the start of 1 April in the years 2007 and 2048 were noted as $4^\circ\,18'\,35''$ and $4^\circ\,43'\,12''$ respectively. These are the extreme values over the years 2004 to 2050. Their average is the value given for APRILIS in the table.

The two months whose starts are closest to the equinoxes are April and October. The declination changes more rapidly at the starts of these months than at the starts of the others. In consequence, the March–April and September–October month separators will be subject to the greatest errors.

The declinations at the starts of April and October may be a little over 0.2° greater than or less than the values shown in the table. In the context of markings on the dial, a month separator may, in some years, be too high or too low by the thickness of its own line.

The start-of-month declinations should be calculated afresh at each repainting to determine the compromise values appropriate for the next 40 years or so.

Scaling

Only one linear dimension has been given so far; all the other values are angles. The linear dimension is the ortho-style distance and this governs the scale of the dial furniture. If the value is reduced, the various curves close up. If it is increased, the curves separate from one another. There seems no good reason to change the scale.

The position of the ball nodus is therefore satisfactory but there is a case for making it larger. The shadow of the present nodus can be hard to see.

Horizontal Widths of Lines and Gaps

With the numerical information given above, all the mathematical lines can be determined but the end-points of many would be indeterminate. A frame and tram-lines are needed.

To specify the dimensions, it is convenient to start by considering a reference horizontal line drawn from the left-hand side of the dial to the right-hand side through the root-of-gnomon origin. This crosses a sequence of lines and gaps. For each line or gap the following table shows the abscissae of its margins and its width.

abscissa	*width*	*line or gap*
−1261.0		
	189.4	blue border, left-hand side
−1071.6		
	22.5	yellow edging
−1049.1		
	15	black edging
−1034.1		
	25.3	gap for left-hand edge tickmarks
−1008.8		
	5	left month tram-line, left-hand side
−1003.8		
	24	gap for months
−979.8		
	5	left month tram-line, right-hand side
−974.8		
	249.8	gap for Signs of the Zodiac, left-hand side
−725.0		
	5	ORTUS SOLIS tram-line, left-hand side
−720.0		
	49.2	gap for times of sunrise
−670.8		
	5	ORTIS SOLIS tram-line, right-hand side
−665.8		
	408.6	gap above HOR of HORIZON
−257.2		
	15	black edging
−242.2		
	22.5	yellow edging
−219.7		
	219.7	half-width of blue spur for stylized sun
0.0		
	219.7	half-width of blue spur for stylized sun
219.7		
	22.5	yellow edging
242.2		
	15	black edging
257.2		
	369.6	gap above ZON of HORIZON
626.8		
	5	LONGITUDO tram-line, left-hand side
631.8		
	49.2	gap for day-lengths
681.0		
	5	LONGITUDO tram-line, right-hand side
686.0		
	274.9	gap for Signs of the Zodiac, right-hand side
960.9		
	5	right month tram-line, left-hand side
965.9		
	24	gap for months
989.9		
	5	right month tram-line, right-hand side
994.9		
	25.3	gap for right-hand edge tickmarks
1020.2		
	15	black edging
1035.2		
	22.5	yellow edging
1057.7		
	189.4	blue border, right-hand side
1247.1		

All values are shown in millimetres. Primary datum values are shown in bold. The other abscissae are calculated with reference to the primary values by using the widths shown in the second column. Widths shown in italics are not used.

Vertical Widths of Lines and Gaps

In like manner, consider a reference vertical line drawn from the top of the dial to the bottom through the root-of-gnomon. This also crosses a sequence of lines and gaps. For each line or gap the following table shows the ordinates of its margins and its width.

ordinate	*width*	*line or gap*
334.2	334.2	top edge of dial to root of gnomon origin
0.0	*304.2*	root-of-gnomon origin to bottom of blue spur
−304.2	22.5	yellow edging
−326.7	15	black edging
−341.7	*1461.9*	gap from bottom of black edging to top of tickmarks
−1803.6	40	width allocated to quarter- and half-hour tickmarks
−1843.6	15	black edging
−1858.6	22.5	yellow edging
−1881.1	189.4	blue border, bottom
−2070.5		

As before, primary datum values are shown in bold. The other ordinates are calculated with reference to the primary values by using the widths shown in the second column. Widths shown in italics are not used.

It is also necessary to consider an offset reference vertical line drawn from the top of the dial to the bottom that is clear of the blue spur that carries the stylized sun. For example, this line might run down the centre of one of the gaps allocated to the Signs of the Zodiac. The following table shows the ordinates and widths associated with this line.

ordinate	*width*	*line or gap*
334.2	156.7	blue border, top
177.5	22.5	yellow edging
155.0	15	black edging
140.0	*1943.6*	gap from bottom of black edging to top of tickmarks
−1803.6	40	width allocated to quarter- and half-hour tickmarks
−1843.6	15	black edging
−1858.6	22.5	yellow edging
−1881.1	189.4	blue border, bottom
−2070.5		

Asymmetries

As already noted in Step 1 and Step 6, there are left-right asymmetries. In particular the bold abscissae −1034.1 and 1020.2 do not correspond. More curiously, the gap allocated to the Signs of the Zodiac on the left, 249.8 mm, is less than the gap allocated to the Signs of the Zodiac on the right, 274.9 mm.

Together, these asymmetries result in the gap above HOR, 408.6 mm, being substantially greater than the gap above ZON, 369.6 mm.

The Vertices of the Central White Area

The central white area of the dial is approximately square with a small square cut out of the top. All the mathematically defined lines and curves are bounded by this region.

Using selected bold values from the three tables above, the coordinates of the eight vertices of the white region are as follows:

(−1034.1, 140.0)	(−257.2, 140.0)	(257.2, 140.0)	(1020.2, 140.0)
	(−257.2, −341.7)	(257.2, −341.7)	
(−1034.1, −1843.6)			(1020.2, −1843.6)

Line thicknesses

Within the white area, four line thicknesses are used as follows:

12 mm	constant-altitude curves, constant-declination curves, hour lines, hour-line tickmarks, horizon line
9 mm	constant-declination curves in the broad tram-lines, half-hour tickmarks
7 mm	temporary hours curves
5 mm	month separators in the narrow tram-lines, constant-azimuth lines, quarter-hour tickmarks, tram-line margins

Other Dimensions

For completeness, the following dimensions are also built into the computer model:

153.4 mm	height of hour labels
18 mm	consequential gap above and below each hour label centred within the blue border
80 mm	height of half-hour lozenges
60 mm	width of half-hour lozenges

Appendix B — Proposed Interpretation

There is no definitive version of the Queens' Dial and at each refurbishment there is necessarily an element of interpretation.

The figure on the back cover presents a proposed interpretation for consideration when planning the 2006–07 refurbishment. This proposal is based on Fig. 21 but it is shown at a scale of 1:15 and there are five other significant differences...

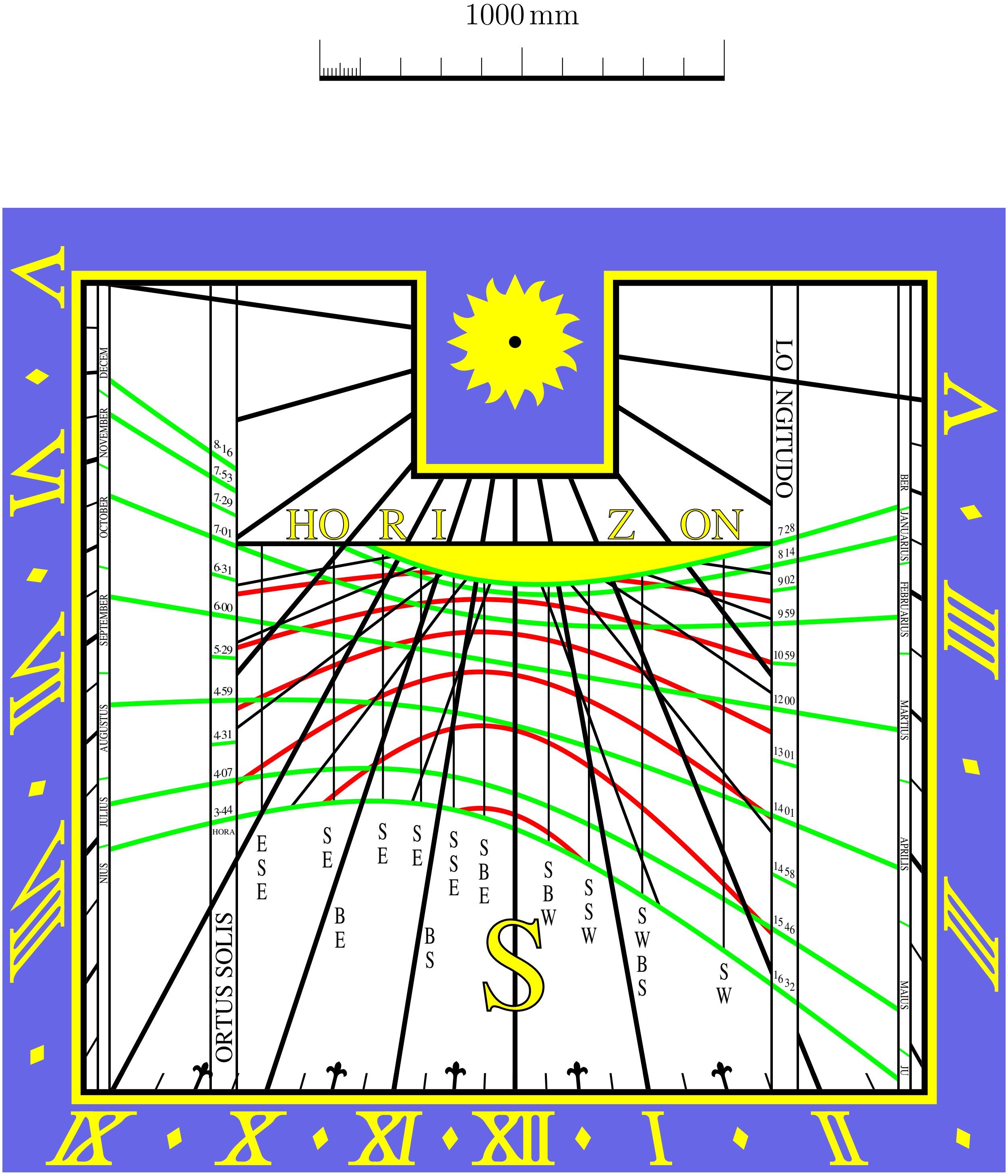
Principal Differences

1. The horizon line is bounded by the inner margins of the broad tram-lines rather than the narrow tram-lines. This clears the spaces in which Libra and Capricorn are to be painted.
2. The broad tram-lines for the LONGITUDO values have been shifted slightly to the right to make the width of the gap for the Signs of the Zodiac on the right the same as the width of the equivalent gap on the left.
3. The broad tram-lines on both sides have been made slightly wider (by the thickness of their marginal lines) to give more space for the labels in these tram-lines.
4. The section of the 2 o'clock hour line where it crosses the LONGITUDO tram-lines has been omitted so that it does not intrude on the 16 32 label on the summer solstice curve.
5. The width of the chapter ring (the blue border at left, bottom and right) is narrower than in previous figures. The width on the 1971 version is not consistent and this narrower width is a revised average.

Minor Differences

The figure also reflects a revised estimate of the position of the root of the gnomon relative to the outer margin of the blue border. As a result of this change of position the summer solstice curve now meets the narrow tram-lines on the right at a slightly higher point.

The Queens' Dial



F.H.K. 5 August 2006